Real-Time Competitive Environments: Truthful Mechanisms for Allocating a Single Processor to Sporadic Tasks

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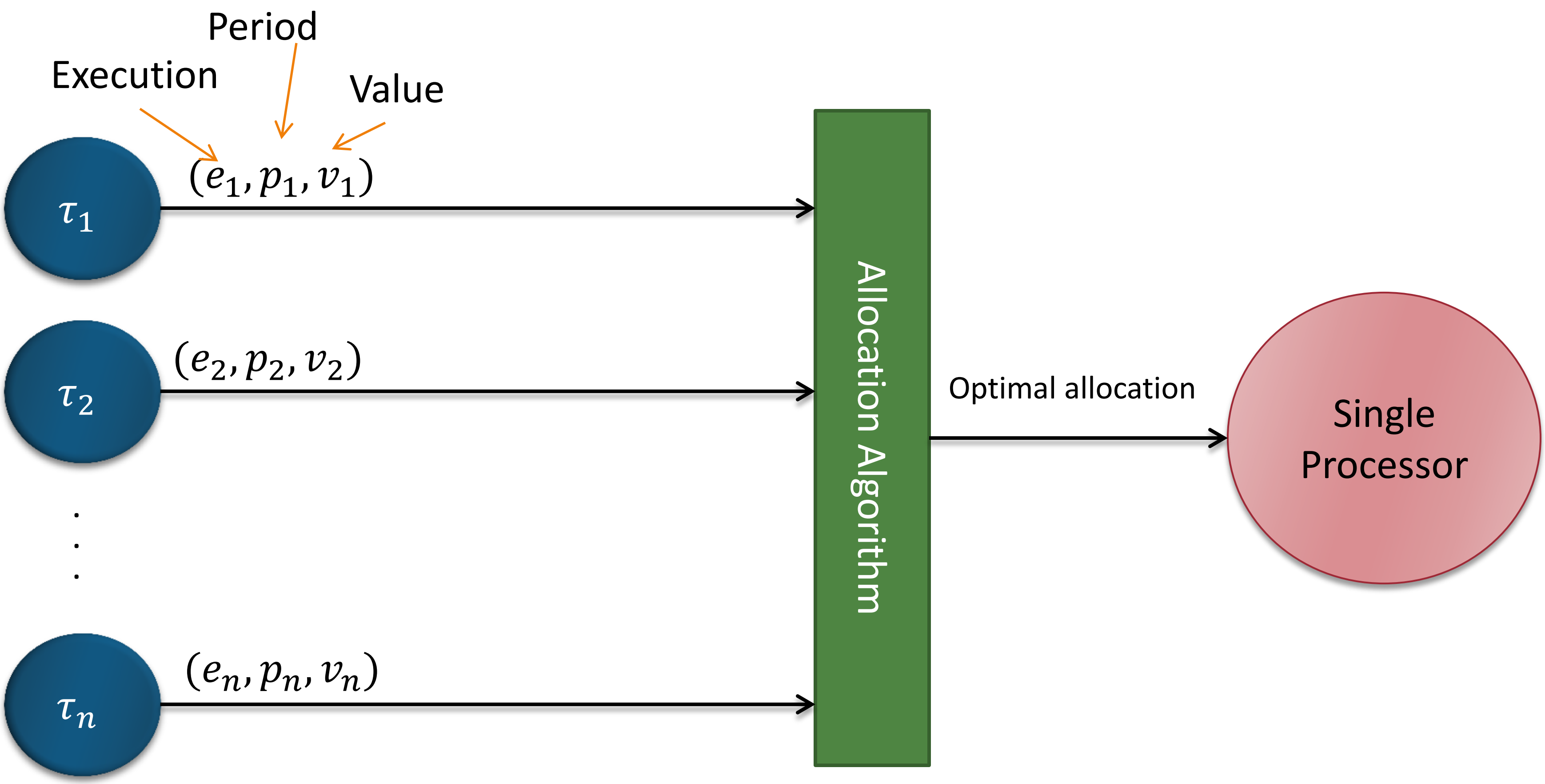
