

The FMLP⁺

An Asymptotically Optimal Real-Time Locking Protocol for Suspension-Aware Analysis

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Suspension-Based Locking Protocols

Semaphores in POSIX

```
pthread_mutex_lock(...)  
// critical section  
pthread_mutex_unlock(...)
```

semaphore:
a waiting task suspends,
makes processor
available to other tasks

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- Locks cause **priority inversions**
≈ extra delay due to lock contention
- Short: **pi-blocking**

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- For a **specific task set**, what is the **maximum duration of pi-blocking** incurred by each task?

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- For a **specific task set**, what is the **maximum duration of pi-blocking** incurred by each task?

Blocking Optimality

- In general, what is the **maximum duration of pi-blocking** incurred by **any task in any task set**?

Multiprocessor Real-Time Locking Optimality Classes

<p>Blocking Optimality [— & Anderson, 2010]</p>	<p>suspension oblivious</p>	<p>suspension aware</p>
<p>How are suspensions analyzed?</p>	<p>CPU demand is over-approximated</p>	<p>CPU demand is modeled accurately</p>
<p>Advantage</p>	<p>simpler analysis</p>	<p>potentially less pessimistic</p>

[— & Anderson, 2010] *Optimality Results for Multiprocessor Real-Time Locking*, RTSS 2010.

Asymptotically Optimal Locking Protocols

<u>JLFP</u> <i>job-level fixed-priority</i>	<u>Suspension Oblivious</u> Any JLFP Scheduler	<u>Suspension Aware</u> EDF w/ Implicit Deadlines	<u>Suspension Aware</u> Any JLFP Scheduler
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Support for **nested critical sections** added by **RNLP**.
 [Ward & Anderson, 2012]

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Remainder of This Talk

What is Suspension-Aware PI-Blocking?

- Assumptions & quick review

Finding a Suitable Progress Mechanism

- How to deal with lock-holder preemptions

Closing the Suspension-Aware Optimality Gap

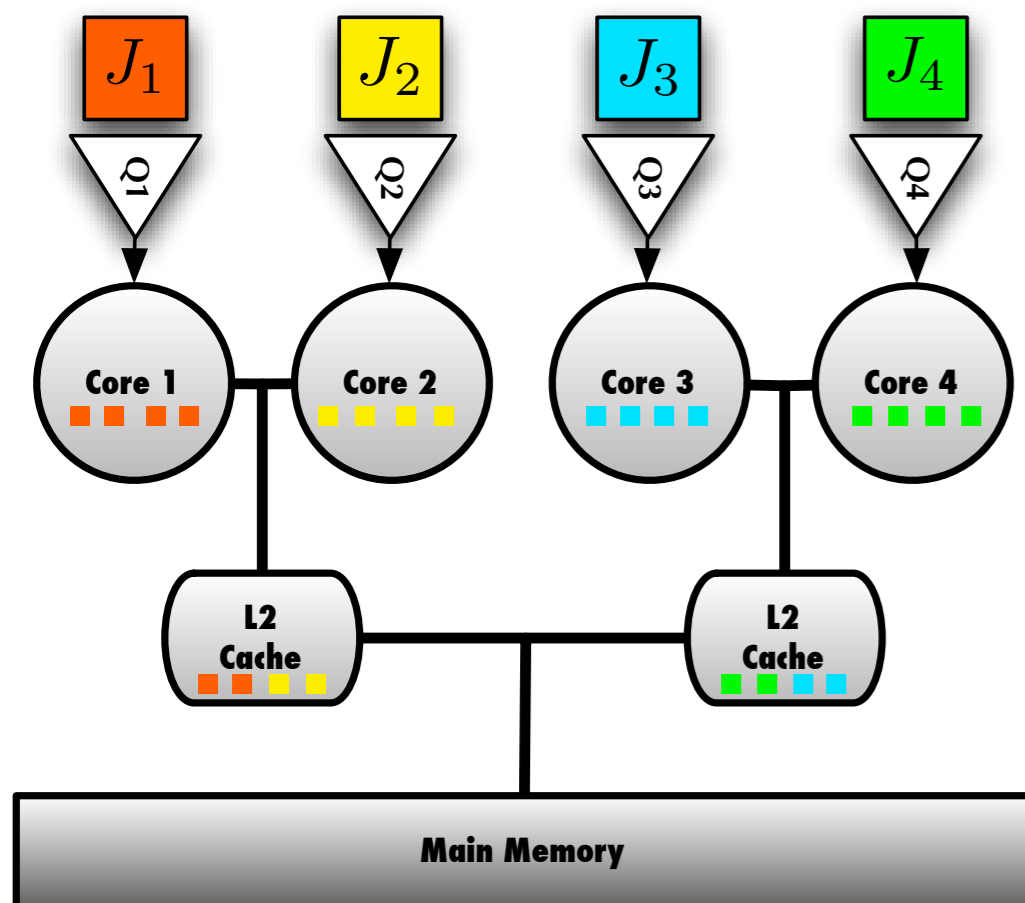
- New progress mechanism:
restricted segment boosting
- Achieving asymptotic optimality with the
generalized FMLP⁺



