TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

University of Tehran

A Framework to Construct Customized Harmonic Periods for Real-Time Systems

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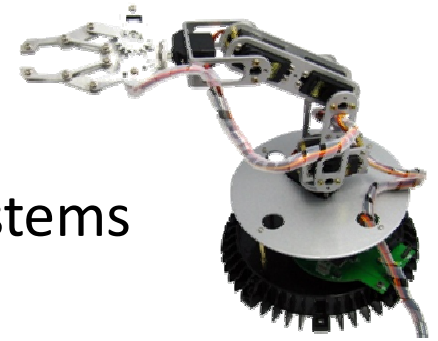
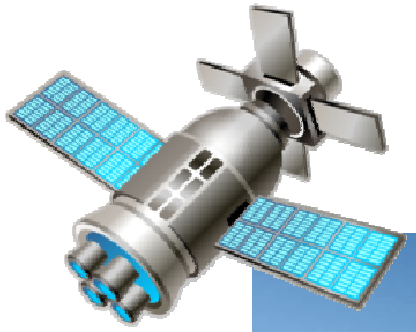
Motivations



- Periodic behavior appears in many real-time systems
  - It guarantees certain levels of safety and QoS

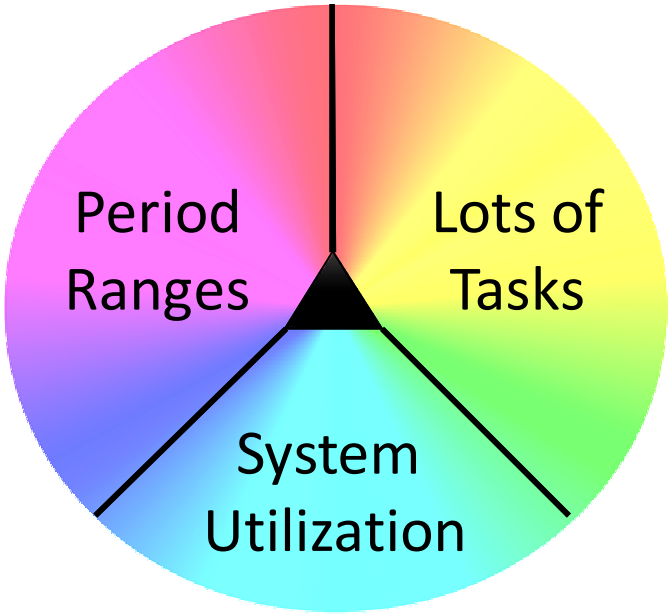


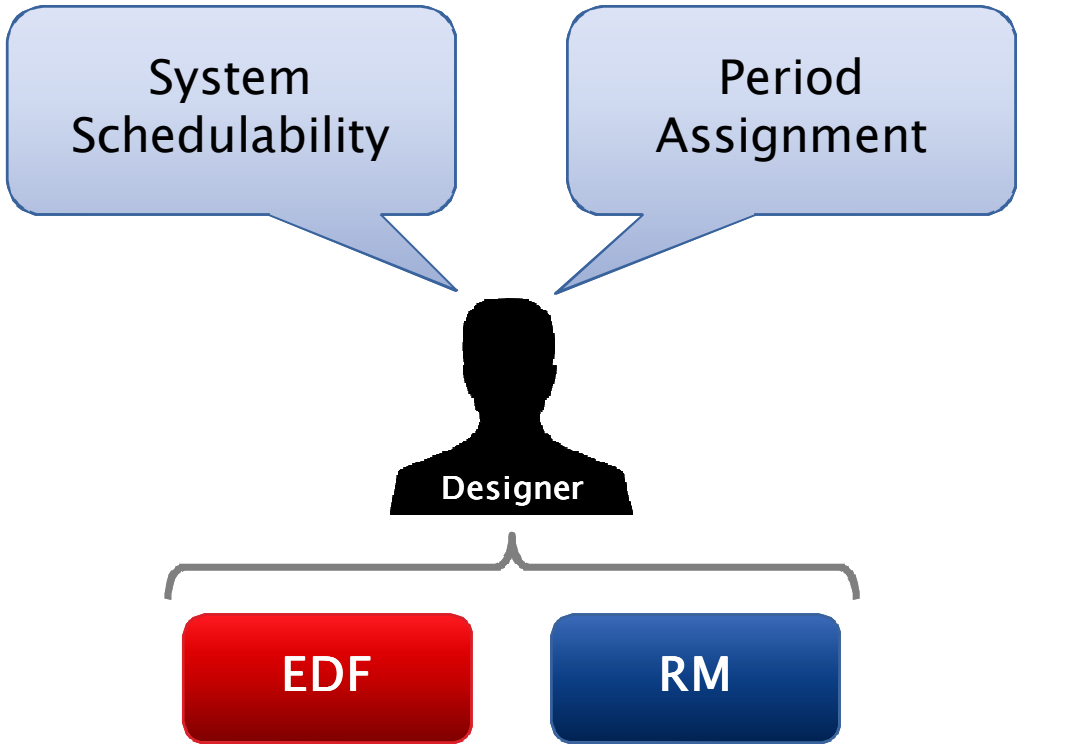
A lot of functionalities

Choice of periods and parameters

System Utilization

Motivations





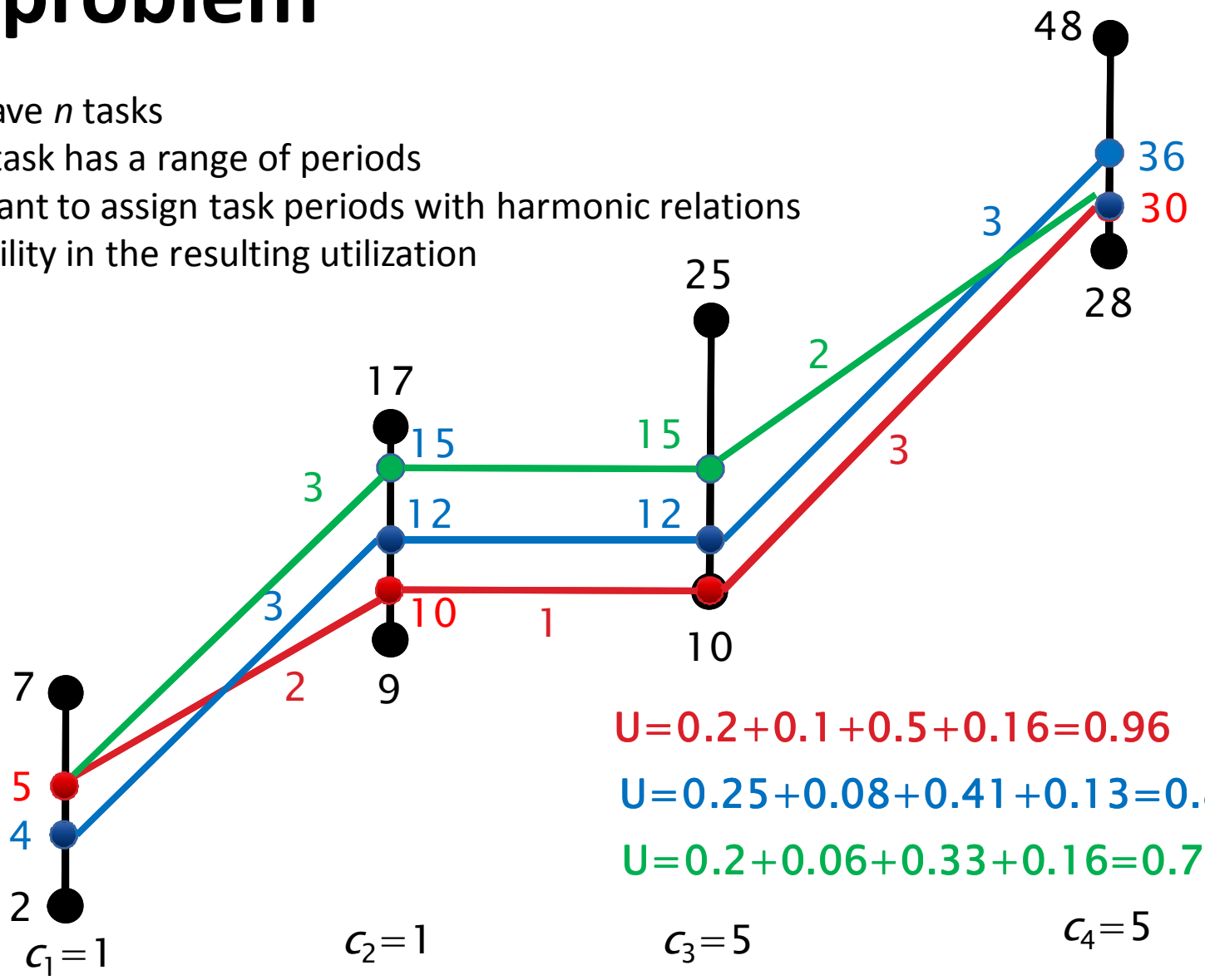
Approach: Finding Harmonic Periods

Harmonic Periods

- Optimally scheduled by **RM**
- Exact **WCRT** has **polynomial-time** computational complexity
- Jitter is efficiently reduced
- **Small hyperperiod**
- Optimal non-preemptive scheduling
- Wide **applicability** in industries:
  Avionics, submarines, robotics

The problem

- We have *n* tasks
- Each task has a range of periods
- We want to assign task periods with harmonic relations
