Efficiently Approximating the Probability of Deadline Misses in Real-Time Systems

Georg von der Brüggen, Nico Piatkowski, Kuan-Hsun Chen,
Jian-Jia Chen, and Katharina Morik

Department of Computer Science
TU Dortmund University, Germany

04 July 2018

Supported by DFG, Collaborative Research Center SFB876, subproject A1.

Table of Contents

Rare Deadline Misses in Real-Time Systems

- Usual assumption: hard real-time constraints

Rare Deadline Misses in Real-Time Systems

- Usual assumption: hard real-time constraints
- Rare deadline misses often acceptable
- Industrial safety standards
  - IEC-61508
  - ISO-26262

Rare Deadline Misses in Real-Time Systems

- Usual assumption: hard real-time constraints
- Rare deadline misses often acceptable
- Industrial safety standards
  - IEC-61508
  - ISO-26262
- Soft real-time systems

Rare Deadline Misses in Real-Time Systems

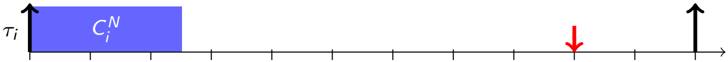
- Usual assumption: hard real-time constraints
- Rare deadline misses often acceptable
- Industrial safety standards
  - IEC-61508
  - ISO-26262
- Soft real-time systems
- Important criteria: probability of deadline miss

Rare Deadline Misses in Real-Time Systems

- Usual assumption: hard real-time constraints
- Rare deadline misses often acceptable
- Industrial safety standards
  - IEC-61508
  - ISO-26262
- Soft real-time systems
- Important criteria: probability of deadline miss
- Safe upper bound

Task Model and Notation

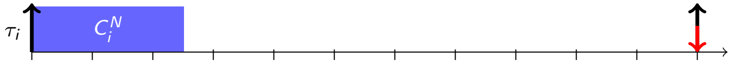
$$\tau_i(C_i,\ D_i,\ T_i)$$



- Uniprocessor, fixed priority
- Sporadic tasks

Task Model and Notation

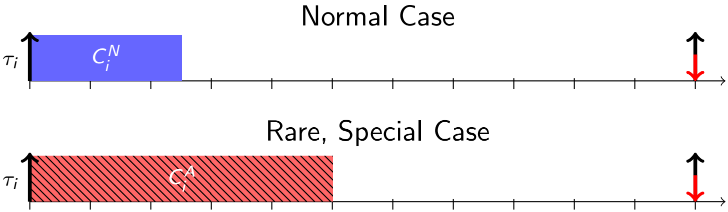
$$\tau_i(C_i,\ D_i,\ T_i)$$



- Uniprocessor, fixed priority
- Sporadic tasks, implicit deadlines: $D_i = T_i\ \forall \tau_i$

Task Model and Notation

$$\tau_i((C_i^N,\ C_i^A),\ D_i,\ T_i)$$



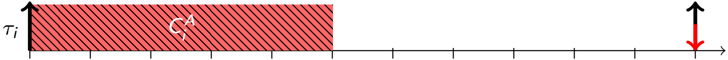
- Uniprocessor, fixed priority
- Sporadic tasks, implicit deadlines: $D_i = T_i\ \forall \tau_i$
- $C_i^A \geq C_i^N$ here: $C_i^A = 2 \cdot C_i^N$

Task Model and Notation

$$\tau_i((C_i^N,\ C_i^A,\ \mathbb{P}(C_i^A),\ \mathbb{P}(C_i^N)),\ D_i,\ T_i)$$

Normal Case

Rare, Special Case



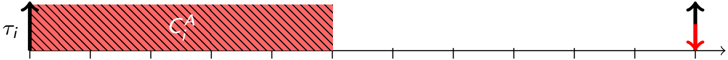
- Uniprocessor, fixed priority
- Sporadic tasks, implicit deadlines: $D_i = T_i \ \forall \tau_i$
- $C_i^A \geq C_i^N$ here: $C_i^A = 2 \cdot C_i^N$
- $\mathbb{P}(C_i^A) \ll \mathbb{P}(C_i^N)$
- $\mathbb{P}(C_i^A) + \mathbb{P}(C_i^N) = 1$

Task Model and Notation

$$\tau_i((C_i^N,\ C_i^A,\ \mathbb{P}(C_i^A),\ \mathbb{P}(C_i^N)),\ D_i,\ T_i)$$

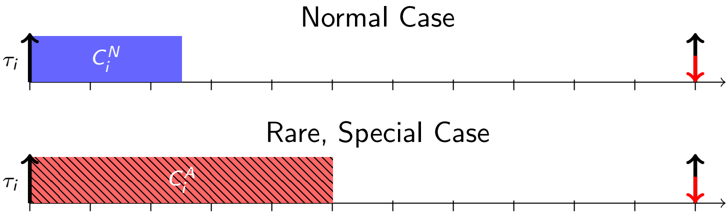
Normal Case

Rare, Special Case



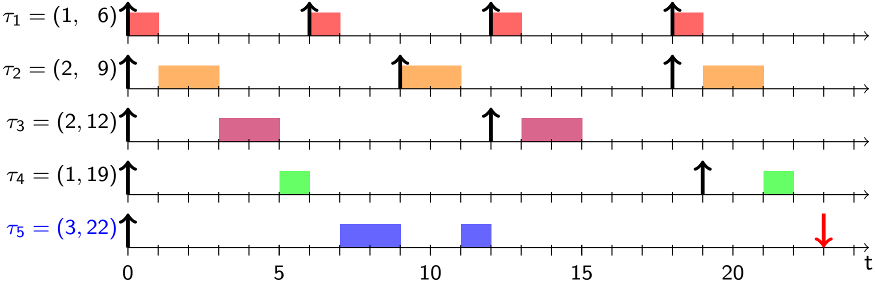
- Uniprocessor, fixed priority
- Sporadic tasks, implicit deadlines: $D_i = T_i\ \forall \tau_i$
- $C_i^A \geq C_i^N$ here: $C_i^A = 2 \cdot C_i^N$
- $\mathbb{P}(C_i^A) \ll \mathbb{P}(C_i^N)$
- $\mathbb{P}(C_i^A) + \mathbb{P}(C_i^N) = 1$
- Probabilities independent

Task Model and Notation



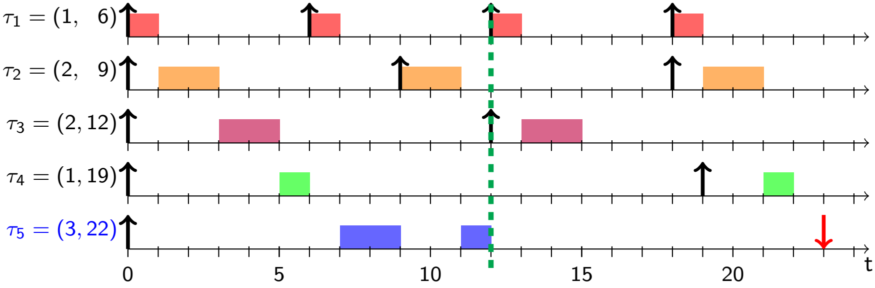
- Uniprocessor, fixed priority
- Sporadic tasks, implicit deadlines: $D_i = T_i \; \forall \tau_i$
- $C_i^A \geq C_i^N$, here: $C_i^A = 2 \cdot C_i^N$
- $\mathbb{P}(C_i^A) \ll \mathbb{P}(C_i^N)$
- $\mathbb{P}(C_i^A) + \mathbb{P}(C_i^N) = 1$
- Probabilities independent

Probability of Deadline Miss



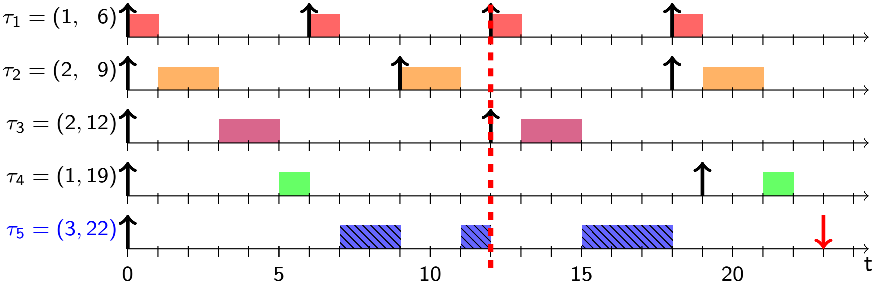
- Looking at lowest priority task

Probability of Deadline Miss



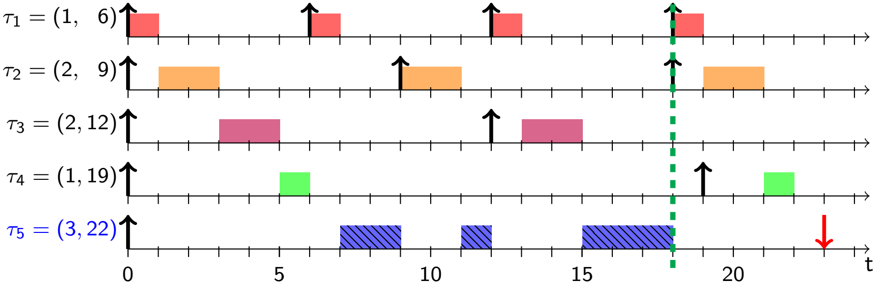
- Looking at lowest priority task
- Normally: TDA binary decision

Probability of Deadline Miss



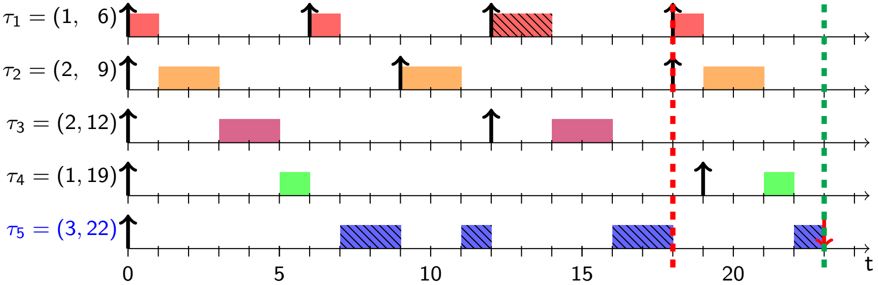
- Looking at lowest priority task
- Normally: TDA binary decision
- $\mathbb{P}(S_t > t)$