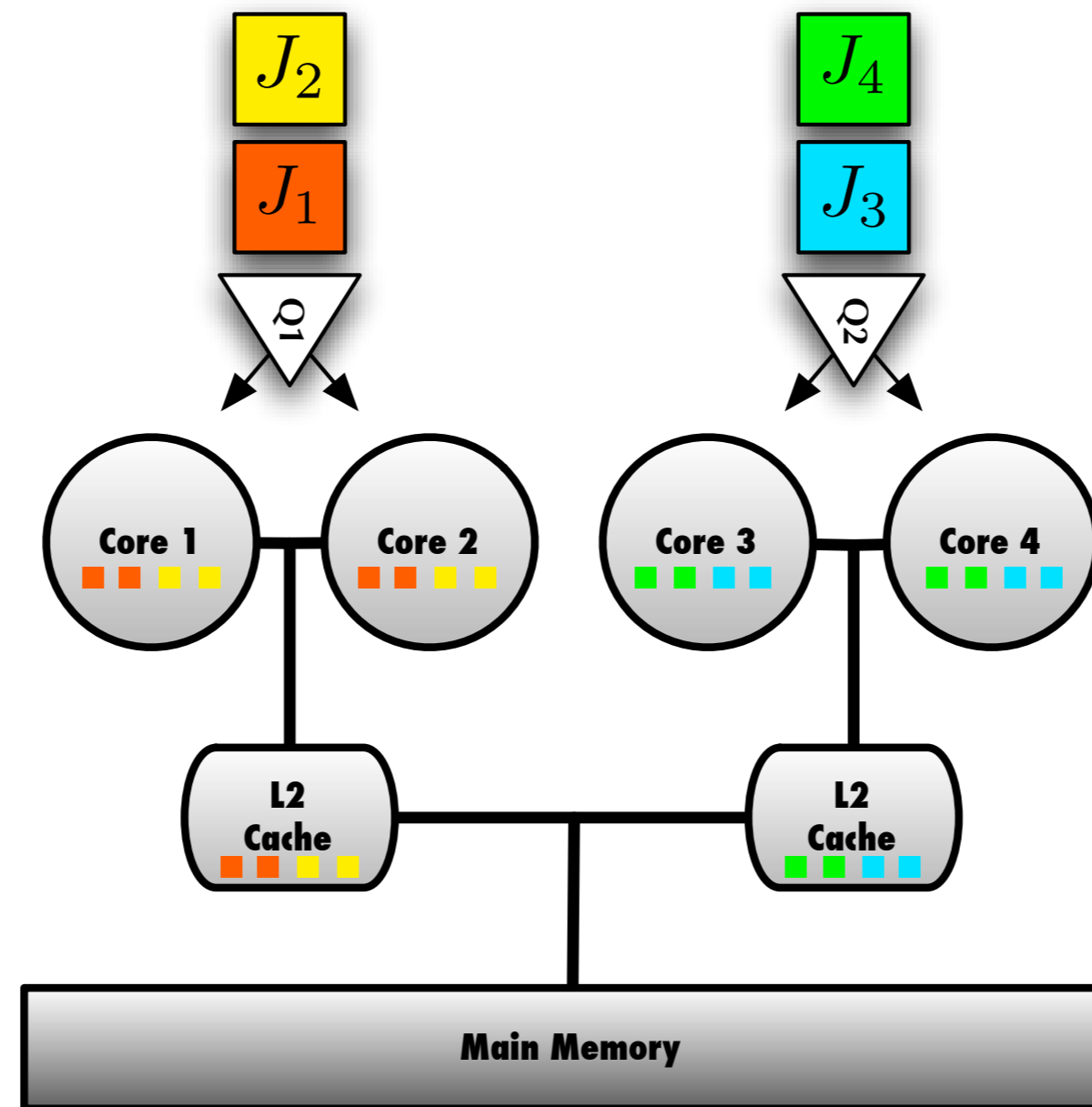
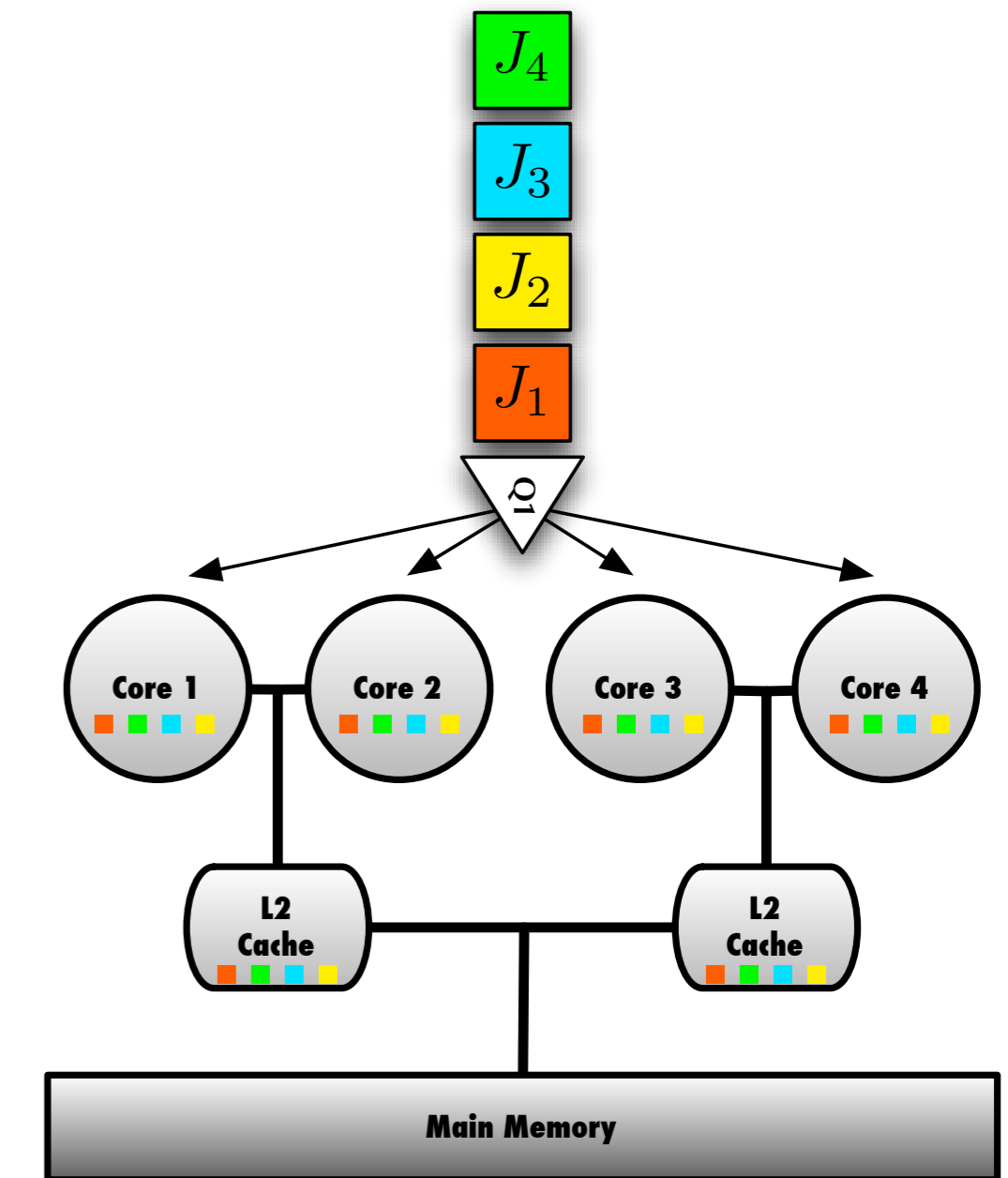
Assumptions & Review of Suspension-Aware PI-Blocking