- Flexibility in the resulting utilization



Contributions

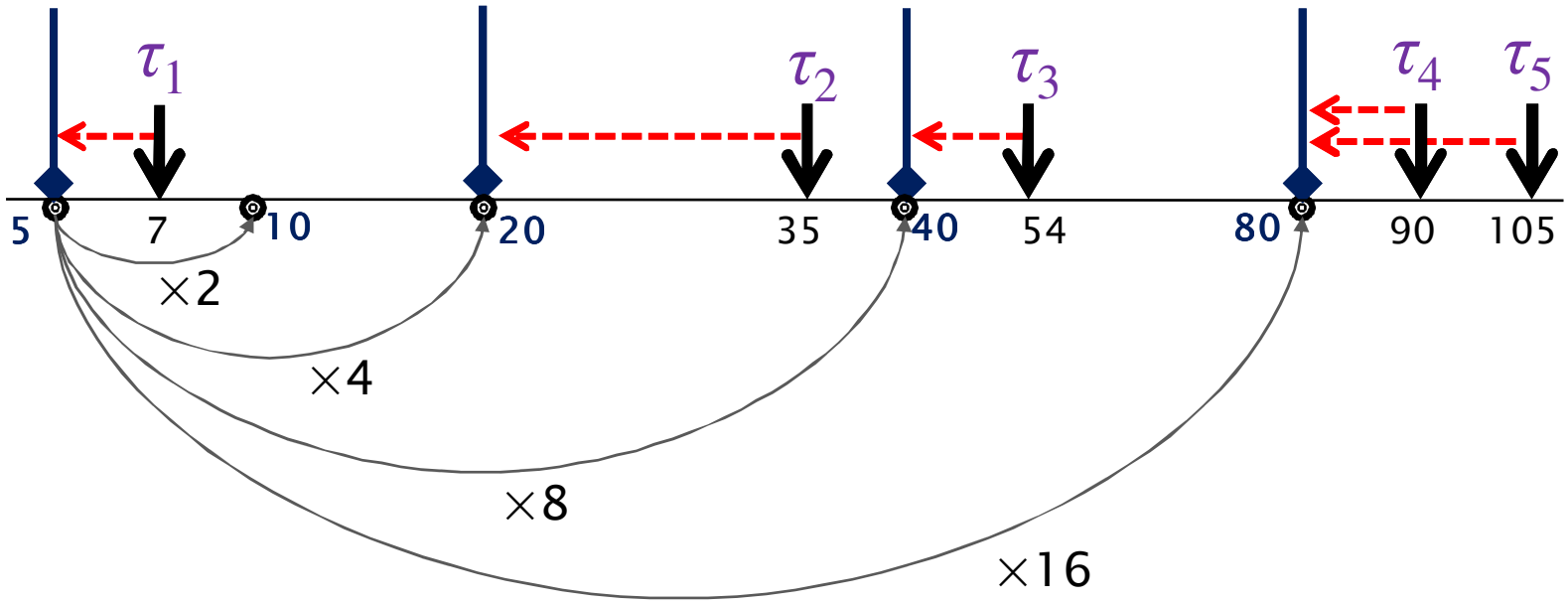
1. A **model** to describe harmonic relations between ranges
2. **Graph** representation of the model
3. Constructing **customized** harmonic relations by pruning the graph
4. Sufficient conditions for existence of a **linear-time solution**
5. **Utilization bound** of the resulting solutions

Agenda

- Introduction and Motivations
- **Related Work**
- Finding Harmonic Relations
- Period Assignment
- Experimental Results
- Conclusion

Schedulability Analysis using Harmonic Periods

- Sr algorithm [Han 1997]



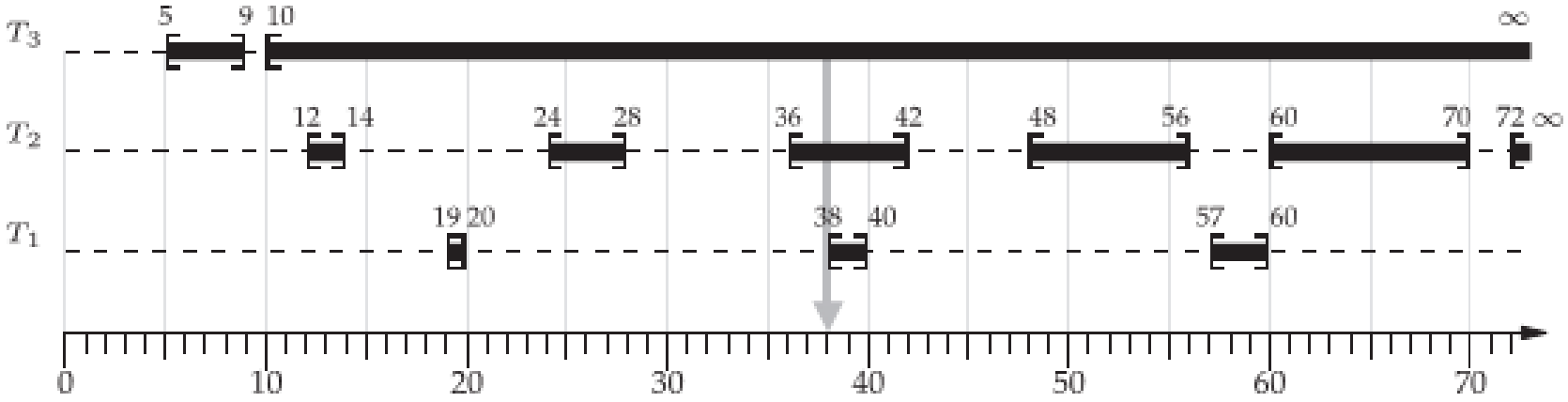
Assuming Period Range in Period Assignment

- Elastic task model
  - Goal: handling overloads
- Parameter assignment in control applications
  - Goal: improving quality of control

- Difference:
  - No utilization bound for the solution
  - None for harmonic periods

Minimizing Hyperperiod

- Ripoll 2013
  - Given Period Ranges, they find the shortest intersection between integer multipliers of the period ranges among all tasks in the system