Probability of Deadline Miss



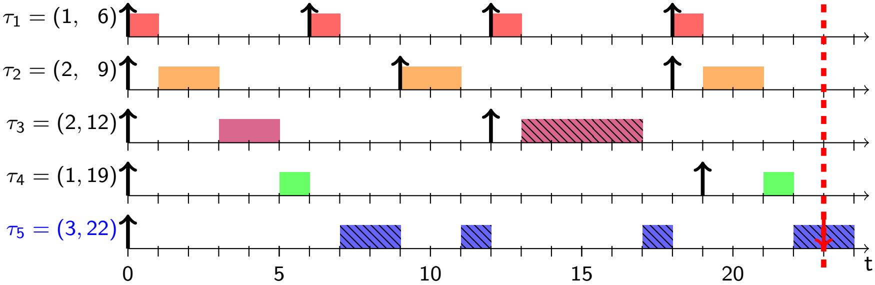
- Looking at lowest priority task
- Normally: TDA binary decision
- $\mathbb{P}(S_t > t)$

Probability of Deadline Miss



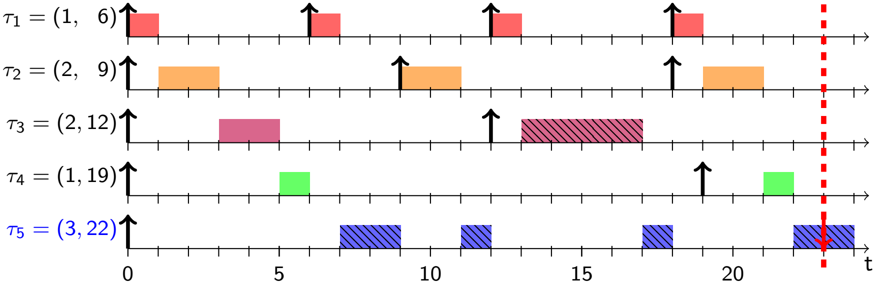
- Looking at lowest priority task
- Normally: TDA binary decision
- $\mathbb{P}(S_t > t)$

Probability of Deadline Miss



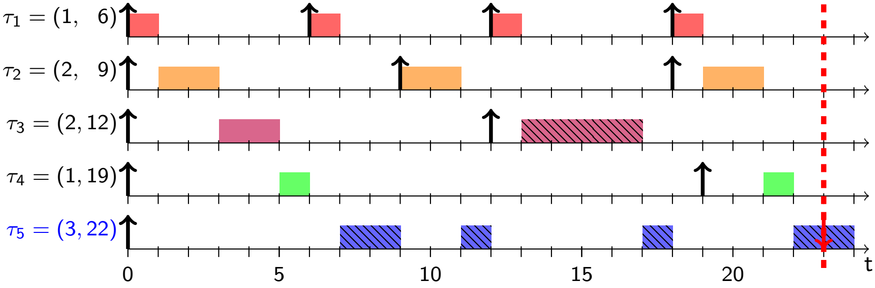
- Looking at lowest priority task
- Normally: TDA binary decision
- $\mathbb{P}(S_t > t)$

Probability of Deadline Miss



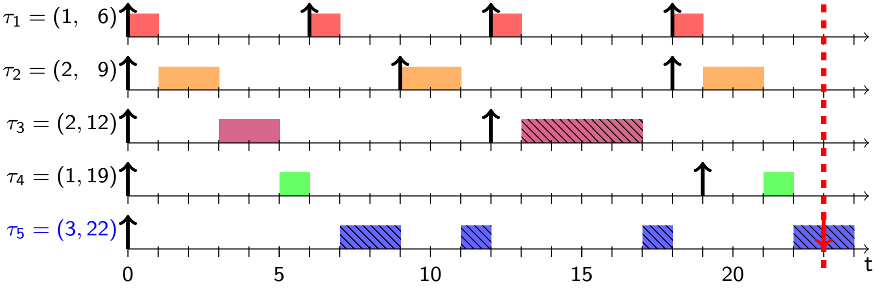
- Looking at lowest priority task
- Normally: TDA binary decision
- $\mathbb{P}(S_t > t)$ for $0 < t \leq D_k$

Probability of Deadline Miss



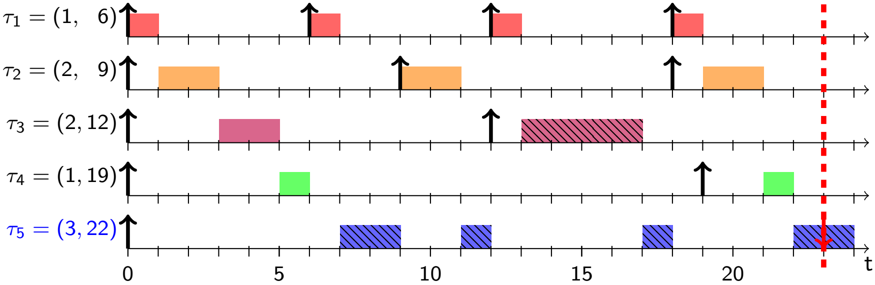
- Looking at lowest priority task
- Normally: TDA binary decision
- $\mathbb{P}(S_t > t)$ for $0 < t \leq D_k$
- Probability of Deadline Miss: $\Phi_k = \min_{0<t\leq D_k} \mathbb{P}(S_t > t)$

Probability of Deadline Miss



- Looking at lowest priority task
- Normally: TDA binary decision
- $\mathbb{P}(S_t > t)$ for $0 < t \leq D_k$
- Probability of Deadline Miss: $\Phi_k = \min_{0<t\leq D_k} \mathbb{P}(S_t > t)$
- Upper bound: any subset of points in $(0, D_k]$

Probability of Deadline Miss



- Looking at lowest priority task
- Normally: TDA binary decision
- $\mathbb{P}(S_t > t)$ for $0 < t \leq D_k$
- Probability of Deadline Miss: $\Phi_k = \min_{0<t\leq D_k} \mathbb{P}(S_t > t)$
- Upper bound: any subset of points in $(0, D_k]$
- Convolution-based approach: enumerate the state space

Convolution

$$\begin{matrix} C_1 \\ \mathbb{P}_1 \end{matrix} = \begin{pmatrix} 3 & 5 \\ 0.9 & 0.1 \end{pmatrix} \otimes \begin{pmatrix} 5 & 6 \\ 0.8 & 0.2 \end{pmatrix} = \begin{matrix} C_2 \\ \mathbb{P}_2 \end{matrix}$$

Convolution

$$\begin{matrix} C_1 \\ \mathbb{P}_1 \end{matrix} = \begin{pmatrix} 3 & 5 \\ 0.9 & 0.1 \end{pmatrix} \otimes \begin{pmatrix} 5 & 6 \\ 0.8 & 0.2 \end{pmatrix} = \begin{matrix} C_2 \\ \mathbb{P}_2 \end{matrix}$$

$$\downarrow$$

$$\begin{pmatrix} \quad \end{pmatrix}$$

Convolution

$$\begin{matrix} C_1 \\ \mathbb{P}_1 \end{matrix} = \begin{pmatrix} 3 & 5 \\ 0.9 & 0.1 \end{pmatrix} \otimes \begin{pmatrix} 5 & 6 \\ 0.8 & 0.2 \end{pmatrix} = \begin{matrix} C_2 \\ \mathbb{P}_2 \end{matrix}$$

$$\downarrow$$

$$\begin{pmatrix} 8 & & \\ 0.72 & & \end{pmatrix}$$

Convolution

$$\begin{matrix} C_1 \\ \mathbb{P}_1 \end{matrix} = \begin{pmatrix} 3 & 5 \\ 0.9 & 0.1 \end{pmatrix} \otimes \begin{pmatrix} 5 & 6 \\ 0.8 & 0.2 \end{pmatrix} = \begin{matrix} C_2 \\ \mathbb{P}_2 \end{matrix}$$

$$\downarrow$$

$$\begin{pmatrix} 8 & 9 & & \\ 0.72 & 0.18 & & \end{pmatrix}$$

Convolution

$$\begin{matrix} C_1 \\ \mathbb{P}_1 \end{matrix} = \begin{pmatrix} 3 & 5 \\ 0.9 & 0.1 \end{pmatrix} \otimes \begin{pmatrix} 5 & 6 \\ 0.8 & 0.2 \end{pmatrix} = \begin{matrix} C_2 \\ \mathbb{P}_2 \end{matrix}$$

$$\Downarrow$$

$$\begin{pmatrix} 8 & 9 & 10 & & \\ 0.72 & 0.18 & 0.08 & & \end{pmatrix}$$

Convolution

$$\begin{matrix} C_1 \\ \mathbb{P}_1 \end{matrix} = \begin{pmatrix} 3 & 5 \\ 0.9 & 0.1 \end{pmatrix} \otimes \begin{pmatrix} 5 & 6 \\ 0.8 & 0.2 \end{pmatrix} = \begin{matrix} C_2 \\ \mathbb{P}_2 \end{matrix}$$

$$\downarrow$$

$$\begin{pmatrix} 8 & 9 & 10 & 11 \\ 0.72 & 0.18 & 0.08 & 0.02 \end{pmatrix}$$

Convolution

$$\begin{matrix} C_1 \\ \mathbb{P}_1 \end{matrix} = \begin{pmatrix} 3 & 5 \\ 0.9 & 0.1 \end{pmatrix} \otimes \begin{pmatrix} 5 & 6 \\ 0.8 & 0.2 \end{pmatrix} = \begin{matrix} C_2 \\ \mathbb{P}_2 \end{matrix}$$

$$\downarrow$$

$$\begin{pmatrix} 8 & 9 & 10 & 11 \\ 0.72 & 0.18 & 0.08 & 0.02 \end{pmatrix}$$

- State-of-the-art: job-wise convolution from 0 to D_k

Job-Level Convolution

τ_1

$$\begin{matrix} C_1 \\ \mathbb{P}_1 \end{matrix} = \begin{pmatrix} 3 & 5 \\ 0.9 & 0.1 \end{pmatrix}$$

$$D_1 = T_1 = 8$$

τ_2

$$\begin{matrix} C_2 \\ \mathbb{P}_2 \end{matrix} = \begin{pmatrix} 5 & 6 \\ 0.8 & 0.2 \end{pmatrix}$$

$$D_2 = T_2 = 14$$

Job-Level Convolution

τ_1

$$\begin{matrix} C_1 \\ \mathbb{P}_1 \end{matrix} = \begin{pmatrix} 3 & 5 \\ 0.9 & 0.1 \end{pmatrix}$$

$D_1 = T_1 = 8$

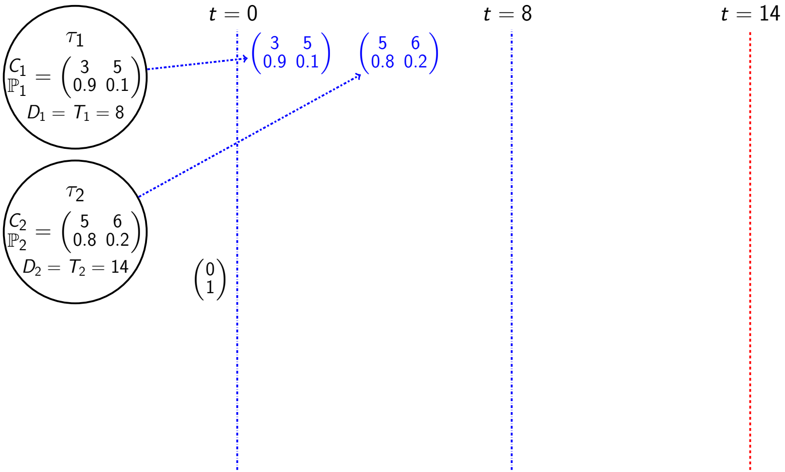
τ_2

$$\begin{matrix} C_2 \\ \mathbb{P}_2 \end{matrix} = \begin{pmatrix} 5 & 6 \\ 0.8 & 0.2 \end{pmatrix}$$