System Model



partitioned scheduling

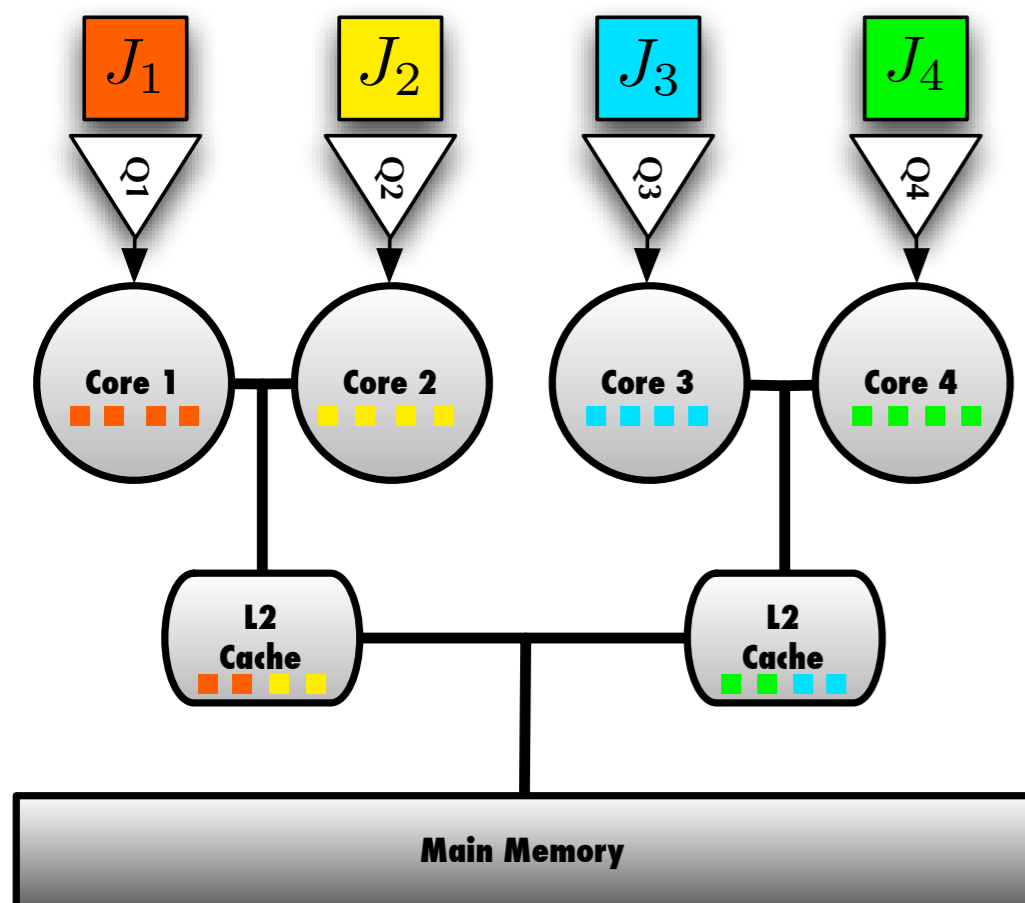