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- Introduction and Motivations
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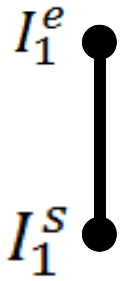
System Model

- Periodic task set
  - A period range for each task
  - Implicit deadline
  - Tasks are indexed according to the T_i^{min}
  - **The goal** is to assign period T_i to each task τ_i
  - An interval is denoted by

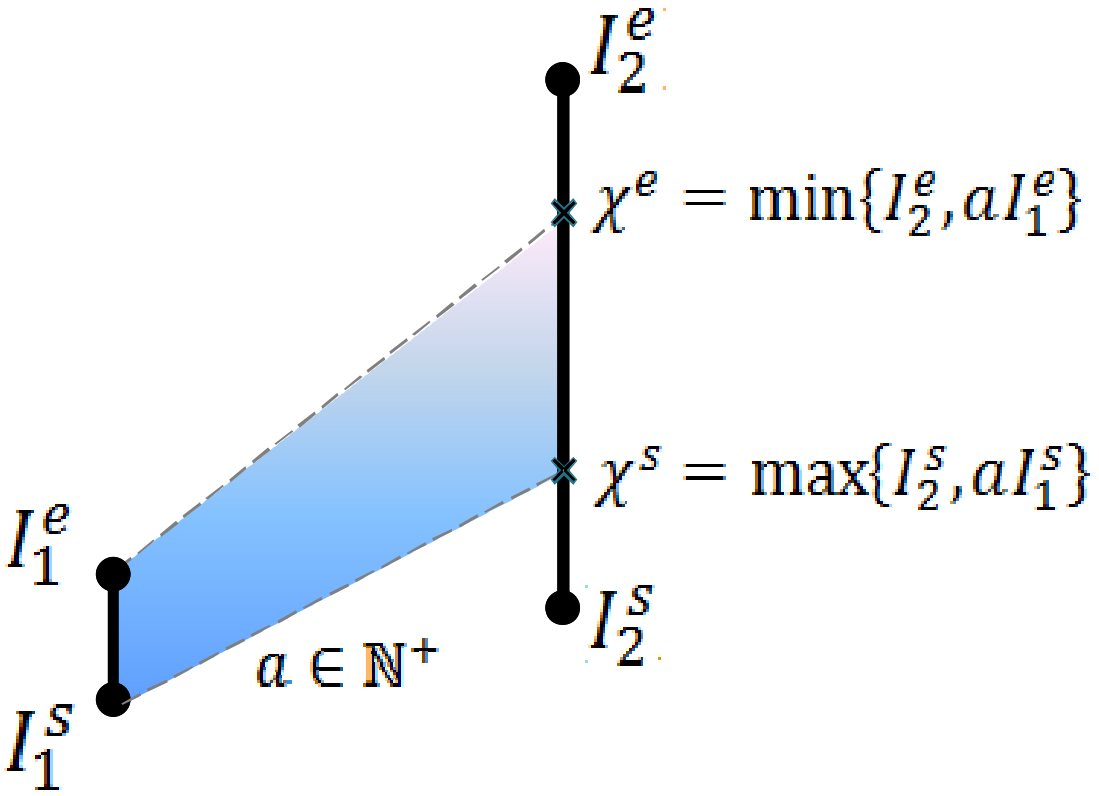
$$\tau = \{\tau_1, \tau_2, \ldots, \tau_n\}$$

$$\tau_i:(c_i, T_i^{min}, T_i^{max})$$



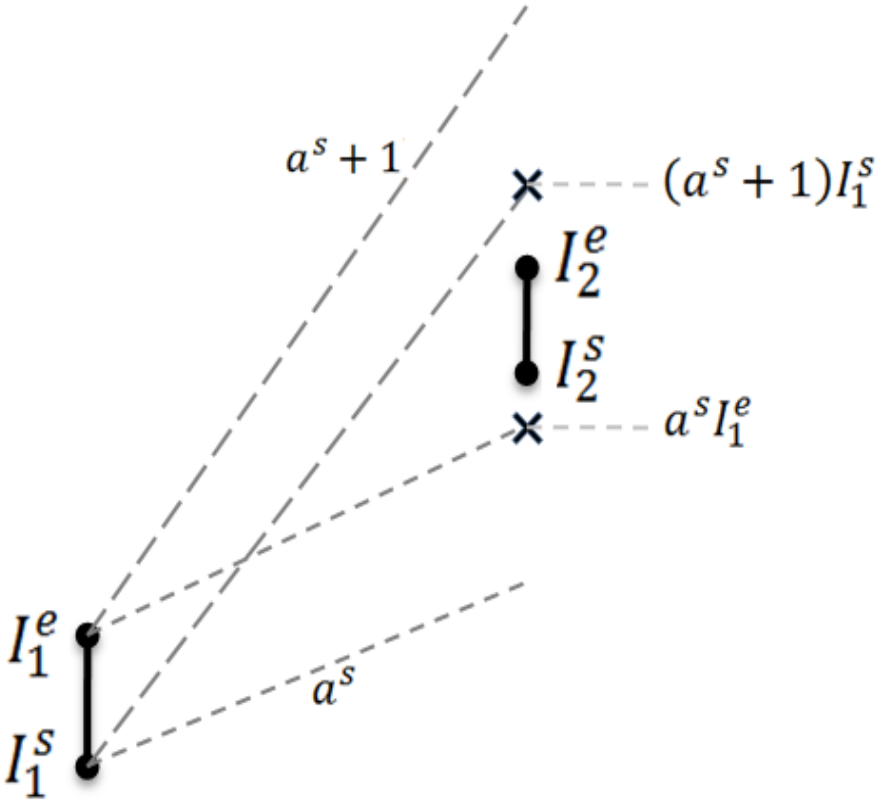
Projected Harmonic Zone

$\chi^a_{I_1 \to I_2}$



Necessary Condition for Projected Harmonic Zones

$$a^s = \left\lfloor \frac{I_2^s}{I_1^e} \right\rfloor$$



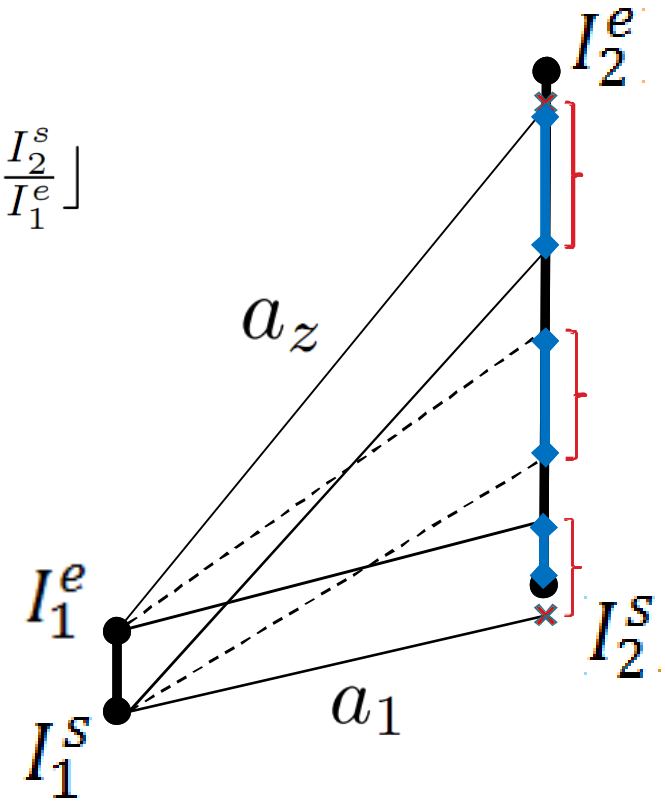
Harmonic Multipliers

$$a_1 = \lfloor \frac{I_2^s}{I_1^e} \rfloor + 1 \quad \text{If } \lfloor \frac{I_2^s}{I_1^e} \rfloor \in \mathbb{N},\ a_1 = \lfloor \frac{I_2^s}{I_1^e} \rfloor$$