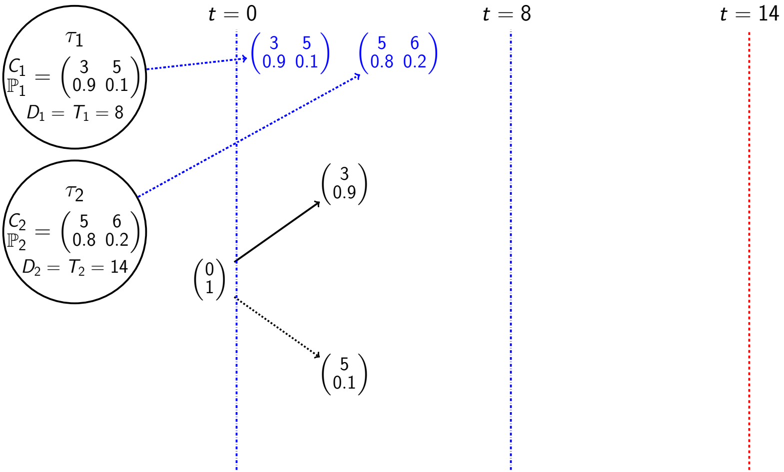
$D_2 = T_2 = 14$

$t = 0$ $t = 8$ $t = 14$

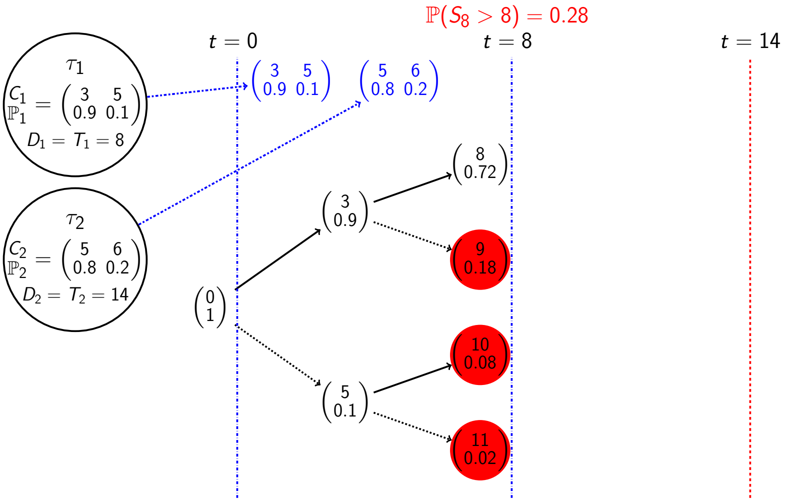
Job-Level Convolution



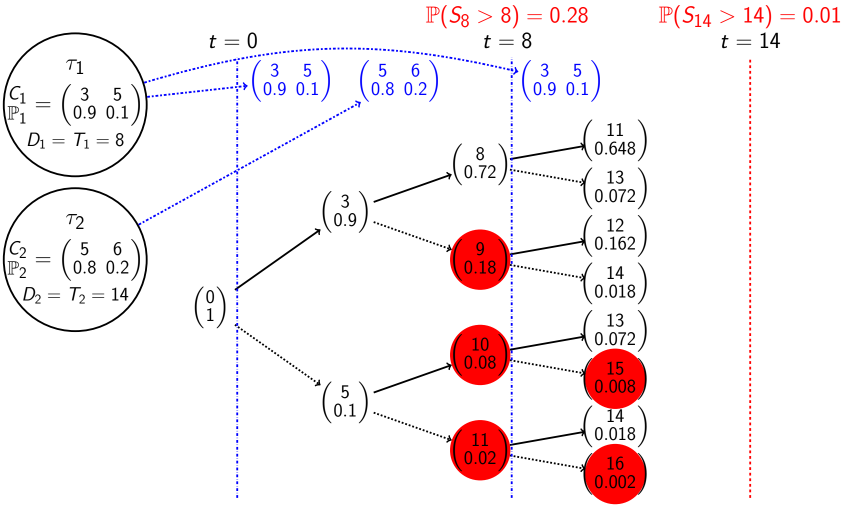
Job-Level Convolution



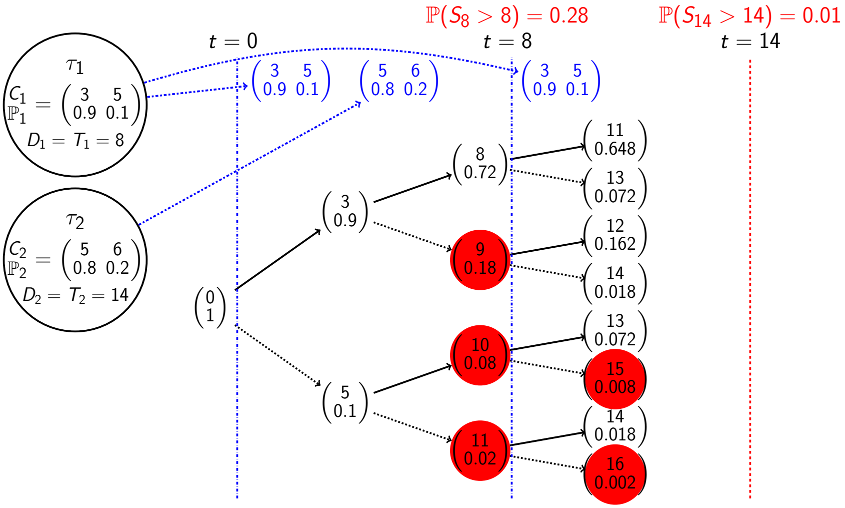
Job-Level Convolution



Job-Level Convolution

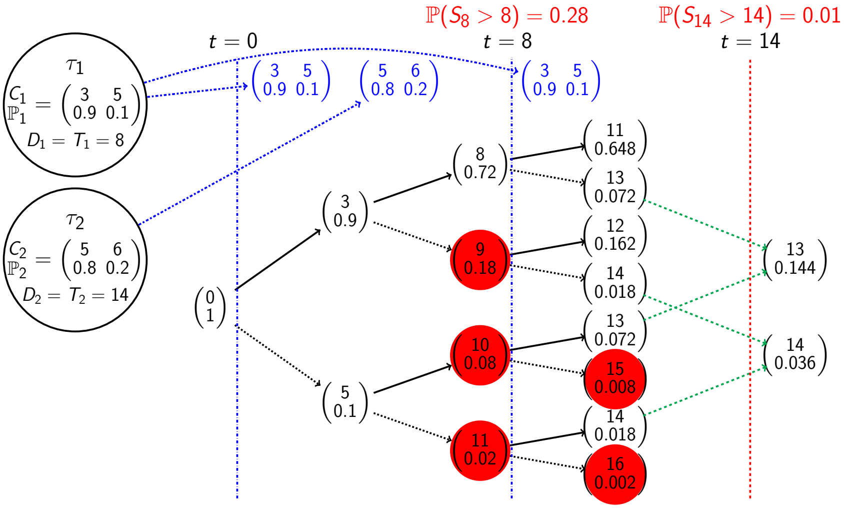


Job-Level Convolution

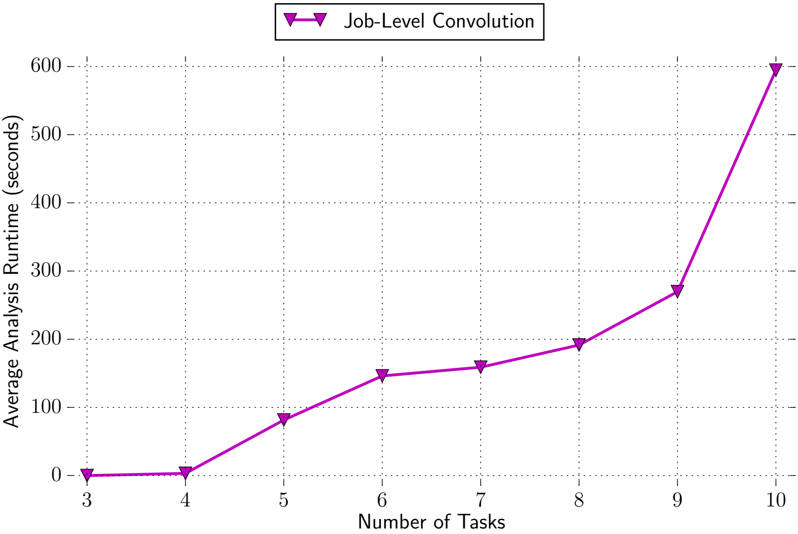


3 tasks $\rightarrow$ 38 jobs $\rightarrow 2^{38} = 274\ 877\ 906\ 944$

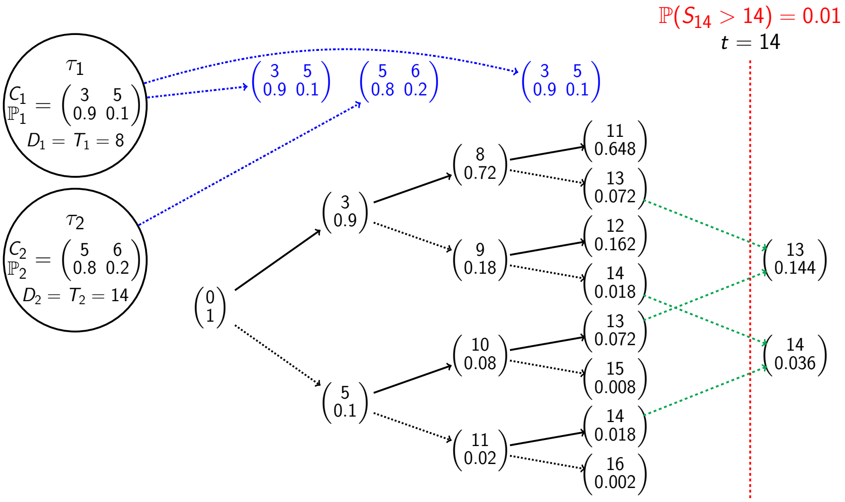
Job-Level Convolution



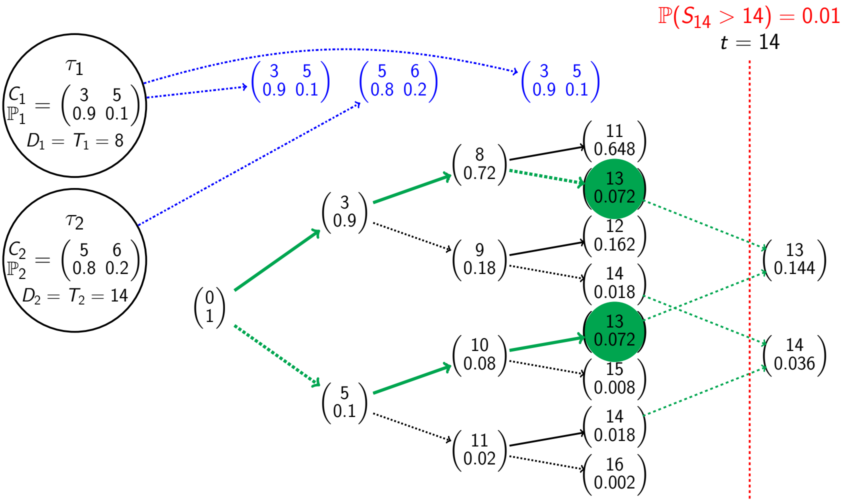
Performance: Job-Level Convolution



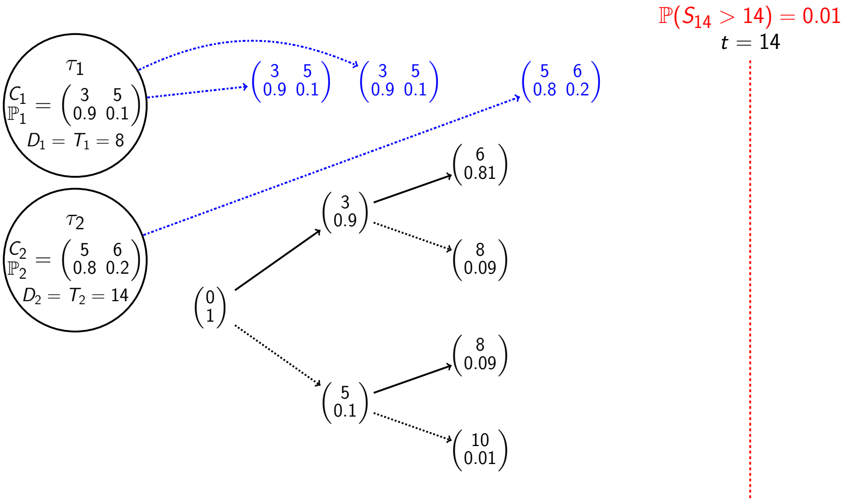
Considering Time Points Individually



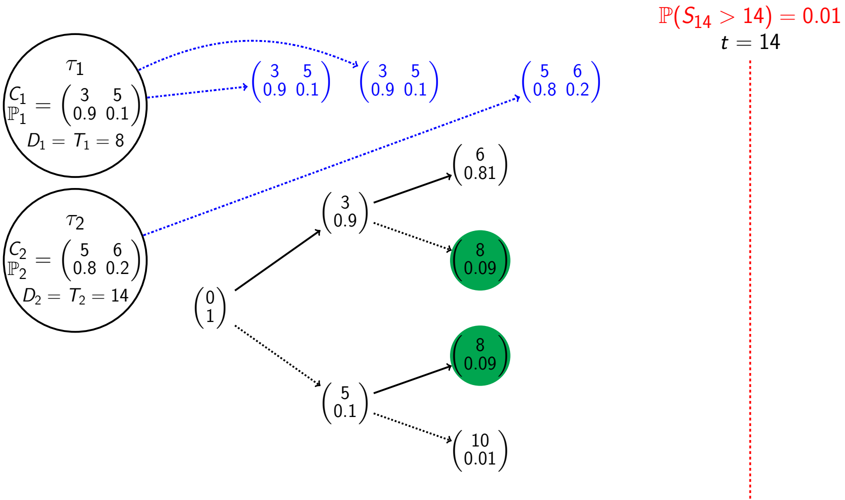
