Push Forward: Global Fixed-Priority Scheduling of Arbitrary-Deadline Sporadic Task Systems

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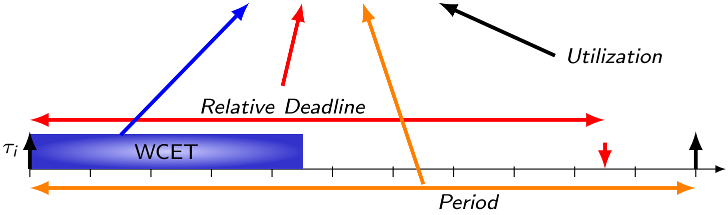
SFB 876 Verfügbarkeit von Information durch Analyse unter Ressourcenbeschränkung

Sporadic Task Model

$$\tau_i(C_i,\ D_i,\ T_i),\ U_i = \frac{C_i}{T_i}$$



Constrained `Deadline`: $D_i \leq T_i, \forall \tau_i$

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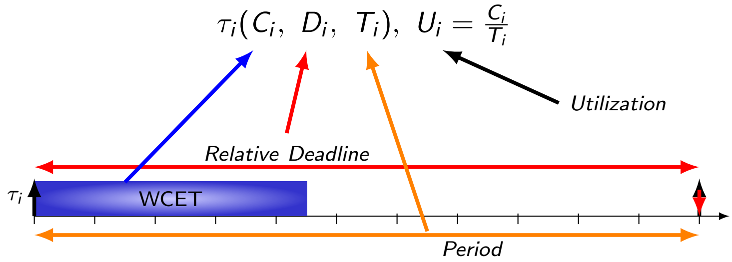

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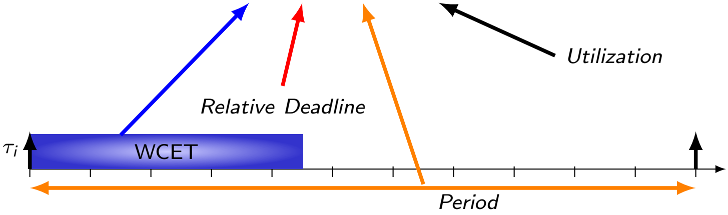


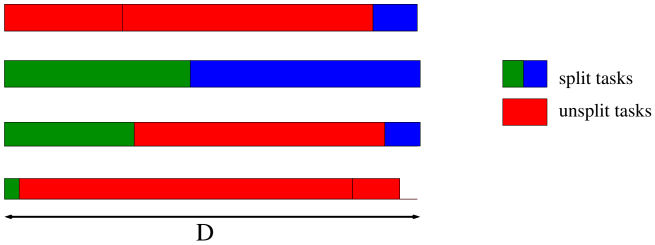
Constrained `Deadline`: $D_i \leq T_i, \forall \tau_i$
Implicit `Deadline`: $D_i = T_i, \forall \tau_i$
Arbitrary `Deadline`: otherwise. The jobs of a task must be executed in the FCFS manner.

Scheduling Model

- M identical multiprocessors: all the processors have the same characteristics
- Global scheduling:
  - A job may execute on any processor
  - The system maintains a global ready queue
  - Execute the M highest-priority jobs in the ready queue
- Basic Terms: EDF, FP, RM, DM
- Global Scheduling: Global-EDF, Global-FP, Global-RM, Global-DM

Good News for Global Scheduling

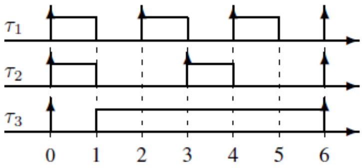
For frame-based task systems, McNaughton's wrap-around rule for $P|pmtn|C_{\max}$ is optimal



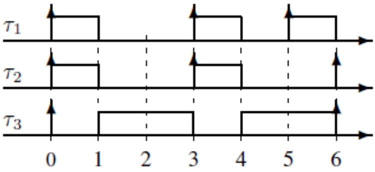
R. McNaughton. Scheduling with deadlines and loss functions. Management Science, 6:1-12, 1959.

Critical Instant?