$$a_2 = a_1 + 1$$

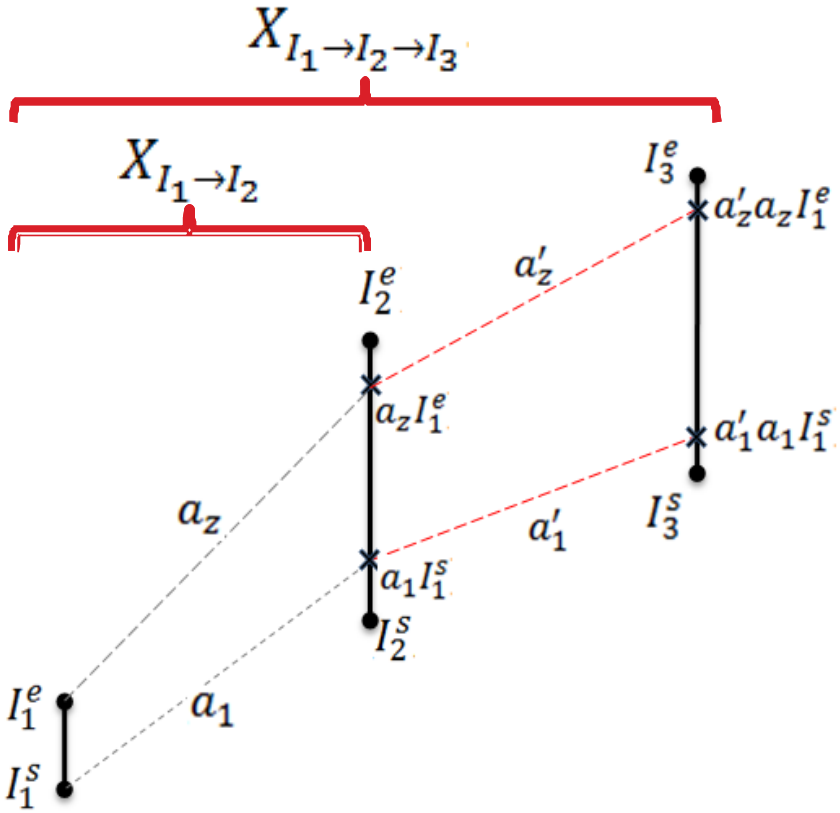
...

$$a_z = \lfloor \frac{I_2^e}{I_1^s} \rfloor$$

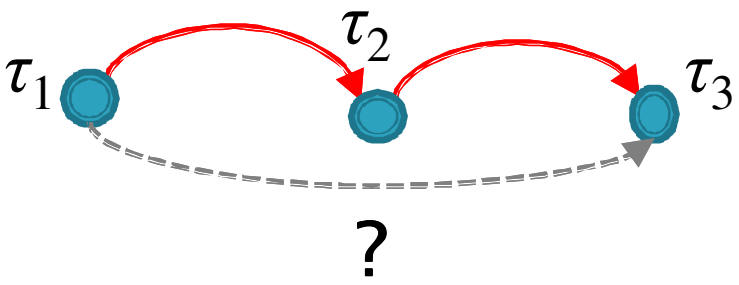


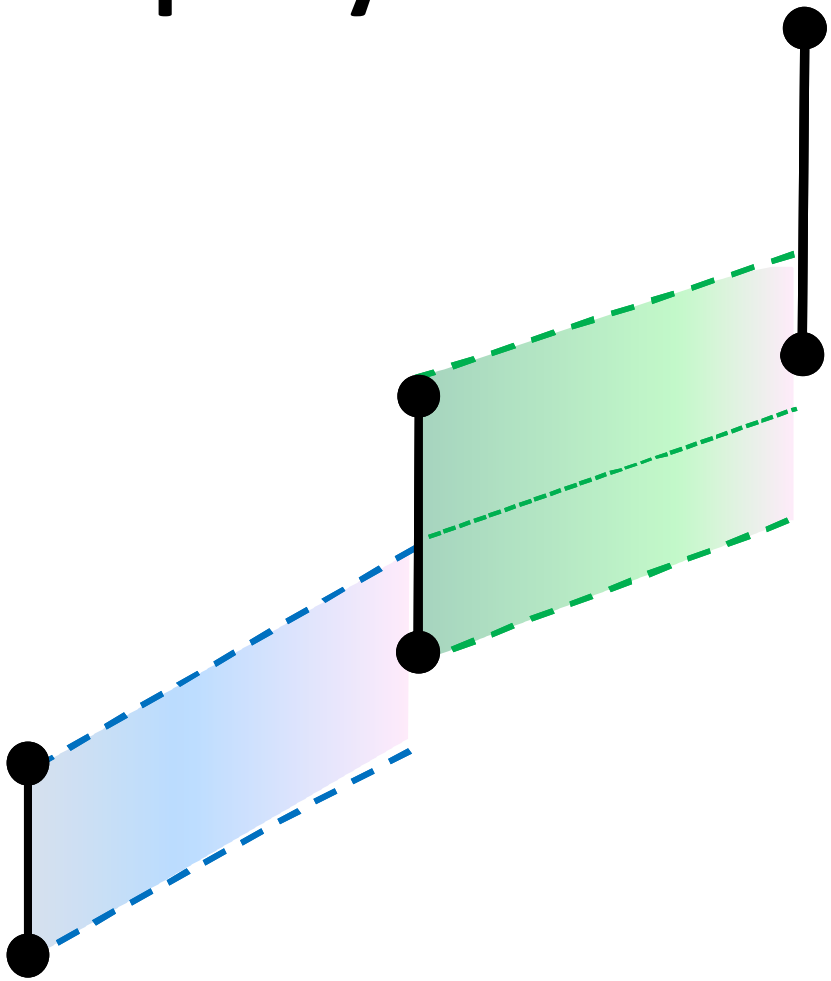
These are the only possible multipliers which can exist