Considering Time Points Individually



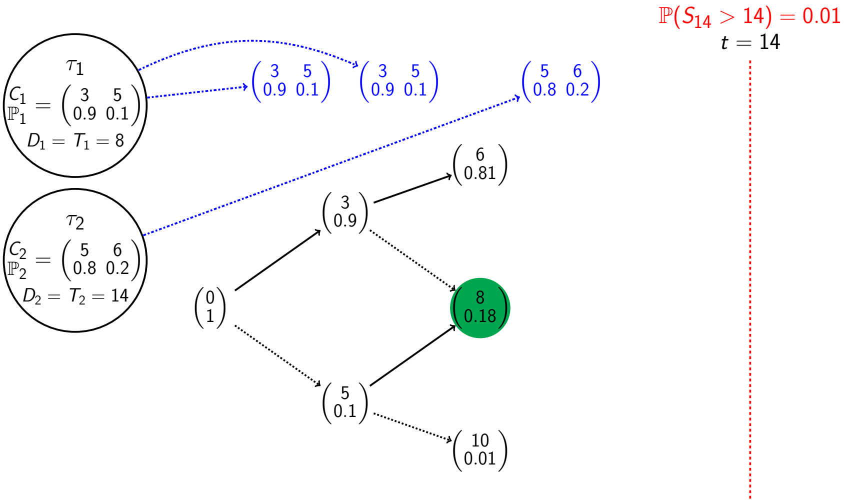
Considering Time Points Individually



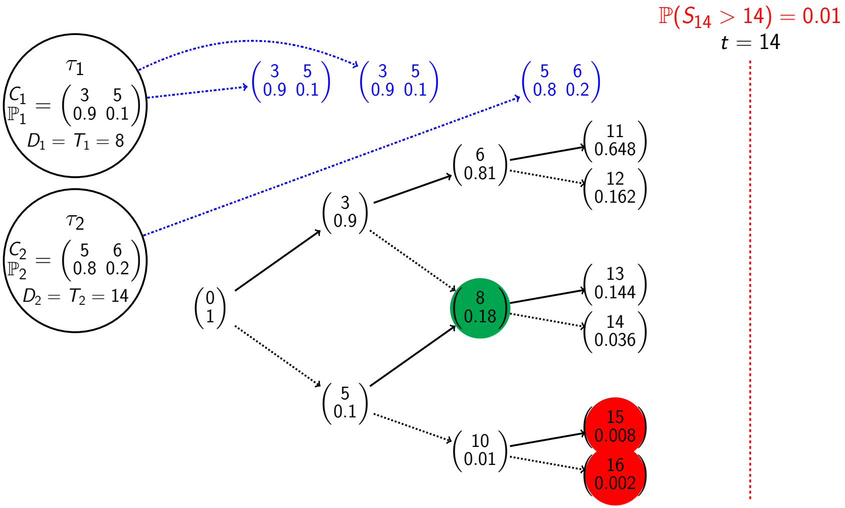
Considering Time Points Individually



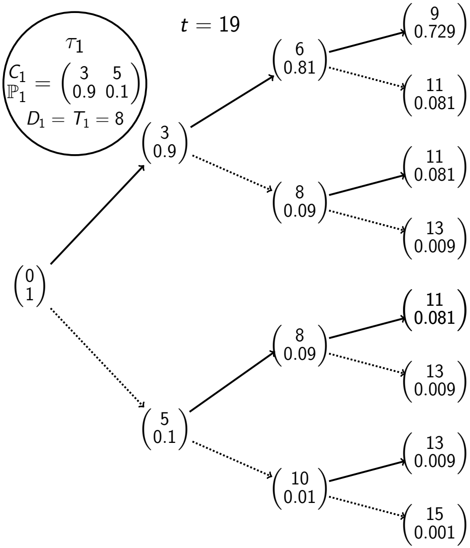
Considering Time Points Individually



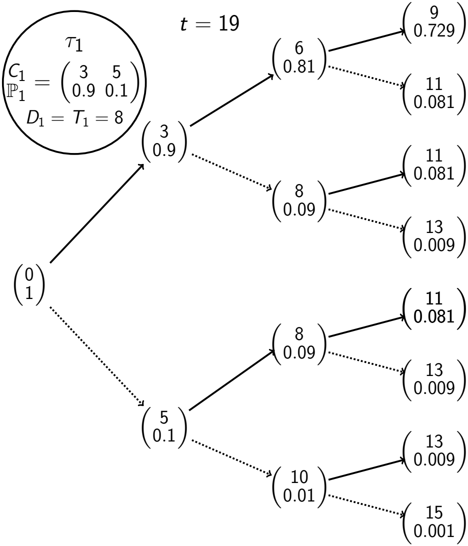
Considering Time Points Individually



Equivalence Classes and Multinomial Distribution

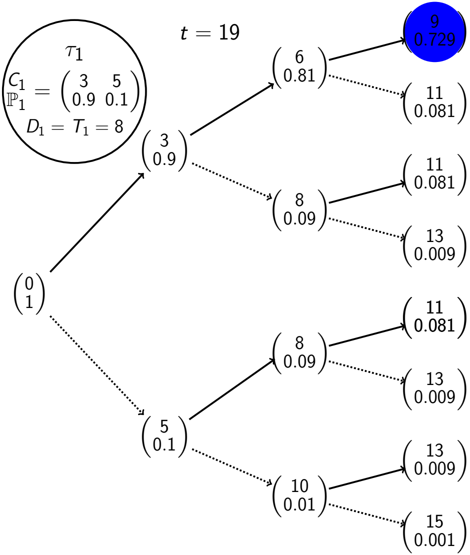


Equivalence Classes and Multinomial Distribution



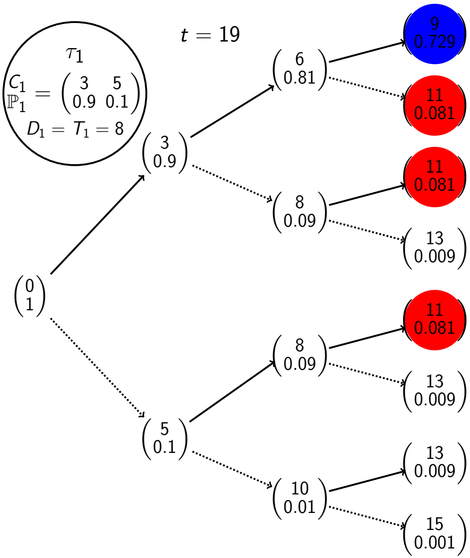
# C_i^A jobs	0	1	2	3
Total C_i	9	11	13	15
Probability				

Equivalence Classes and Multinomial Distribution



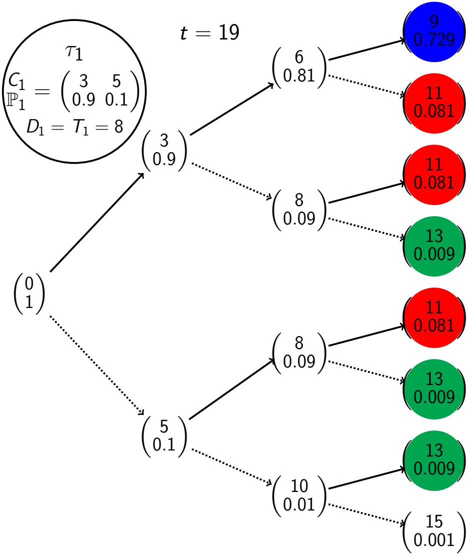
# C_i^A **jobs**	0	1	2	3
Total C_i	9	11	13	15
Probability	0.729			