- Synchronous release of higher-priority tasks and as early as possible for the following jobs
- Example: M=2 and 3 tasks: (C_i, D_i, T_i) are $\tau_1 = (1, 2, 2), \tau_2 = (1, 3, 3), \tau_3 = (5, 6, 6)$



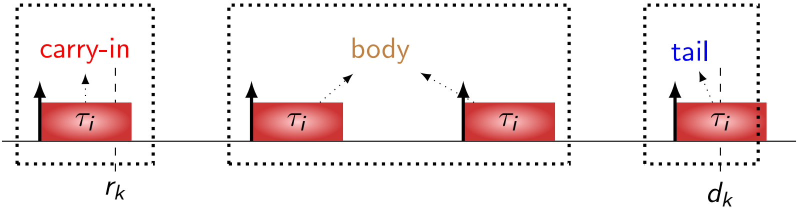
Feasible for τ_3.



Infeasible for τ_3.

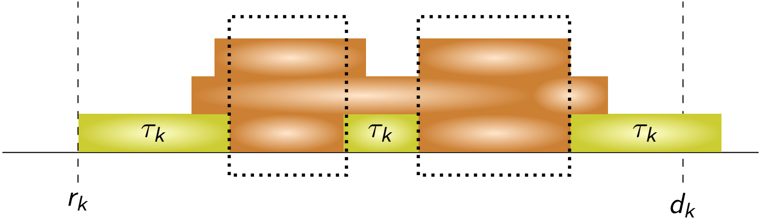
Baruah in RTSS 2007.

Identifying Interference for Constrained Deadlines



- For contrapositive, assume that a job of task τ_k misses its absolute deadline at time d_k with release time r_k
- Problem window (interval) is defined in $[r_k, d_k)$

Necessary Condition for Deadline Misses



- *demand* $E(\Delta)$: high-priority computation *executed* in the interval of length Δ.
- If τ_k misses its deadline at time d_k, then

$$E(D_k) > M \times (D_k - C_k) + C_k$$

Carry-In Demand

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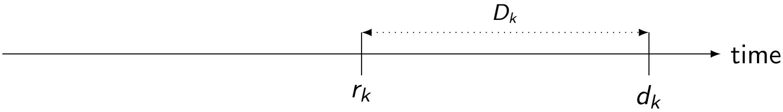

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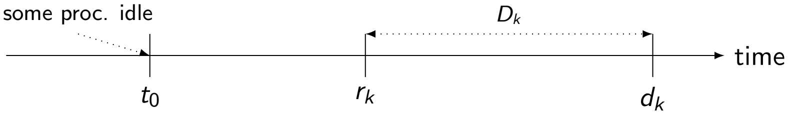

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- Let t_0 be the earliest time instant such that the system executes jobs on M processors from t_0 to r_k.
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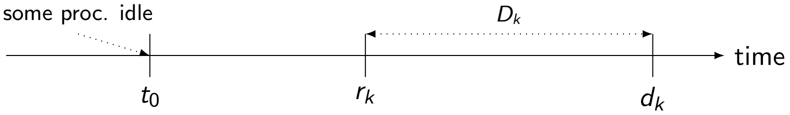

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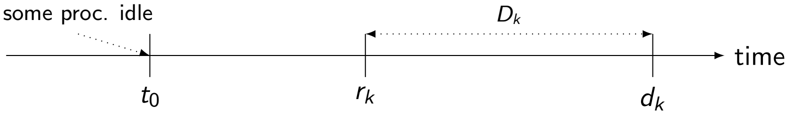

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  - Prior to t_0, at least one processor idles
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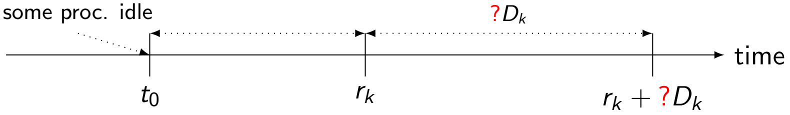
  - For arbitrary-deadline systems, at most M-1 carry-in tasks and there may be multiple jobs in a carry-in task

Global-FP: Arbitrary-Deadline



- Essential Problems
  - How to define the window of interest?
  - How many jobs should be considered in the window of interest?