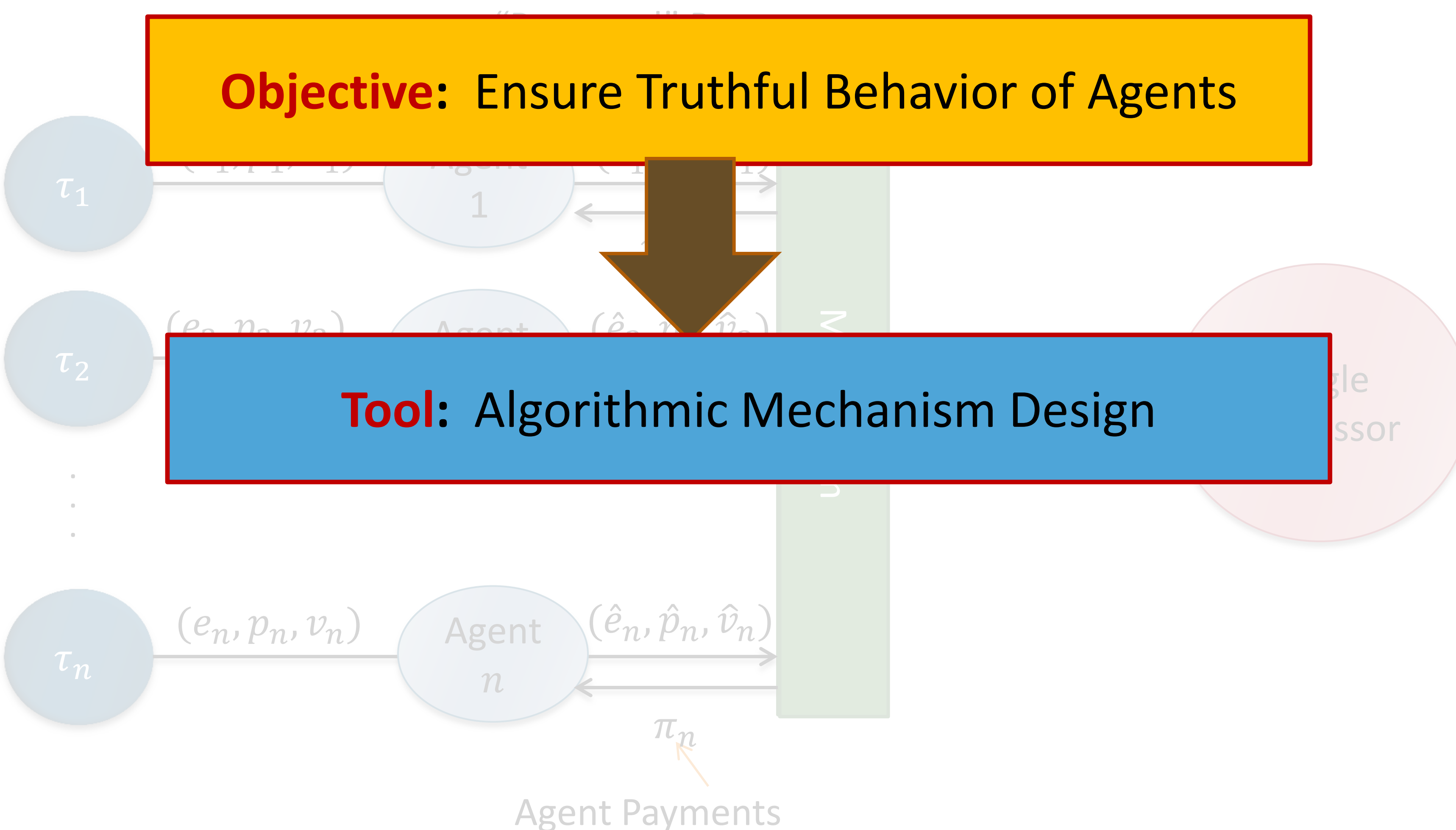
Non-Competitive vs. Competitive



Non-Competitive vs. Competitive

Objective: Ensure Truthful Behavior of Agents

Tool: Algorithmic Mechanism Design



Mechanism Design

What is Mechanism Design?