Equivalence Classes and Multinomial Distribution



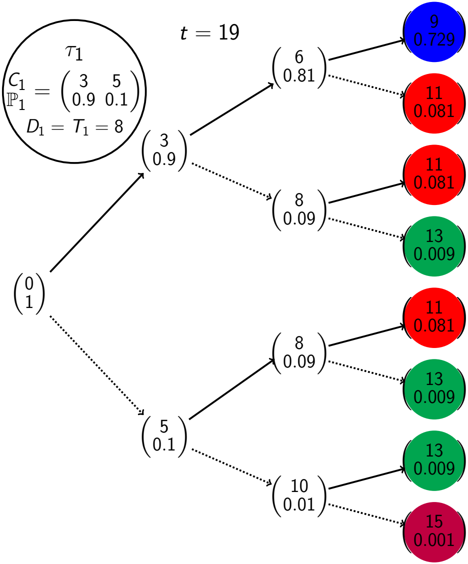
# C_i^A jobs	0	1	2	3
Total C_i	9	11	13	15
Probability	0.729	0.243		

Equivalence Classes and Multinomial Distribution



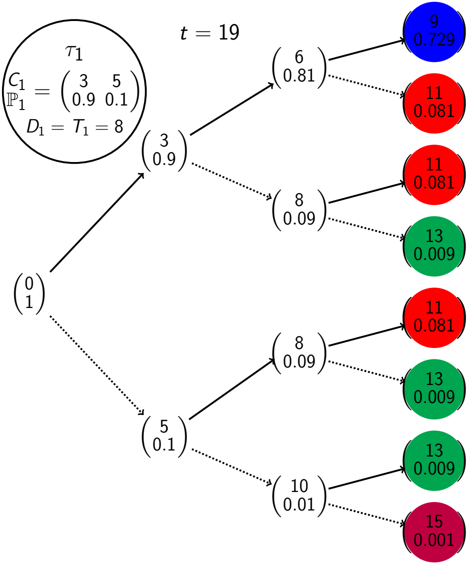
# C_i^A jobs	0	1	2	3
Total C_i	9	11	13	15
Probability	0.729	0.243	0.027	

Equivalence Classes and Multinomial Distribution



# C_i^A jobs	0	1	2	3
Total C_i	9	11	13	15
Probability	0.729	0.243	0.027	0.001

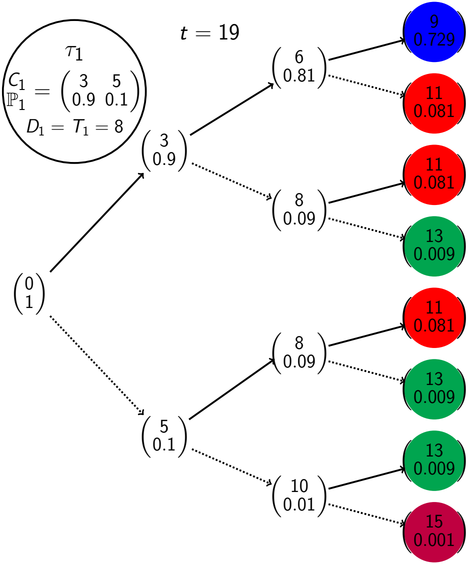
Equivalence Classes and Multinomial Distribution



# C_i^A **jobs**	0	1	2	3
Total C_i	9	11	13	15
Probability	0.729	0.243	0.027	0.001

- Set of equivalence classes

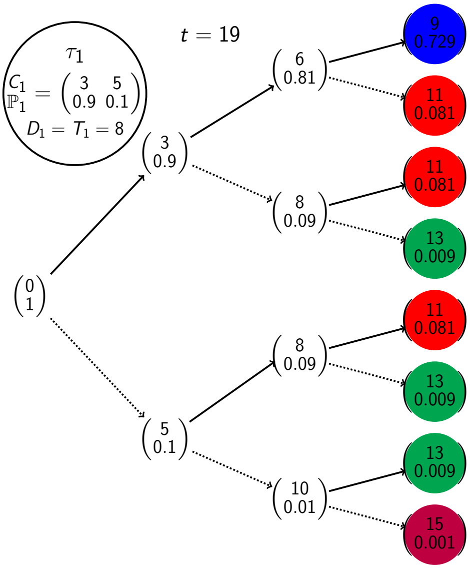
Equivalence Classes and Multinomial Distribution



# C_i^A jobs	0	1	2	3
Total C_i	9	11	13	15
Probability	0.729	0.243	0.027	0.001

- Set of equivalence classes
- For one equivalence class:
  - Execution time
  - Number of paths
  - Probability of each path

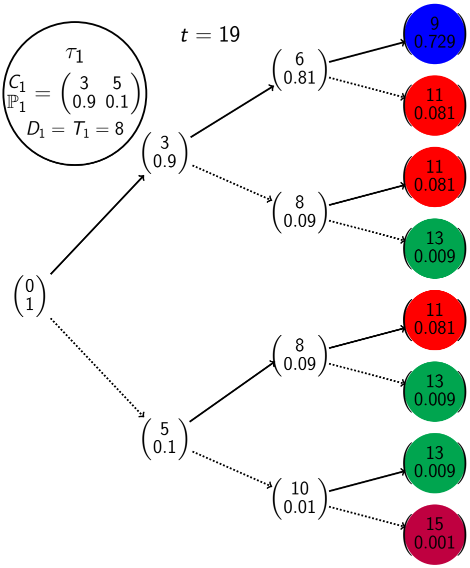
Equivalence Classes and Multinomial Distribution



# C_i^A jobs	0	1	2	3
Total C_i	9	11	13	15
Probability	0.729	0.243	0.027	0.001

- Set of equivalence classes
- For one equivalence class:
  - Execution time
  - Number of paths
  - Probability of each path
- Multinomial distribution
  - Probability of each path: $\mathbb{P}_i(1)^{\ell_{i,1}} \cdot \mathbb{P}_i(2)^{\ell_{i,2}} \cdot \ldots \cdot \mathbb{P}_i(h)^{\ell_{i,h}}$

Equivalence Classes and Multinomial Distribution



# C_i^A jobs	0	1	2	3
Total C_i	9	11	13	15
Probability	0.729	0.243	0.027	0.001

- Set of equivalence classes
- For one equivalence class:
  - Execution time
  - Number of paths
  - Probability of each path
- Multinomial distribution
  - Probability of each path: $\mathbb{P}_i(1)^{\ell_{i,1}} \cdot \mathbb{P}_i(2)^{\ell_{i,2}} \cdot \ldots \cdot \mathbb{P}_i(h)^{\ell_{i,h}}$
  - Number of paths: $\frac{\rho_{i,t}!}{\ell_{i,1}!\ell_{i,2}!\ldots\ell_{i,h}!}$

Task-Level Convolution

$t = 24$

τ_1

$$\begin{matrix} C_1 \\ \mathbb{P}_1 \end{matrix} = \begin{pmatrix} 3 & 5 \\ 0.9 & 0.1 \end{pmatrix}$$

$D_1 = T_1 = 15$

τ_2

$$\begin{matrix} C_2 \\ \mathbb{P}_2 \end{matrix} = \begin{pmatrix} 6 & 10 \\ 0.8 & 0.2 \end{pmatrix}$$

$D_2 = T_2 = 30$

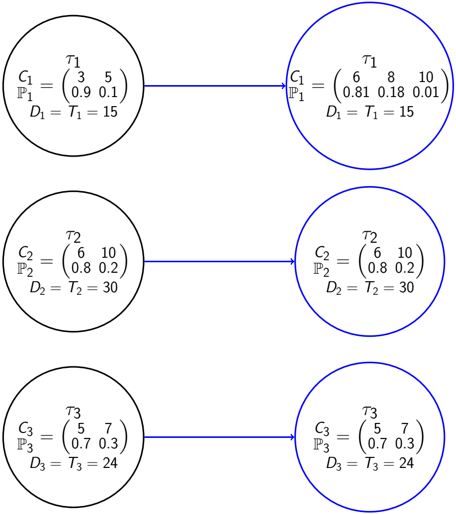
τ_3

$$\begin{matrix} C_3 \\ \mathbb{P}_3 \end{matrix} = \begin{pmatrix} 5 & 7 \\ 0.7 & 0.3 \end{pmatrix}$$