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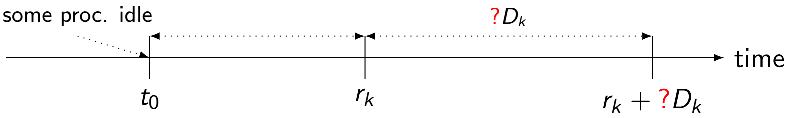

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- Existing Results
  - Baker (RTSJ 2006): downward extension
  - Baruah and Fisher (OPODIS 2007, ICDCN 2008): to-be-detailed later
  - Guan et al. (RTSS 2009): M-1 carry-in jobs (flawed)
  - Sun et al. (RTCSA 2014): complex carry-in workload functions
  - Huang and Chen (RTNS 2015): a more precise quantification for the number of carry-in jobs

Resource Augmentation

Speedup (resource augmentation) factor ρ ($\rho \geq 1$):

$\Rightarrow$

If the task set (system) is schedulable (feasible), Algorithm A also returns a schedulable (feasible) answer when speeding up the system by a factor ρ.

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$\Leftarrow$

If Algorithm A does not return a schedulable (feasible) answer, the system is also unschedulable (infeasible) when slowing down by a factor ρ, i.e., at speed $1/\rho$.

Resource Augmentation of Global DM

	implicit deadlines	constrained deadlines	arbitrary deadlines
upper bounds	2.668 (Lundberg 2002) 2.823 (Chen et al. 2016)	$3-1/M$ (Baruch et al. 2011) $3-1/M$ (Chen et al. 2016)	$\frac{2(M-1)}{4M-1-\sqrt{12M^2-8M+1}} \leq 3.73$ (Baruah and Fisher 2007)
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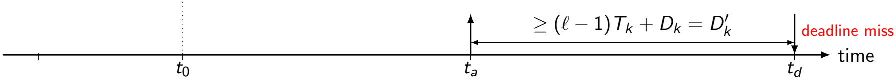
Outline

Push Forward (Basic Idea from Baruah and Fisher)



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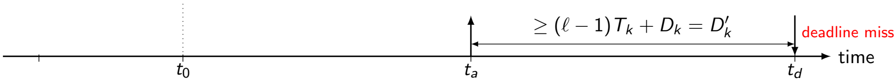

- τ_k is continuously active from t_a to t_d with deadline miss at t_d
- t_0 is the smallest value of $t \leq t_a$ such that $\Omega(t, t_d) \geq \mu_k$
  - $E(t, t_d)$: the amount of workload (sum of the execution times) of the higher-priority jobs, i.e., from $\tau_1, \tau_2, \ldots, \tau_{k-1}$, executed in the time interval $[t, t_d)$
  - C_k^*: amount of time that task τ_k is executed from t_a to t_d
  - ℓ-th job of task τ_k misses its deadline, i.e., $C_k^* < \ell C_k = C'_k$
  - $\Omega(t, t_d)$ is $\frac{C_k^* + E(t, t_d)}{t_d - t}$

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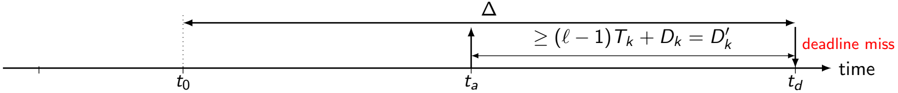

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- time instant t_i is the arrival time of a higher-priority carry-in task τ_i if τ_i is continuously active in time interval $[t_i, t_0 + \varepsilon]$, where $t_i < t_0$ and $\varepsilon > 0$ is an arbitrarily small number

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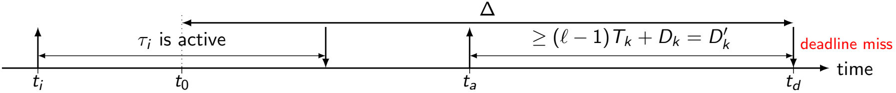

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Properties of Pushing Forward

Suppose that $\mu_k = M - (M-1)\rho$ for a certain ρ with $1 \geq \rho \geq \frac{C'_k}{D'_k}$.

Lemma

If τ_k misses its deadline at t_d, for any ρ with $1 \geq \rho \geq \frac{C'_k}{D'_k}$, the time t_0 always exists with $\Omega(t_0, t_d) \geq \mu_k$ and $t_0 \leq t_a$.

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A task τ_i is a carry-in task in the schedule S, if τ_i is continuously active in a time interval $[t_i, t_0 + \varepsilon]$, for $t_i < t_0$ and an arbitrarily small $\varepsilon > 0$.