The art of designing rules in a competitive environment to achieve

- ***Truthfulness*** ← Design rules such that reporting **true task**
- ***Efficiency*** ← Optimizing some **system-wide objective funtion**.

Why Mechanism Design for Real-Time Systems?

- Real-time systems are becoming more **open**.
- **Many applications in computer science**:
  - network routing
  - human-computer interaction
  - parallel & distributed systems (e.g., grid/cluster computing)
  - internet advertisements
- **Spectacular commercial success:**
  - Google: $6 billion in 2005!
  - Yahoo!: $2.6 billion in 2005!

Related Work

- **Value/Utility Allocation in Non-Competitive Real-Time Systems**:
  - Baruah et al. [1991]: On-line scheduling in the presence of overload.
  - Rajkumar et al. [1997]: QoS-based Resource Allocation Model (QRAM).
  - Aydin et al. [2001]: Optimal reward-based scheduling.
  - …
- **Non-Real-Time Mechanism Design**:
  - Initiated by Nisan and Ronen [2001]
  - Aggarwal [2006] studied knapsack auctions.
- **Game theory in real-time systems**:
  - Sheikh et al. [2011]: Multiprocessor periodic scheduling using game-theoretic concepts
  - Porter [2004]: Mechanism design for online real-time scheduling
  - **None of these prior works on scheduling considers *traditional recurring tasks* in competitive environments.**

Model (Non-Competitive)

Implicit-deadline Sporadic Task System (with value)

Relative Deadline

- $\boldsymbol{T} = \{T_1, T_2, \dots, T_n\}$
- Each task is denoted by $T_i = (e_i, d_i, p_i, v_i)$
- Metrics:
  - Task utilization: $u_i = \frac{e_i}{p_i}$.
  - System utilization: $U(\boldsymbol{T}) = \sum_{T_i \in \boldsymbol{T}} u_i$.
- Implicit-Deadlines: $d_i = p_i$.
- Tasks are scheduled by earliest-deadline-first (EDF).
- Schedulability Test: $U(\boldsymbol{T}) \leq 1$.

Implication: Each task is completely characterized by utilization u_i.

Problem Statement

EDF-MAXVAL Problem:

Maximize $\sum_{i=1}^{n} v_i x_i$

Economics: "Social Welfare"

Subject to: $\sum_{i=1}^{n} u_i x_i \leq 1$

$x_i \in \{0,1\}$

$x_i = 1$ if task T_i is selected and $x_i = 0$, otherwise.

Model (Competitive)

Competitive Environment

- Each task T_i is owned by Agent i.
- Each Agent i is characterized by a ***type*** $\theta_i = (u_i, v_i)$.
- Agent i may have chose a different **declared type** $\hat{\theta}_i = (\hat{u}_i, \hat{v}_i)$.
- Set of agents $N = \{1,2, \ldots, n\}$.
- **Efficiency Assumption**: *Resource owner* is trying to maximize social welfare.

Motivational Example

Agent 1
$u_1 = 0.1$
$v_1 = 2$

Agent 2
$u_2 = 0.2$
$v_2 = 7$

Agent 3
$u_3 = 0.4$
$v_3 = 8$

Agent 4
$u_4 = 0.6$
$v_4 = 9$

Agent 5
$u_5 = 0.7$
$v_5 = 11$

	Agent 1	Agent 2	Agent 3	Agent 4	Agent 5
True Parameters	(0.1, 2)	(0.2, 7)	(0.4, 8)	(0.6, 9)	(0.7, 11)
Declared Parameters	(0.1,2)	(0.2,7)	(0.4,8)	(0.6,9) / (0.6,20)	(0.7,11) / (0.8,11)

Allocation Algorithm

Winners:

Implication: Lying by agents can affect overall system objective!