$D_3 = T_3 = 24$

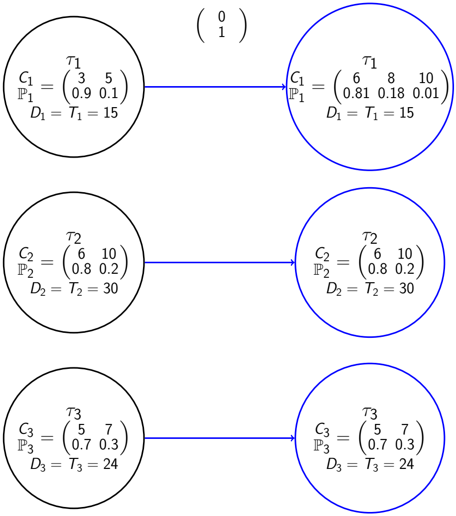
Task-Level Convolution

$t = 24$

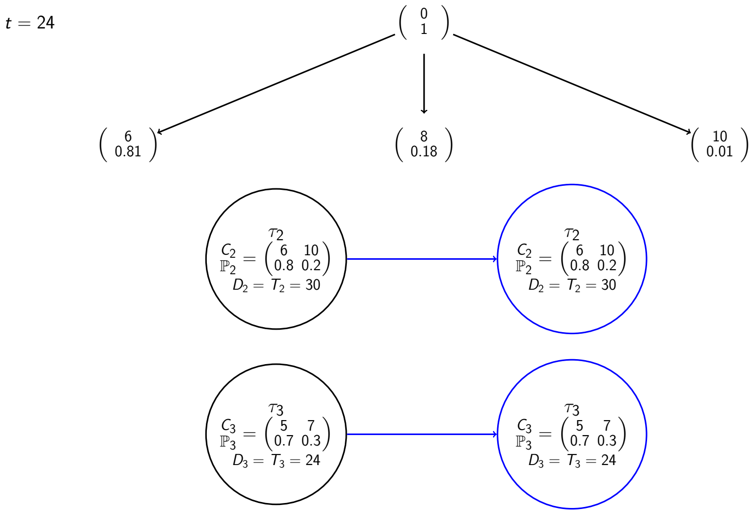


Task-Level Convolution

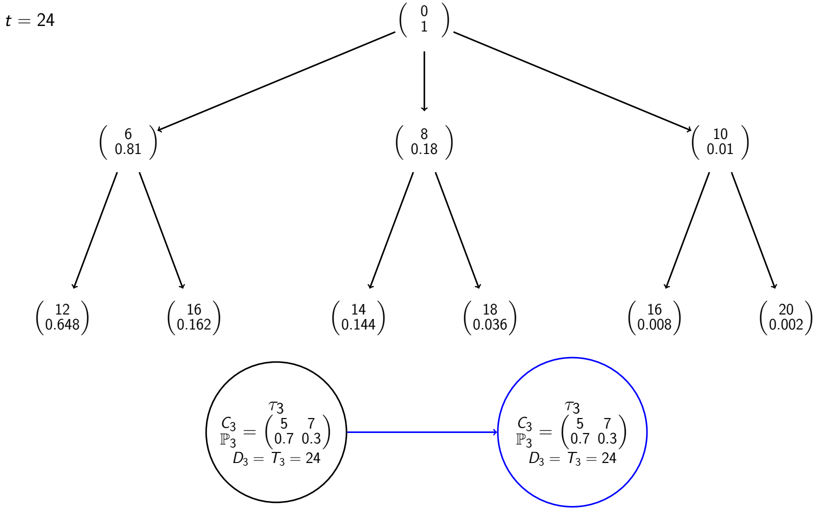
$t = 24$



Task-Level Convolution

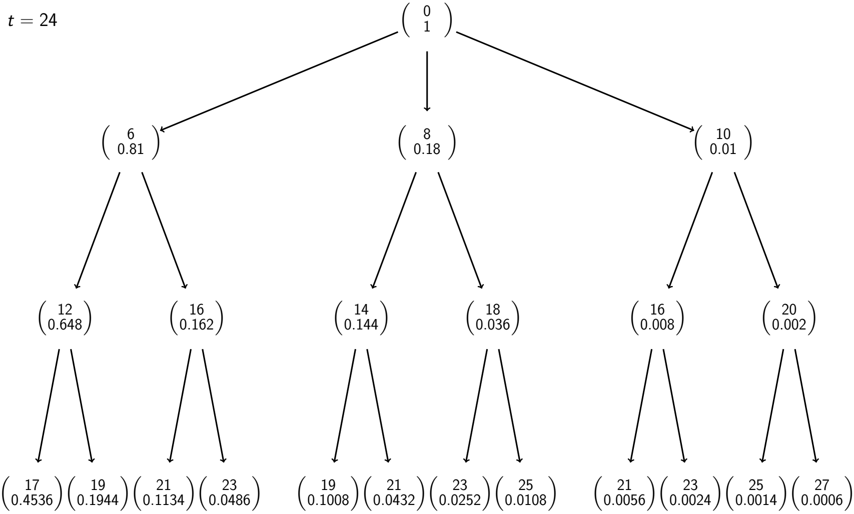


Task-Level Convolution

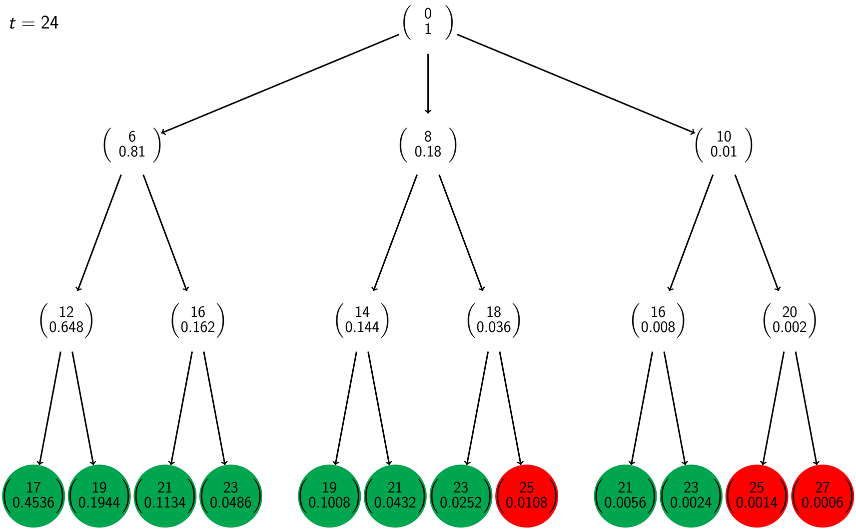


Task-Level Convolution

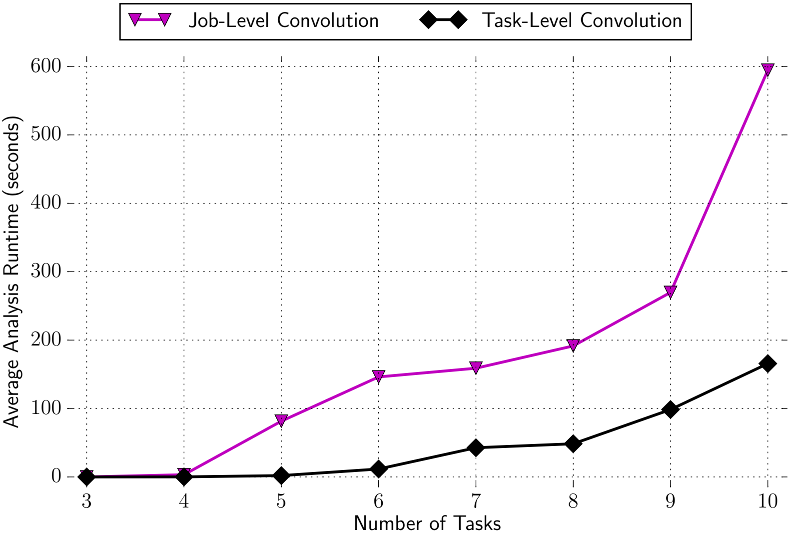
$t = 24$



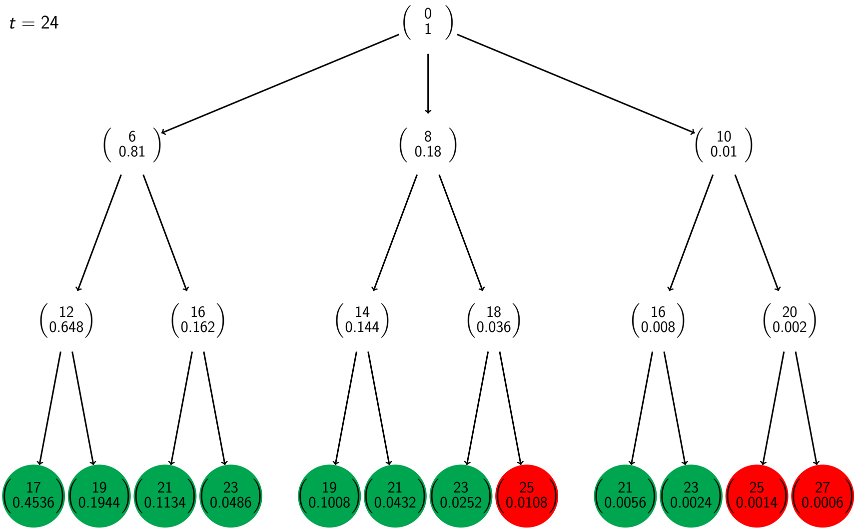
Task-Level Convolution



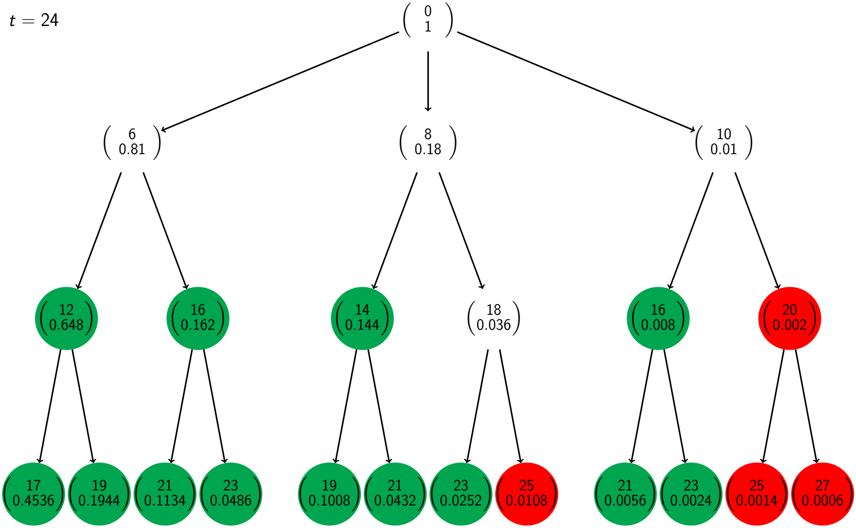
Performance: Task-Level Convolution



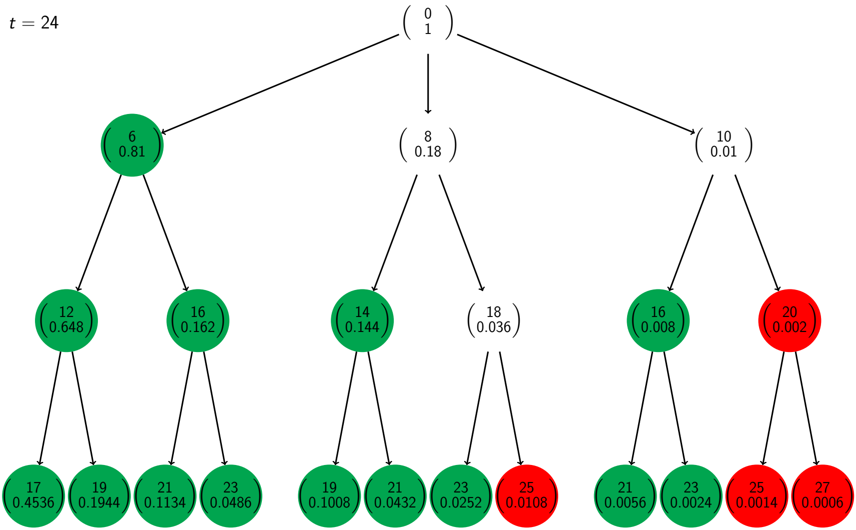
State Space Pruning



State Space Pruning

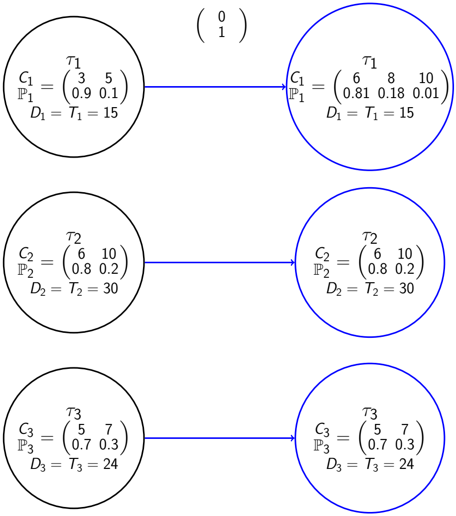


State Space Pruning



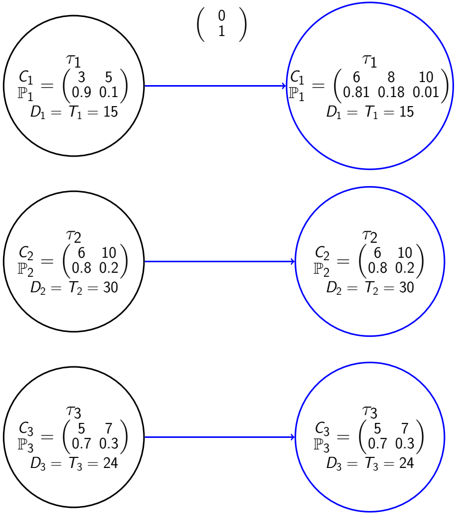
State Space Pruning

$t = 24$



State Space Pruning

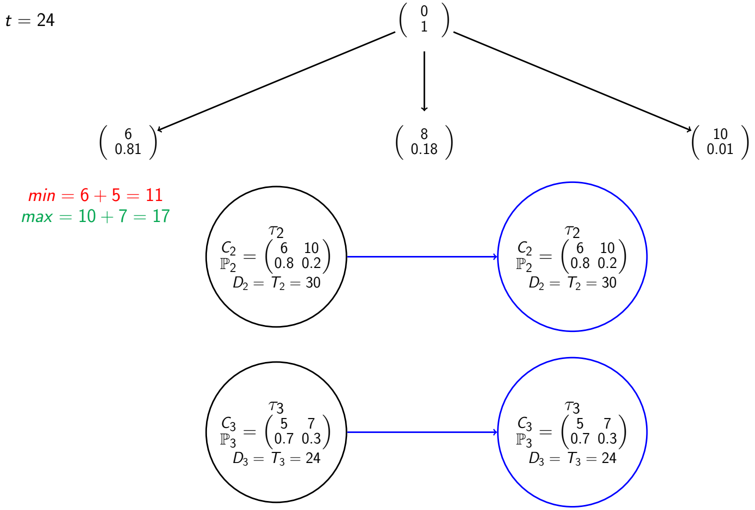
$t = 24$



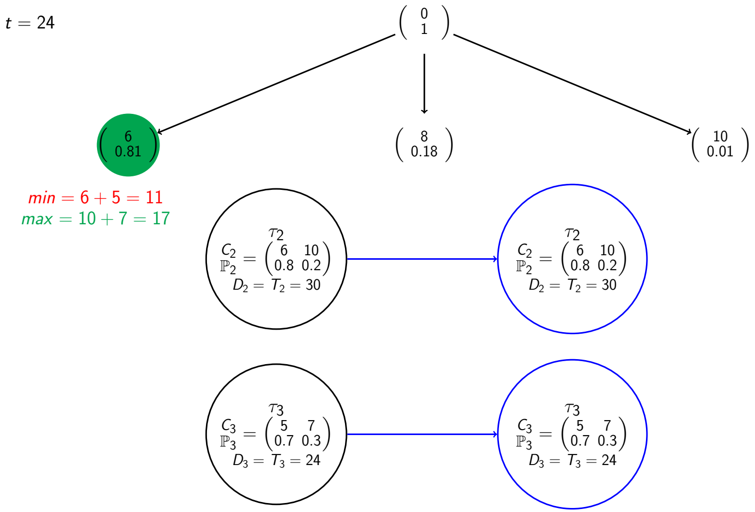
$min = 6 + 5 = 11$
$max = 10 + 7 = 17$

$min = 5$
$max = 7$