Harmonic Relations of More Tasks

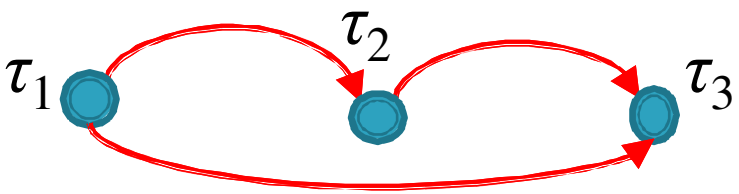


Transitive Closure Property

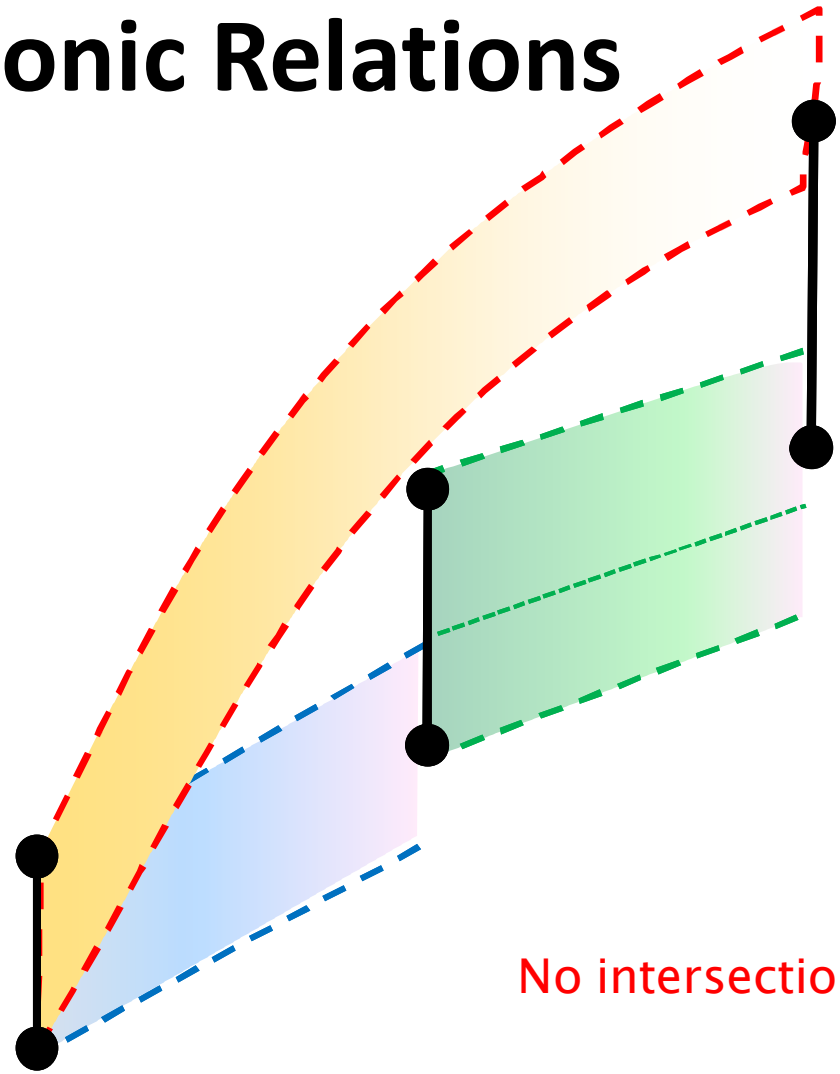




Existence of Harmonic Relations

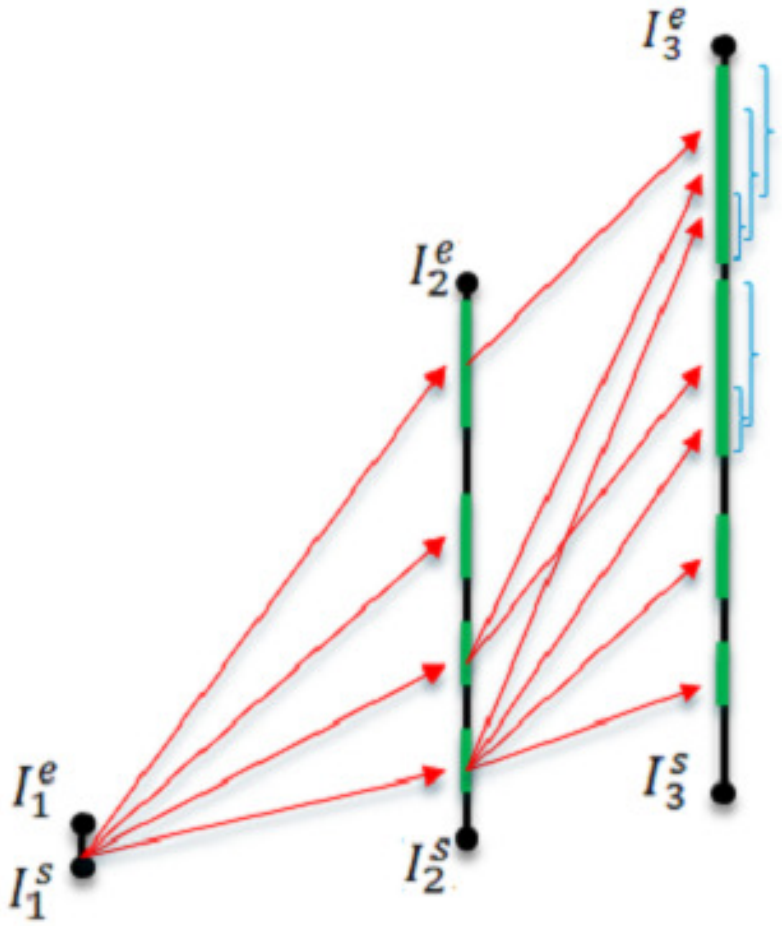


But it does not imply that you have a harmonic relation between all of tasks!

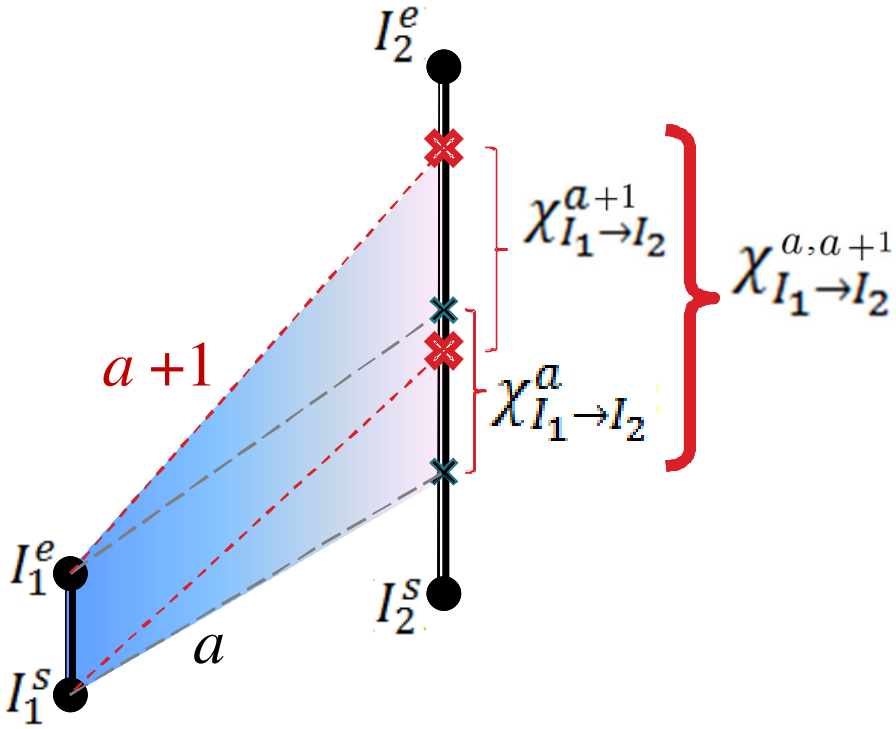


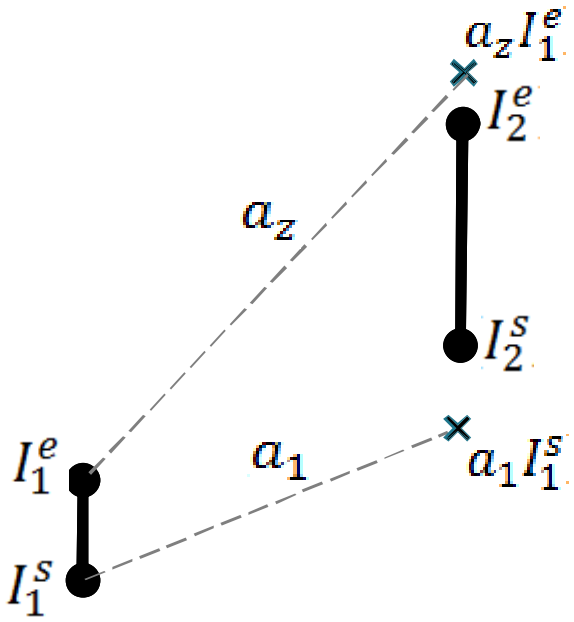
No intersection!