Truthful Social Welfare: 20

Social Welfare: 20 / 17 / 18

Mechanism Design Concepts

Mechanism

A mechanism is composed of

- **Allocation Algorithm $\boldsymbol{A}$**: determines which agents obtain the processor according to efficiency assumption.
- **Payment Scheme $\boldsymbol{\pi} = (\boldsymbol{\pi_1}, \dots, \boldsymbol{\pi_n})$**: calculates the payment of each agent.

Agent's strategy

- The strategy of an agent is her declared type.
- Agent's utility is $\mu_j = \begin{cases} v_j - \pi_j, & \text{if Agent j is a winner,} \\ -\pi_j, & \text{otherwise.} \end{cases}$
- **Selfish Assumption:** Each agent tries to maximize her utility.
- An agent may strategically declare a different type from her true type to maximize her utility.

Truthful Mechanisms

A mechanism is called **truthful** if for each agent, truthful revelation is a **dominant strategy**, that is agents maximize their utilities by reporting their true types.

Vickrey-Clarke-Groves (VCG) Mechanism: a truthful mechanism given an **optimal** allocation algorithm.

Truthful Mechanisms

EDF-MAXVAL-VCG Mechanism

- **Allocation algorithm:** The pseudo-polynomial-time algorithm EDF-MAXVAL-DP.
- **Payment scheme:**

$$\pi_j^{VCG} = \sum_{i \in A(\hat{\boldsymbol{\theta}}_{-j})} \hat{v}_i - \sum_{i \in A(\hat{\boldsymbol{\theta}}),\, i \neq j} \hat{v}_i$$

VCG Payment ≈ Total marginal loss of value of including Agent j (w.r.t. other agents).

Computable in Pseudo-Polynomial Time (dependent upon maximum task value).

Approximate Mechanisms

- Applying VCG payments to standard knapsack approximation algorithm [Kim & Ibarra, 1975] is <u>not</u> truthful!
  - **Reason**: VCG requires a **monotonic** allocation algorithm.

If Agent j wins (using allocation algorithm A) declaring $\hat{\theta}_i = (\hat{u}_i, \hat{v}_i)$, then she should also win declaring $\hat{\theta}'_i = (\hat{u}'_i, \hat{v}'_i)$ where

$$\hat{u}'_i \leq \hat{u}_i \text{ and } \hat{v}'_i \geq \hat{v}_i.$$

- We apply technique by Briest et al. [2005] to obtain truthful approximation called EDF-MAXVAL-APROX.

Approximate Mechanisms

EDF-MAXVAL-APROX Mechanism (FPTAS)

For any $\varepsilon > 0$, EDF-MAXVAL-APROX is **truthful** and returns an allocation with social welfare **no less than** $(1 - \varepsilon)$ times the optimal obtainable social welfare in time polynomial in $1/\varepsilon$ and n.

Reserve Prices

Theorem: EDF-MAXVAL-APROX remains truthful even if the **resource owner requires** that each Agent j report $\hat{v}_j \geq C\hat{u}_j$ for some constant C. ← **Reserve Price per Unit**

Frugality Metric

Measurement of over/under-payment by agents.

The frugality ratio of a mechanism is the **total payments** divided by the **second disjoint optimum value**. [Talwar, 2003]

Upper Bound (EDF-MAXVAL-VCG)

Theorem: Given k winning agents, the maximum frugality ratio is k.

The bound is "tight".

Upper Bound (EDF-MAXVAL-APROX)

Theorem: Given k winning agents and $\varepsilon > 0$, the maximum frugality ratio is at most $\big(1 + \varepsilon(n+2)\big)k$.