State Space Pruning

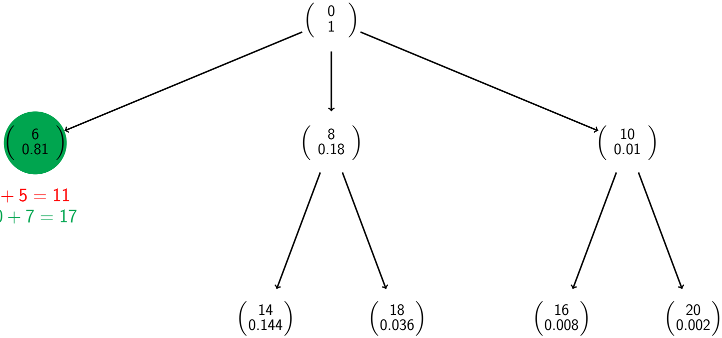


State Space Pruning



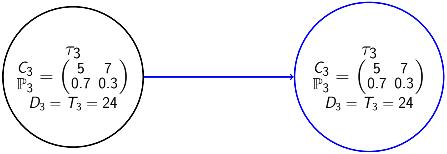
State Space Pruning

$t = 24$



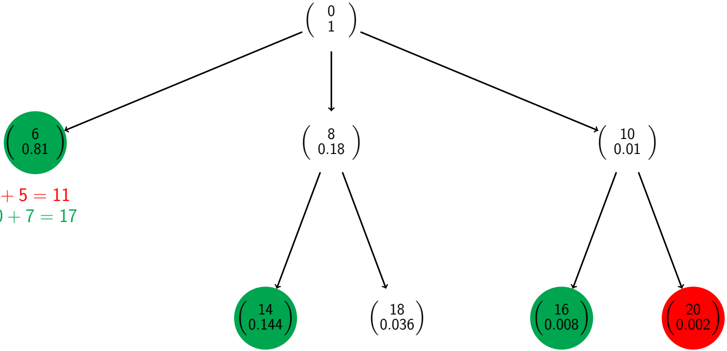
$min = 6 + 5 = 11$
$max = 10 + 7 = 17$

$min = 5$
$max = 7$



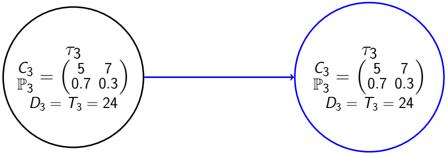
State Space Pruning

$t = 24$



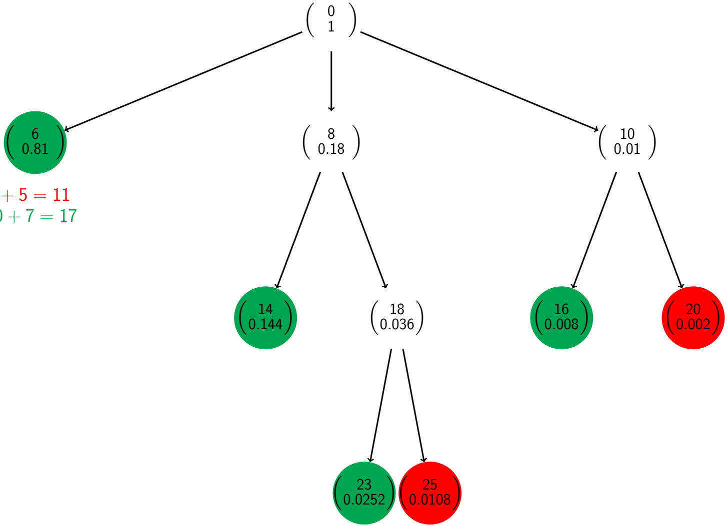
$min = 6 + 5 = 11$
$max = 10 + 7 = 17$

$min = 5$
$max = 7$



State Space Pruning

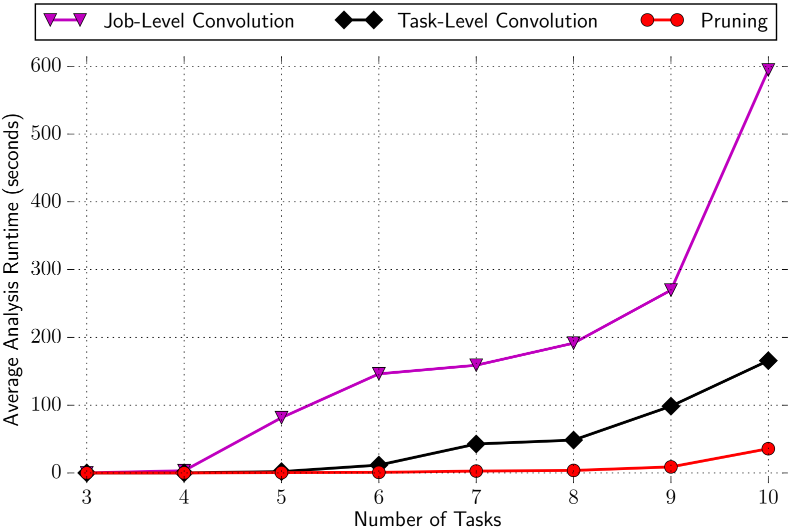
$t = 24$



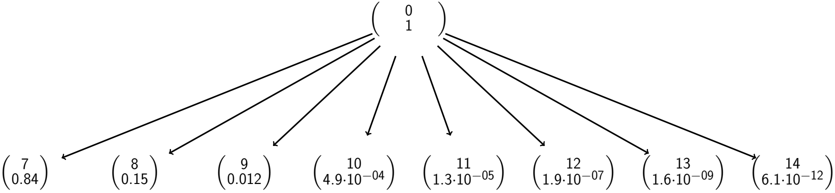
$min = 6 + 5 = 11$
$max = 10 + 7 = 17$

$min = 5$
$max = 7$

Performance: Job-Level Convolution with Pruning

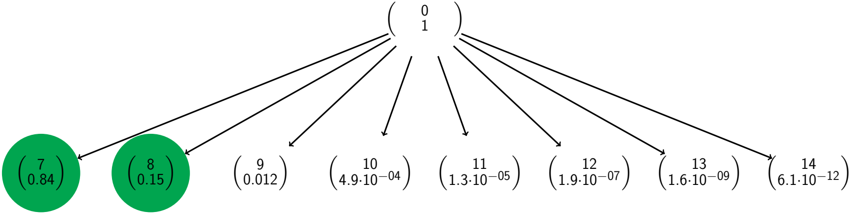


Union of Equivalence Classes



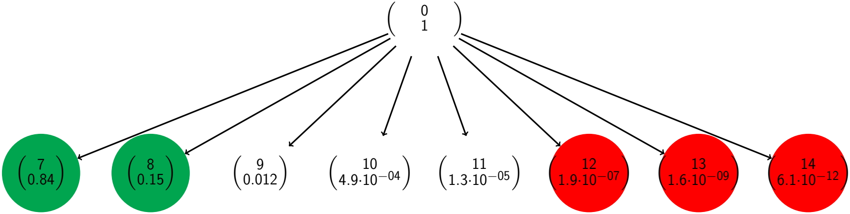
# C_i^A jobs	0	1	2	3	4	5	6	7
Total C_i	7	8	9	10	11	12	13	14
Probability	0.84	0.15	0.012	$4.9 \cdot 10^{-04}$	$1.3 \cdot 10^{-05}$	$1.9 \cdot 10^{-07}$	$1.6 \cdot 10^{-09}$	$6.1 \cdot 10^{-12}$