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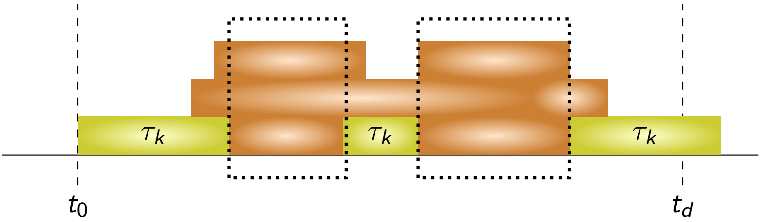
For $1 \geq \rho \geq \frac{C'_k}{D'_k}$, there are at most $\lceil M - (M-1)\rho \rceil - 1$ carry-in tasks at t_0 in schedule S.

Necessary Condition for Deadline Misses



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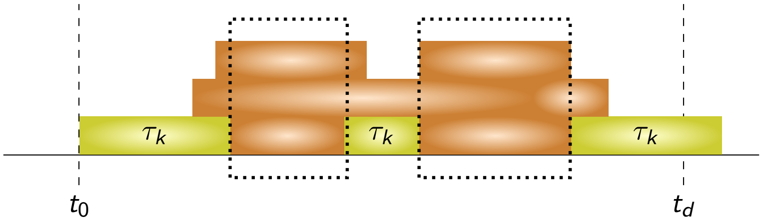

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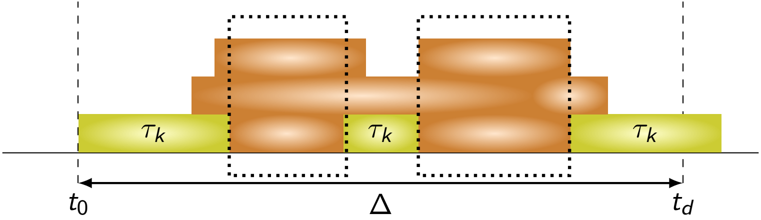

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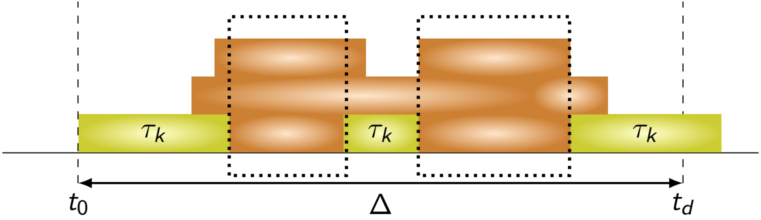

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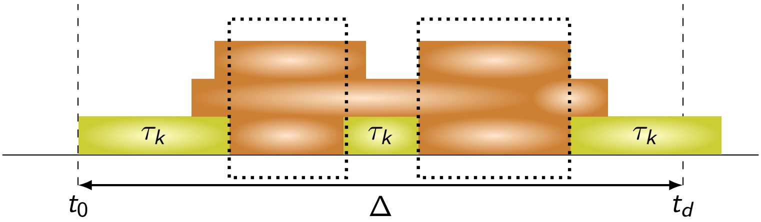
$E(\Delta)$: high-priority computation *executed* in interval length Δ.

Sufficient Condition for τ_k



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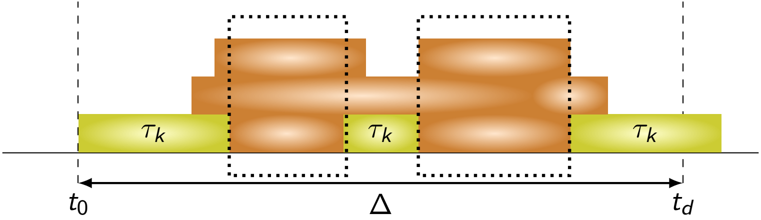

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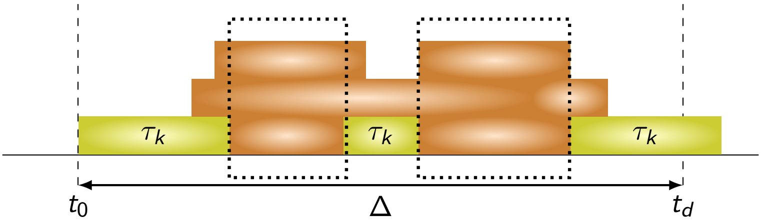

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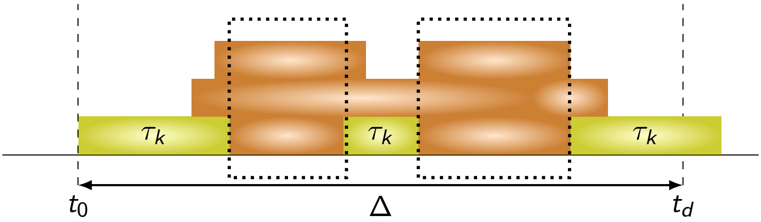