Evaluation

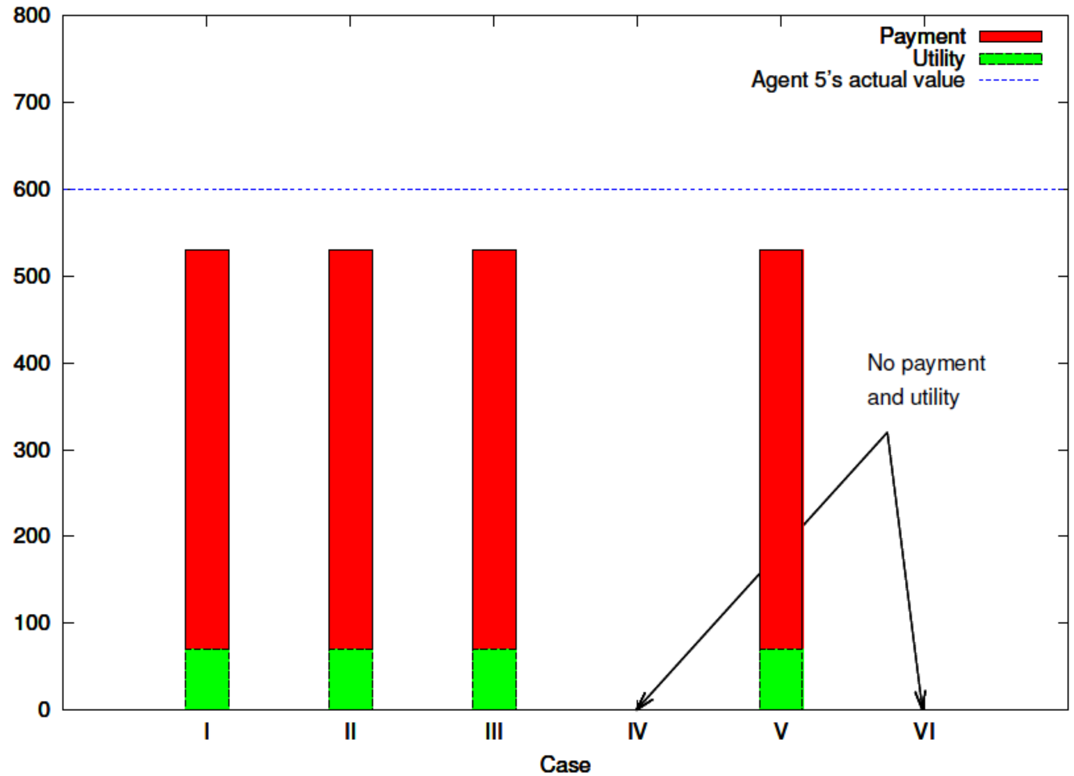
Type of Comparisons:

- Non-Truthful Type Declaration (Case Analysis)
- Frugality Ratios (Simulation)
- Execution Time (Simulation)

Non-Truthful Type Declaration (Case Analysis)

- We consider an environment of 10 agents.
- All agents are truthful except Agent 5.
- We consider 6 cases:
  - Case I: Agent 5 is truthful.
  - Cases II, III, IV: Agent 5 is declaring non-true values
  - Cases V and VI: Agent 5 is declaring non-true utilization

Non-Truthful Type Declaration (Case Analysis)