Union of Equivalence Classes



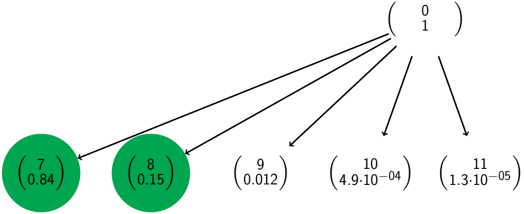
# C_i^A jobs	0	1	2	3	4	5	6	7
Total C_i	7	8	9	10	11	12	13	14
Probability	0.84	0.15	0.012	$4.9 \cdot 10^{-04}$	$1.3 \cdot 10^{-05}$	$1.9 \cdot 10^{-07}$	$1.6 \cdot 10^{-09}$	$6.1 \cdot 10^{-12}$

Union of Equivalence Classes



# C_i^A jobs	0	1	2	3	4	5	6	7
Total C_i	7	8	9	10	11	12	13	14
Probability	0.84	0.15	0.012	$4.9 \cdot 10^{-04}$	$1.3 \cdot 10^{-05}$	$1.9 \cdot 10^{-07}$	$1.6 \cdot 10^{-09}$	$6.1 \cdot 10^{-12}$

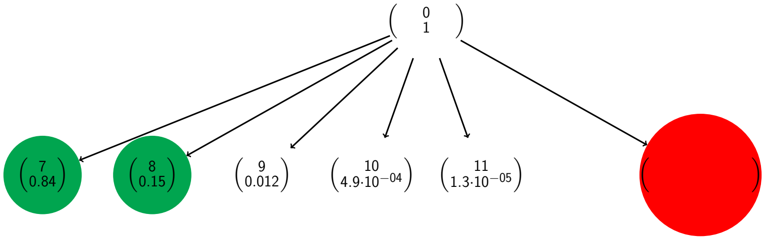
Union of Equivalence Classes



# C_i^A jobs	0	1	2	3	4	5	6	7
Total C_i	7	8	9	10	11	12	13	14
Probability	0.84	0.15	0.012	$4.9 \cdot 10^{-04}$	$1.3 \cdot 10^{-05}$	$1.9 \cdot 10^{-07}$	$1.6 \cdot 10^{-09}$	$6.1 \cdot 10^{-12}$

# C_i^A jobs	0	1	2	3	4
Total C_i	7	8	9	10	11
Probability	0.84	0.15	0.012	$4.9 \cdot 10^{-04}$	$1.3 \cdot 10^{-05}$

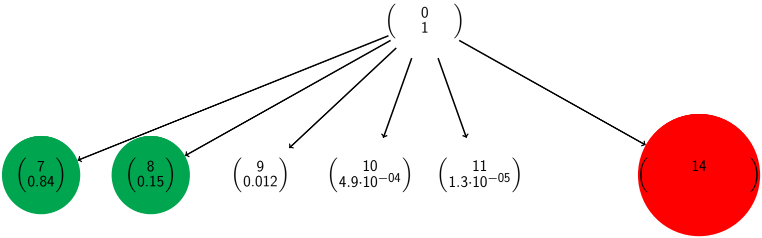
Union of Equivalence Classes



# C_i^A jobs	0	1	2	3	4	5	6	7
Total C_i	7	8	9	10	11	12	13	14
Probability	0.84	0.15	0.012	$4.9 \cdot 10^{-04}$	$1.3 \cdot 10^{-05}$	$1.9 \cdot 10^{-07}$	$1.6 \cdot 10^{-09}$	$6.1 \cdot 10^{-12}$

# C_i^A jobs	0	1	2	3	4	5, 6, or 7
Total C_i	7	8	9	10	11	
Probability	0.84	0.15	0.012	$4.9 \cdot 10^{-04}$	$1.3 \cdot 10^{-05}$	

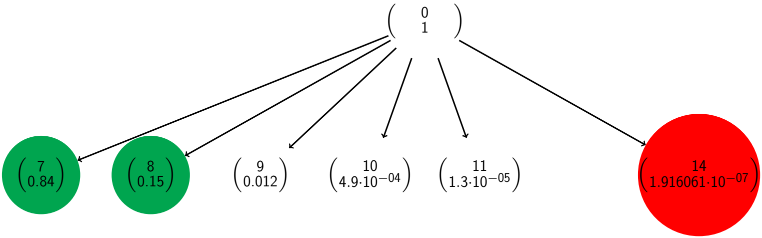
Union of Equivalence Classes



# C_i^A jobs	0	1	2	3	4	5	6	7
Total C_i	7	8	9	10	11	12	13	14
Probability	0.84	0.15	0.012	$4.9 \cdot 10^{-04}$	$1.3 \cdot 10^{-05}$	$1.9 \cdot 10^{-07}$	$1.6 \cdot 10^{-09}$	$6.1 \cdot 10^{-12}$

# C_i^A jobs	0	1	2	3	4	5, 6, or 7
Total C_i	7	8	9	10	11	14
Probability	0.84	0.15	0.012	$4.9 \cdot 10^{-04}$	$1.3 \cdot 10^{-05}$	

Union of Equivalence Classes



# C_i^A jobs	0	1	2	3	4	5	6	7
Total C_i	7	8	9	10	11	12	13	14
Probability	0.84	0.15	0.012	$4.9 \cdot 10^{-04}$	$1.3 \cdot 10^{-05}$	$1.9 \cdot 10^{-07}$	$1.6 \cdot 10^{-09}$	$6.1 \cdot 10^{-12}$

# C_i^A jobs	0	1	2	3	4	5, 6, or 7
Total C_i	7	8	9	10	11	14
Probability	0.84	0.15	0.012	$4.9 \cdot 10^{-04}$	$1.3 \cdot 10^{-05}$	$1.916061 \cdot 10^{-07}$

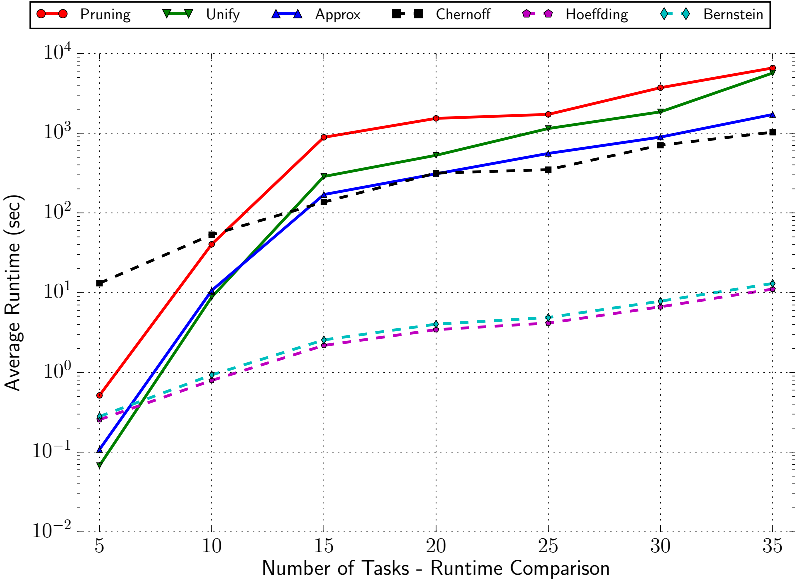
Evaluation: Setup

- Utilization: 70%
- Periods: UUniFast, 10ms-1000ms
- $\mathbb{P}_i(A) = 0.025$, $\mathbb{P}_i(N) = 0.975$

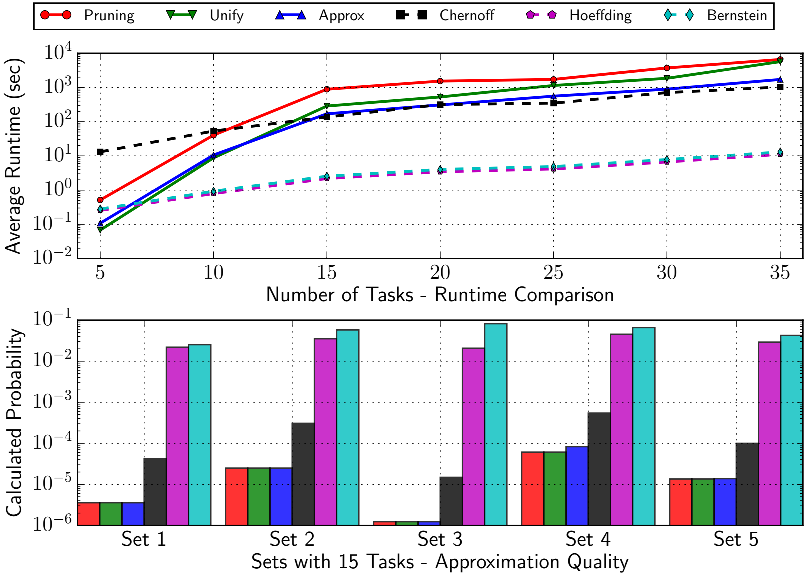
- For 5-20 tasks: 20 task sets
- For 25-35 tasks: 5 task sets

1. **Pruning:** multinomial-based task-level convolution with pruning
2. **Unify:** pruning and union of equivalence classes (max error 10^{-6})
3. **Approx:** only considering D_k and last release times
4. Analytical approach with **Chernoff** *bounds* (Chen and Chen)
5. Analytical approach with **Hoeffding***'s inequality* (this paper)
6. Analytical approach with **Bernstein** *inequalities* (this paper)

Evaluation: Runtime



Evaluation: Precision



Conclusion

- Probability of deadline miss important in system design
- For multiple execution times: no binary schedulability decision
- Job-level convolution not scalable (not more than 10 tasks)
- Idea: Task-level convolution
  - Multinomial distribution
  - Pruning improves runtime without precision loss
  - Union improves runtime with bounded precision loss
- With pruning: approach scalable
  - 75 tasks: average 621.6 sec per time point
  - 100 tasks: average 791.1 sec per time point
  - Easy parallelization
- Analytical bounds: *Hoeffding*'s and *Bernstein* inequality
- Precision roughly proportional to runtime

Conclusion

- Probability of deadline miss important in system design
- For multiple execution times: no binary schedulability decision
- Job-level convolution not scalable (not more than 10 tasks)
- Idea: Task-level convolution
  - Multinomial distribution
  - Pruning improves runtime without precision loss
  - Union improves runtime with bounded precision loss
- With pruning: approach scalable
  - 75 tasks: average 621.6 sec per time point
  - 100 tasks: average 791.1 sec per time point
  - Easy parallelization
- Analytical bounds: *Hoeffding*'s and *Bernstein* inequality
- Precision roughly proportional to runtime

Thank You!