Harmonic Relations of More Tasks



Continuous Harmonic Zones





Tight Harmonic Relation

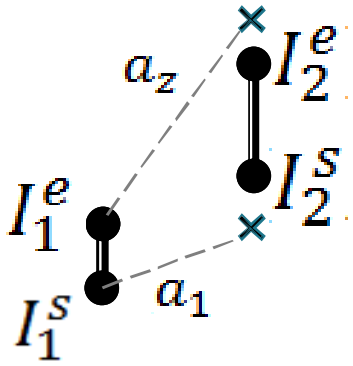
Necessary Conditions for Tight Harmonic Relations

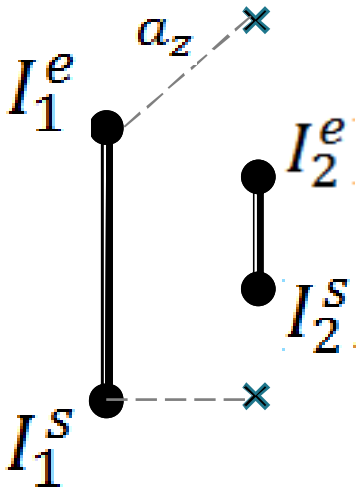
$$a_1(I_1^e - I_1^s) \geq I_1^s$$

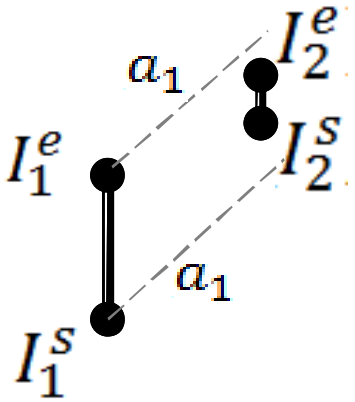
$$\frac{I_1^s I_1^e}{I_1^e - I_1^s} \leq I_2^s$$

$$I_2^e \leq I_1^e$$

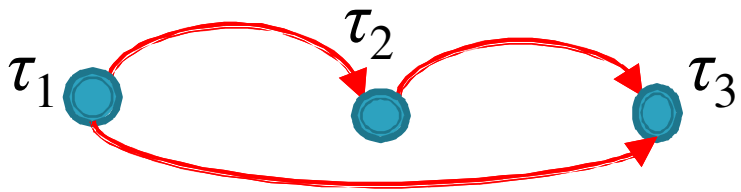
$$a_1 I_1^s \leq I_2^s \leq I_2^e \leq a_1 I_1^e$$

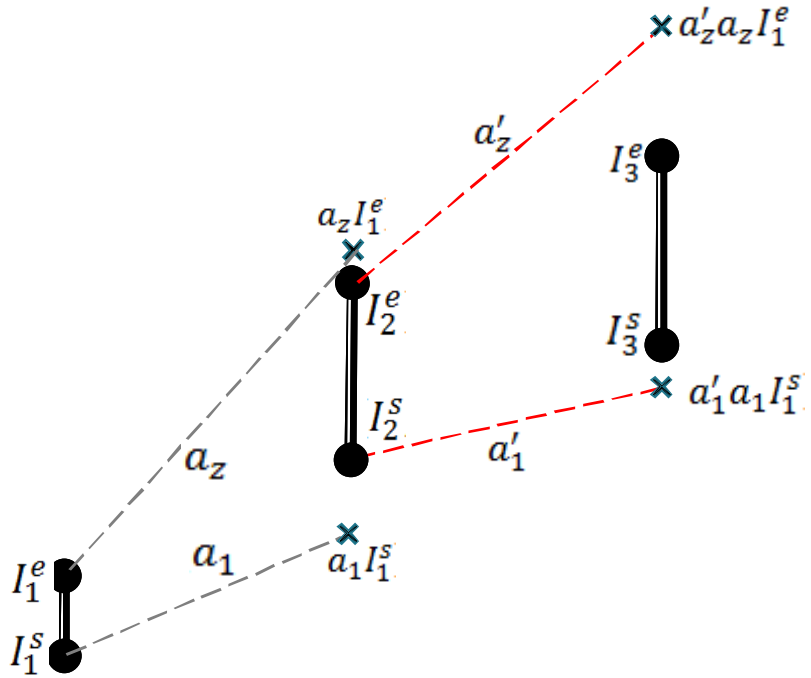






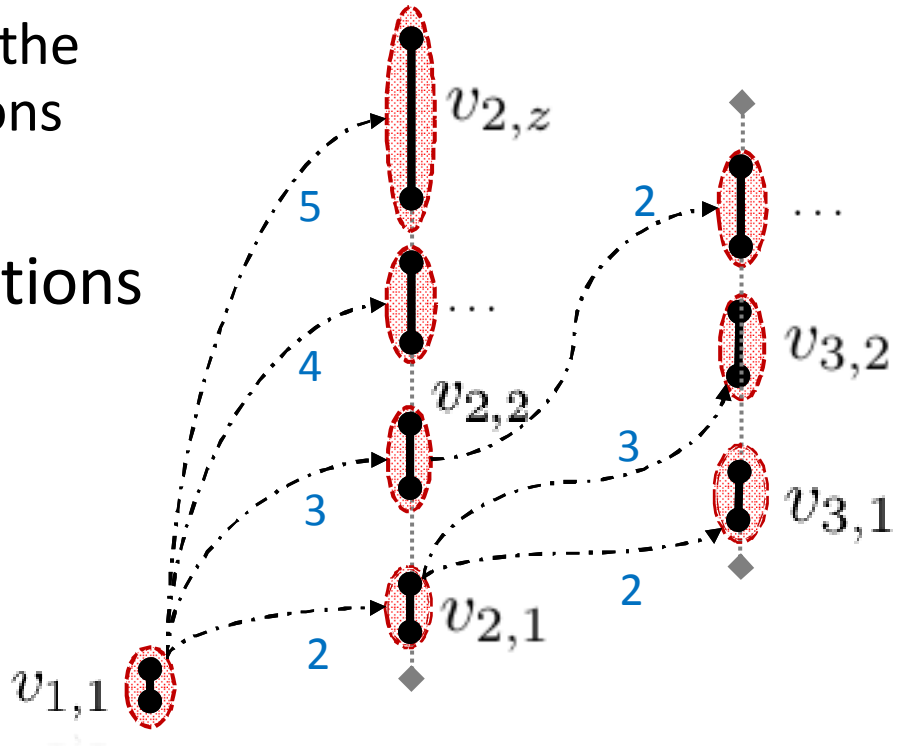
Transitive Closure Property in Tight Harmonic Relations