Utilities and payments of Agent 5

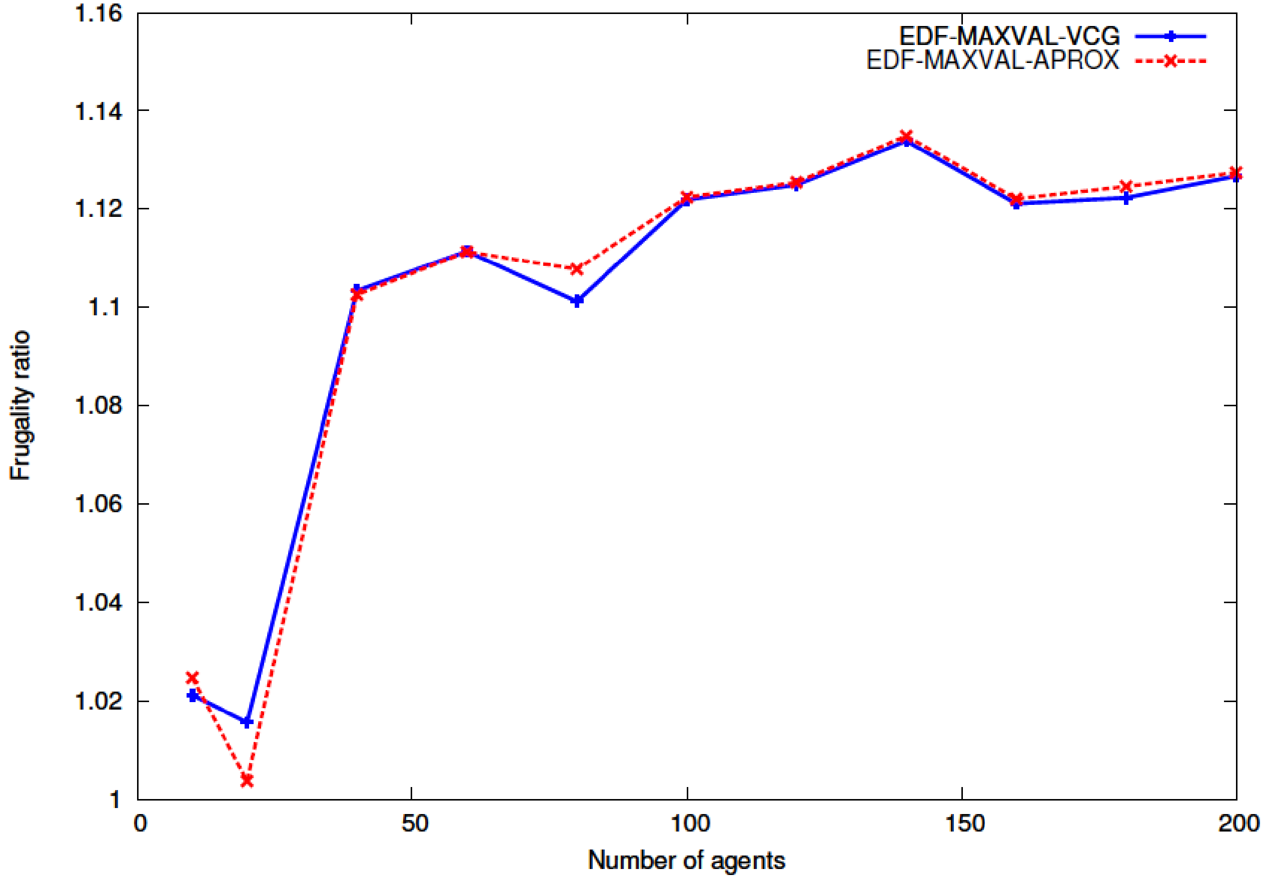
Agent 5 can not obtain better utility by lying.