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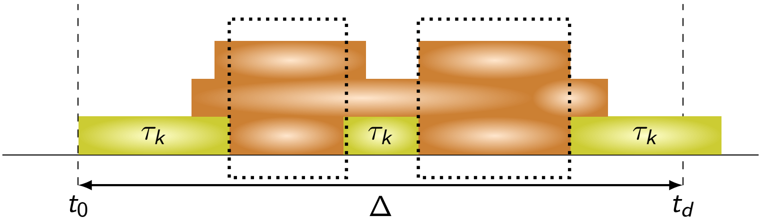

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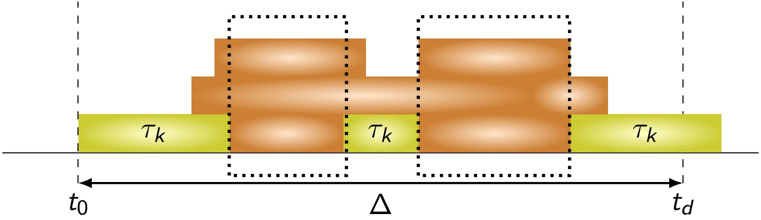

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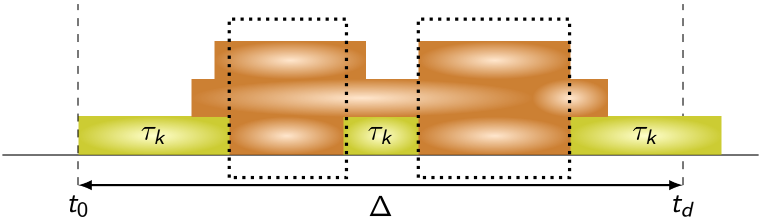
Baruah and Fisher's Analysis for Global-DM



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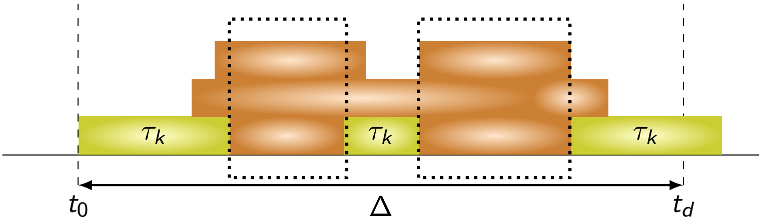

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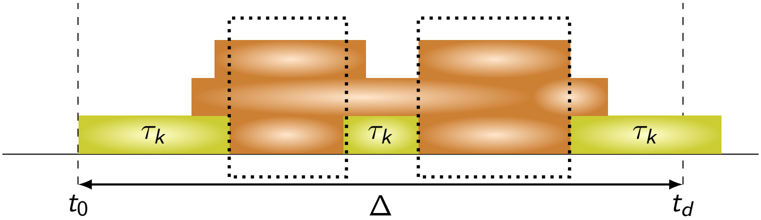

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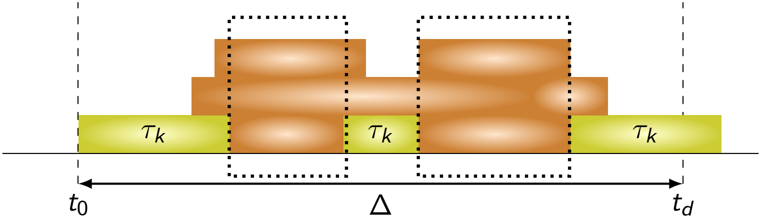
- such that

$$\ell C_k + (\lceil M - (M-1)\rho \rceil - 1)\rho\Delta + \left(\sum_{i=1}^{k-1} dbf_i(\Delta + D_i)\right) \leq (M-(M-1)\rho)\Delta$$