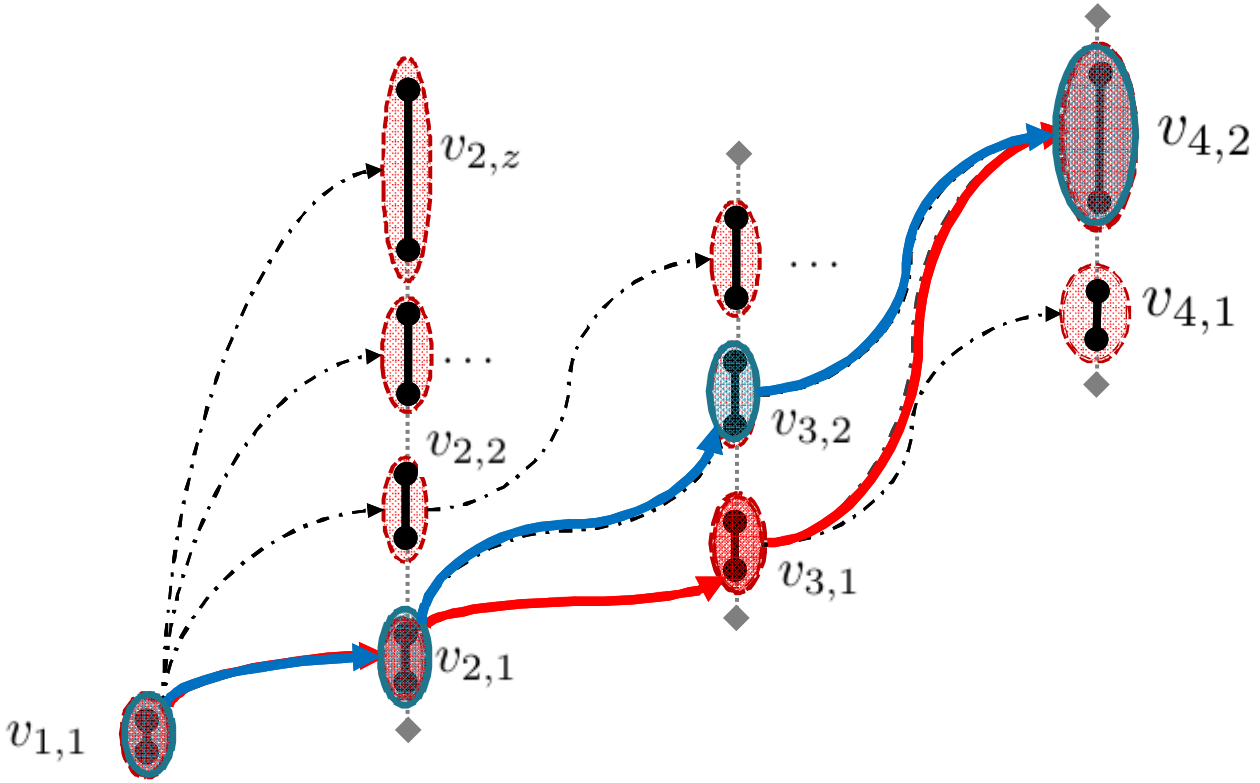


Graph Representation

- GCA algorithm
  - A BFS algorithm to construct the graph of the harmonic relations
- Customized Harmonic Relations by Pruning the Graph



Each Path is a Solution



Linear-time computational complexity for tight harmonic relations

Agenda

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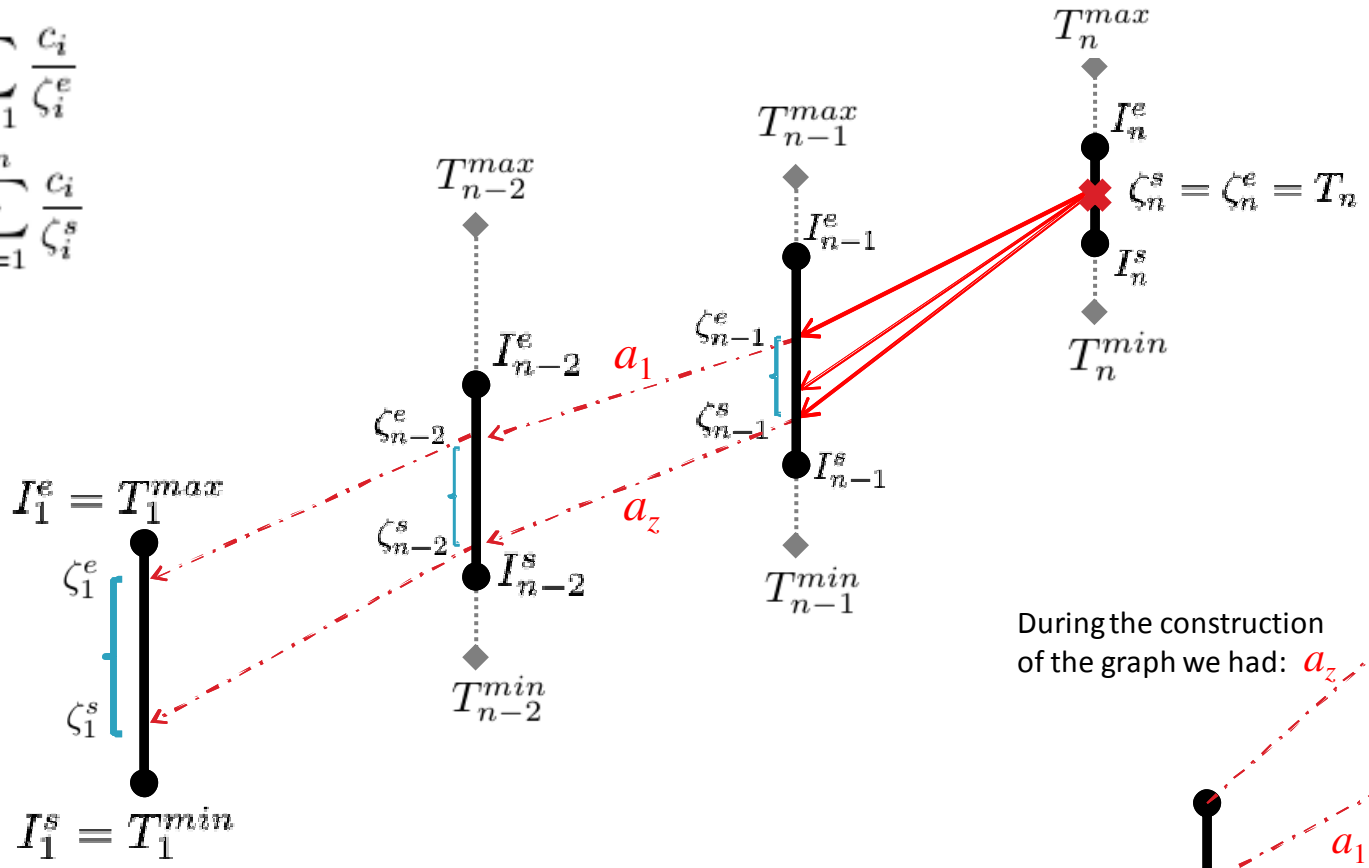
Utilization Bound of a Solution

$$U^{low} = \sum_{i=1}^{n} \frac{c_i}{\zeta_i^e}$$

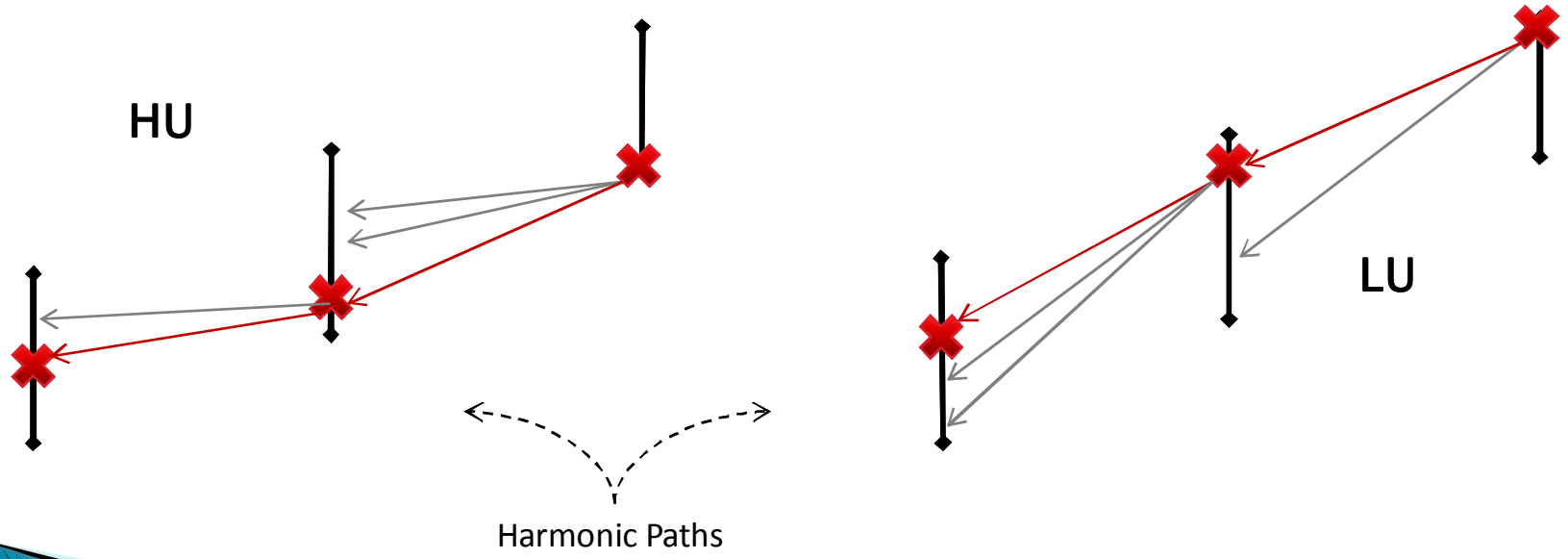
$$U^{high} = \sum_{i=1}^{n} \frac{c_i}{\zeta_i^s}$$



During the construction of the graph we had: a_z

Period Assignment

- Heuristic Algorithms
  - Period assignment with high and low utilization (HU and LU)
  - They might not provide the highest or the lowest utilization.