Simulation Settings

U > 1

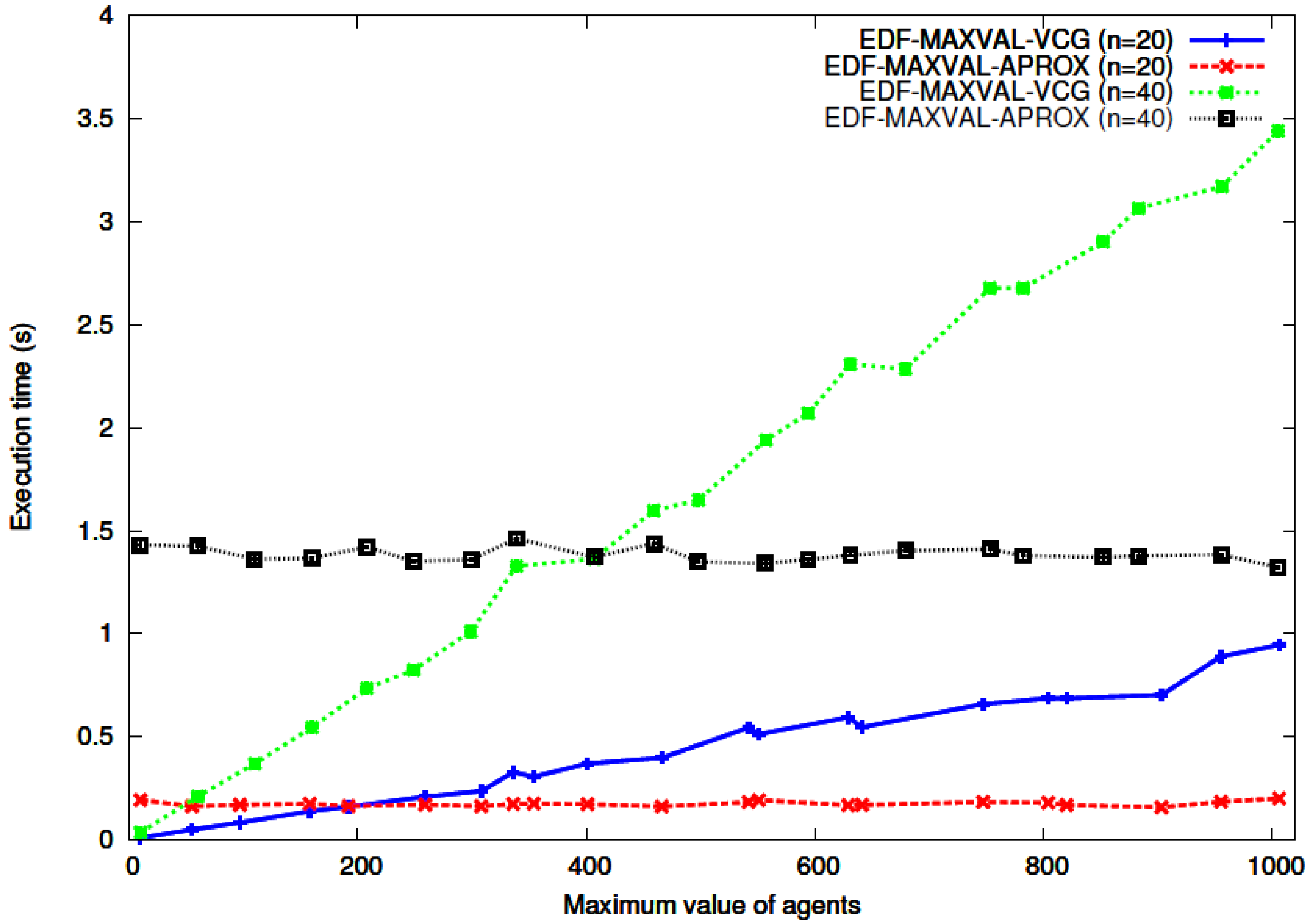
- Generate the utilizations using **UUniFast-Discard** [Davis & Burns 2009].
- Generate values using a random uniform number generator within [1, 1000].
- MATLAB environment on an 8-core Intel Core i7 (1.73GHz) machine was used.
- **Approximation Error:** $\varepsilon = 0.1$.

Frugality Ratios



- More competition, higher frugality ratio.
- Frugality ratios for exact and approx mechanism are close.

Execution Time of Mechanisms



Conclusion

- **Goal**: Introduce notion of *competition* to real-time scheduling/allocation.
  - **Reason**: systems are becoming more *open*.
  - **Challenge**: game theory often “well-behaved” utility functions.
- We extended existing algorithms to obtain **truthful** exact and approximate mechanisms with
  - *bounded* frugality ratios
  - reserve prices
- **Future Plans**:
  - Different resource owner objectives
  - Compositional systems
  - Multi-processor settings
  - Group-strategy proof mechanisms

Questions?

Thank you!

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