Baruah and Fisher's Analysis for Global-DM



Task τ_k always meets its deadline, if

- $\forall$ positive integer ℓ, i.e., ℓ-th job of τ_k (in a continuously actively interval) misses its deadline
- for $\rho = \max_i^k\{\frac{C_i}{T_i}, \frac{C_i}{D_i}\}$, $\forall \Delta \geq D'_k = (\ell - 1)T_k + D_k$
- such that

$$(\lceil M - (M-1)\rho \rceil - 1)\,\rho\Delta + \sum_{i=1}^{k} 2\times dbf_i(\Delta) \leq (M - (M-1)\rho)\Delta$$

since $D_i \leq \Delta$

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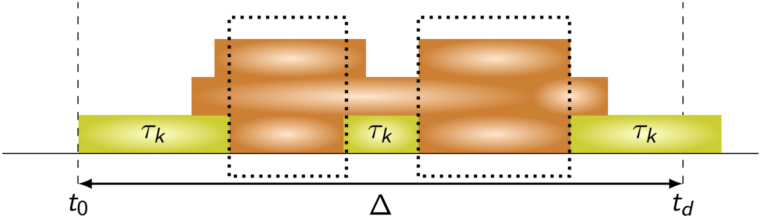

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Enforcement Techniques (My Slide at ECRTS'17)

- Strong and/or early-design/analytical enforcements
  - simplify the analysis

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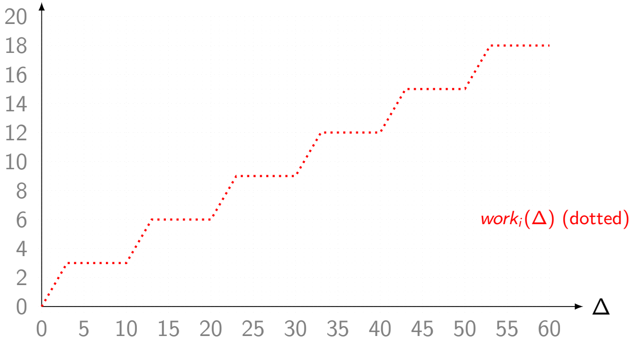
Observation 6

Adding enforcements tailoring the design of a scheduling algorithm or test to facilitate the derivation of a bounded speedup factor can be counterproductive; it may severely compromise performance in practical settings.

Workload Upper Bounds

$T_i = 10, C_i = 3$



- Executed workload of a non-carry-in task τ_i from t_0 to $t_d = t_0 + \Delta$ is at most $work_i(\Delta)$
- Executed workload of a carry-in task τ_i from t_0 to $t_d = t_0 + \Delta$ is at most $work_i(\Delta + D_i)$

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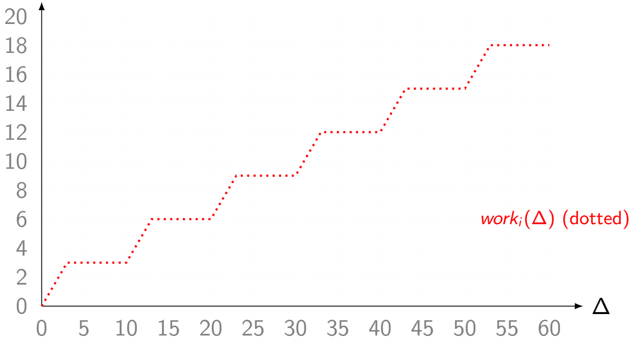

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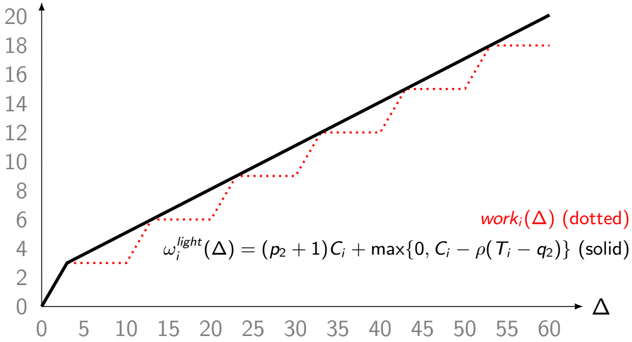
Workload Upper Bounds

$T_i = 10, C_i = 3$ $\quad p_2 = \lceil (\Delta - C_i)/T_i \rceil - 1,\ q_2 = \Delta - C_i - p_2 T_i$



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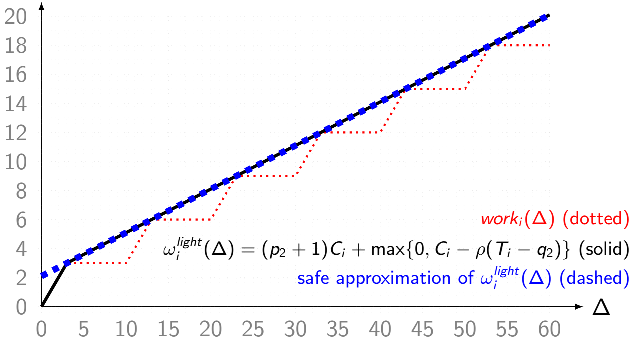