Agenda

- Introduction and Motivations
- Related Work
- Finding Harmonic Relations
- Period Assignment
- **Experimental Results**
- Conclusion

Experiments

- Algorithms
  - Sr [Han 1997]
  - Optimal period assignment: searches all possible solutions to find maximum feasible utilization (Optimal)
  - Period assignment with High Utilization (HU)
  - Period assignment with Low Utilization (LU)
  - Lower Bound of the utilization
  - Upper Bound of the utilization

LU and HU are heuristic

Experiments

- Workload Model
  - Based on random period ranges (not necessarily harmonic) [Ripoll 2013]

$$T_i^{max} = rand[100, 5000]$$
$$T_i^{min} = T_i^{max}(1 - \sigma)$$
$$\sigma \in [0.3, 0.8]$$

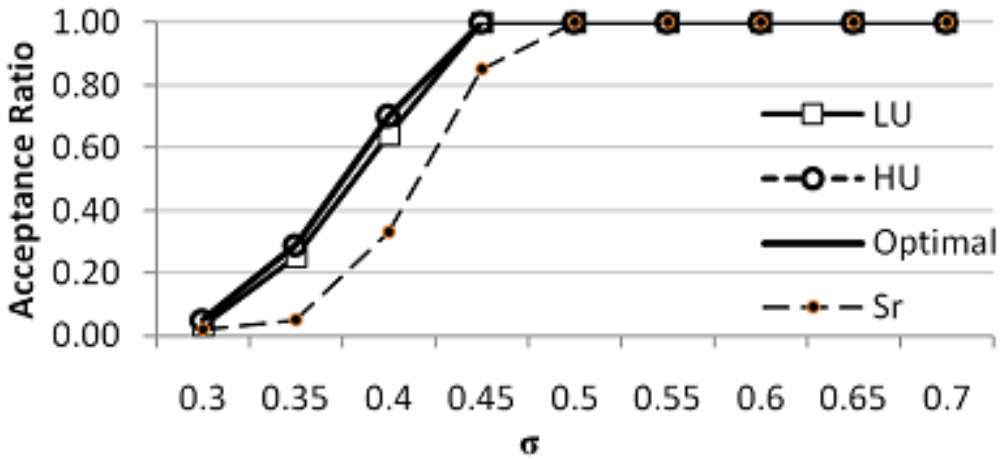
  - More experiments in the paper

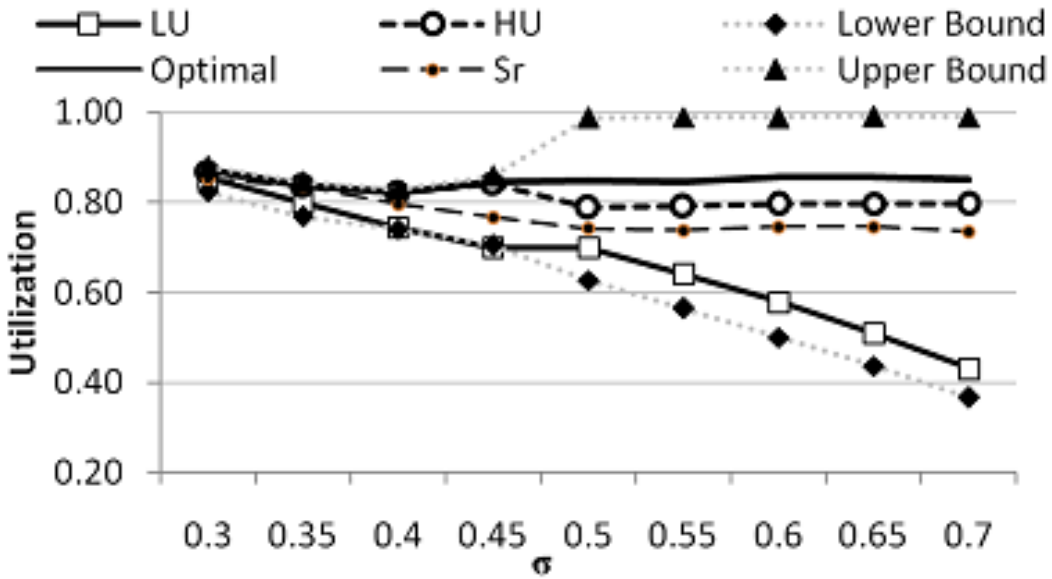
- Target measure
  - Acceptance Ratio: Ratio of accepted fully harmonic task sets to all generated task sets
  - Utilization

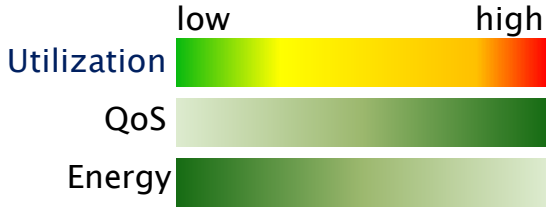
Experiments

Random Tasks with Workload Model [Ripoll13]

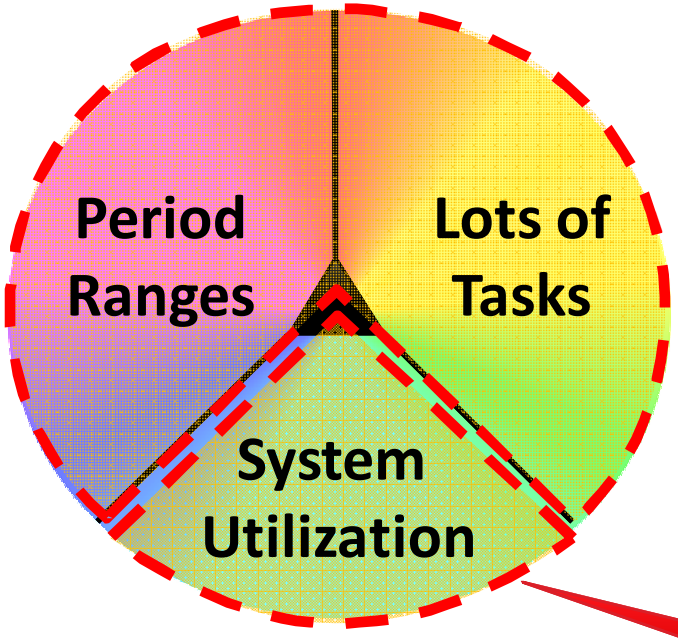
σ shows how wide are period ranges.







Conclusions



Harmonic Periods

A Model

A Graph

Sufficient Conditions of Linear-Time Solution

Customized Harmonic Relations by Pruning the Graph

Utilization Bound of Each Resulting Solution

Future Works

- The error bound of period rounding
- Period assignment algorithm with user defined utilization
- Slack-aware graph pruning algorithm
  - Provides sufficient conditions for optimal scheduling of non-preemptive harmonic tasks

Thank you