- Executed workload of a non-carry-in task τ_i from t_0 to $t_d = t_0 + \Delta$ is at most $work_i(\Delta) \leq (C_i - U_i D_i) + U_i \Delta$
- Executed workload of a carry-in task τ_i from t_0 to $t_d = t_0 + \Delta$ is at most $\omega_i^{heavy}(\Delta) = work_i(\Delta + D_i) \leq (C_i - U_i D_i) + U_i \Delta + U_i D_i$ if $U_i > \rho$
- Executed workload of a carry-in task τ_i from t_0 to $t_d = t_0 + \Delta$ is at most $\omega_i^{light}(\Delta) \leq (C_i - U_i D_i) + U_i \Delta$ if $U_i \leq \rho$

Putting Them Together (Theorem 4.4)

Task τ_k is schedulable by the given Global FP if

$$\forall \ell \in \mathbb{N}, \exists 1 \geq \rho \geq \ell C_k/((\ell-1)T_k + D_k), \forall \Delta \geq D'_k = (\ell-1)T_k + D_k,$$

$$\ell C_k + \sum_{\tau_i \in \mathbf{T}^{carry-approx}} \gamma_i U_i D_i + \sum_{i=1}^{k-1} (C_i - C_i U_i + U_i \Delta) \leq \mu_k \Delta$$

where $\mu_k = M - (M-1)\rho$,

$$\gamma_i = \begin{cases} 1 & \text{if } U_i > \rho \\ 0 & \text{if } U_i \leq \rho \end{cases}$$

and $\mathbf{T}^{carry-approx}$ is the set of the $\lceil \mu_k \rceil - 1$ tasks among the $k-1$ higher-priority tasks with the largest values of $\gamma_i U_i D_i$.

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The worst case of Δ happens when $\Delta = D'_k$

Further Approximations

- Which ρ? (Theorem 4.6)

  Task τ_k is schedulable by the given Global-FP if

$$\forall \ell \in \mathbb{N}, \frac{\ell C_k}{D'_k} + \sum_{i=1}^{k-1} \left(\frac{C_i - C_i U_i}{D'_k} + U_i \right) \leq (M - (M-1) U_{\delta,k}^{\max})$$

D'_k is $(\ell - 1) T_k + D_k$ and $U_{\delta,k}^{\max} = \max\{\max_{i=1}^{k-1} U_i, \frac{C_k}{T_k}, \frac{C_k}{D_k}\}$

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- Which ℓ? (Theorem 4.7)

  Task τ_k is schedulable by the given Global-FP if

$$\max\left\{ \frac{C_k}{T_k}, \frac{C_k}{D_k} \right\} + \sum_{i=1}^{k-1} \left(\frac{C_i - C_i U_i}{D_k} + U_i \right) \leq M - (M-1) U_{\delta,k}^{\max}$$

Conclusion

- We prove that Sanjoy's idea can yield a speedup factor of 3 for Global-DM

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Future work

- Priority assignment: OPA-Compatible
- Proofs for the speedup bound of the optimal Global-FP
- Soft real-time tasks

(Have you seen Sanjoy during the talk?)

More materials: (links are in the paper)

- Impl. of Thm. 4.4 in $O(N \log N)$ for testing whether τ_k meets its deadline
- Evaluation results with many figures

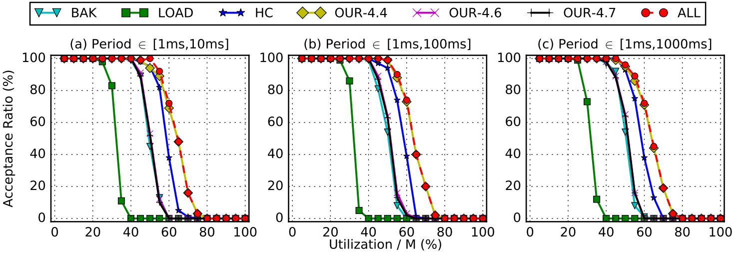

Figure: Global-DM, $M = 8$, $N = 40$, $\frac{D_i}{T_i} \in [0.8, 2]$.