**clustered scheduling**

global scheduling

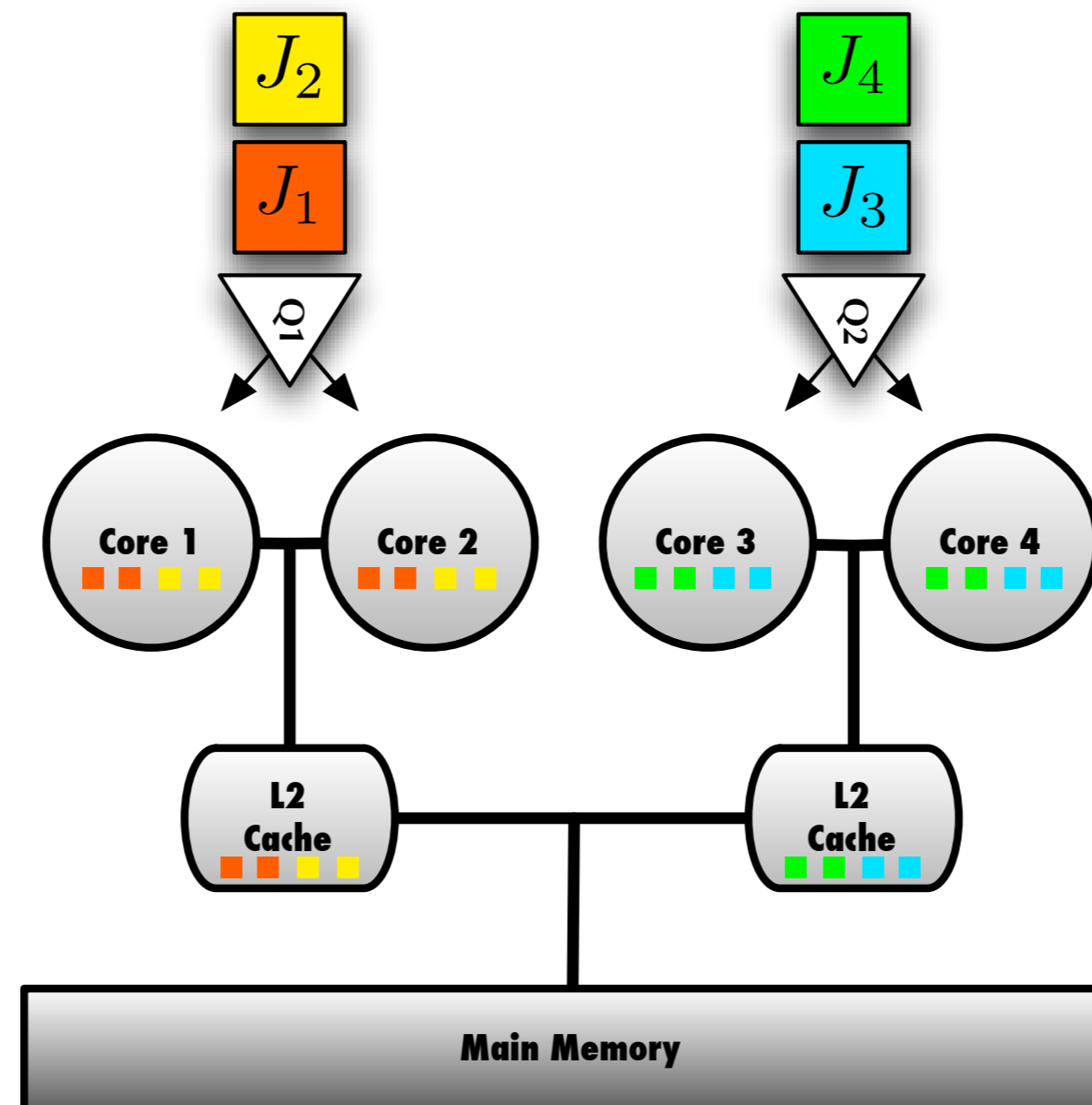
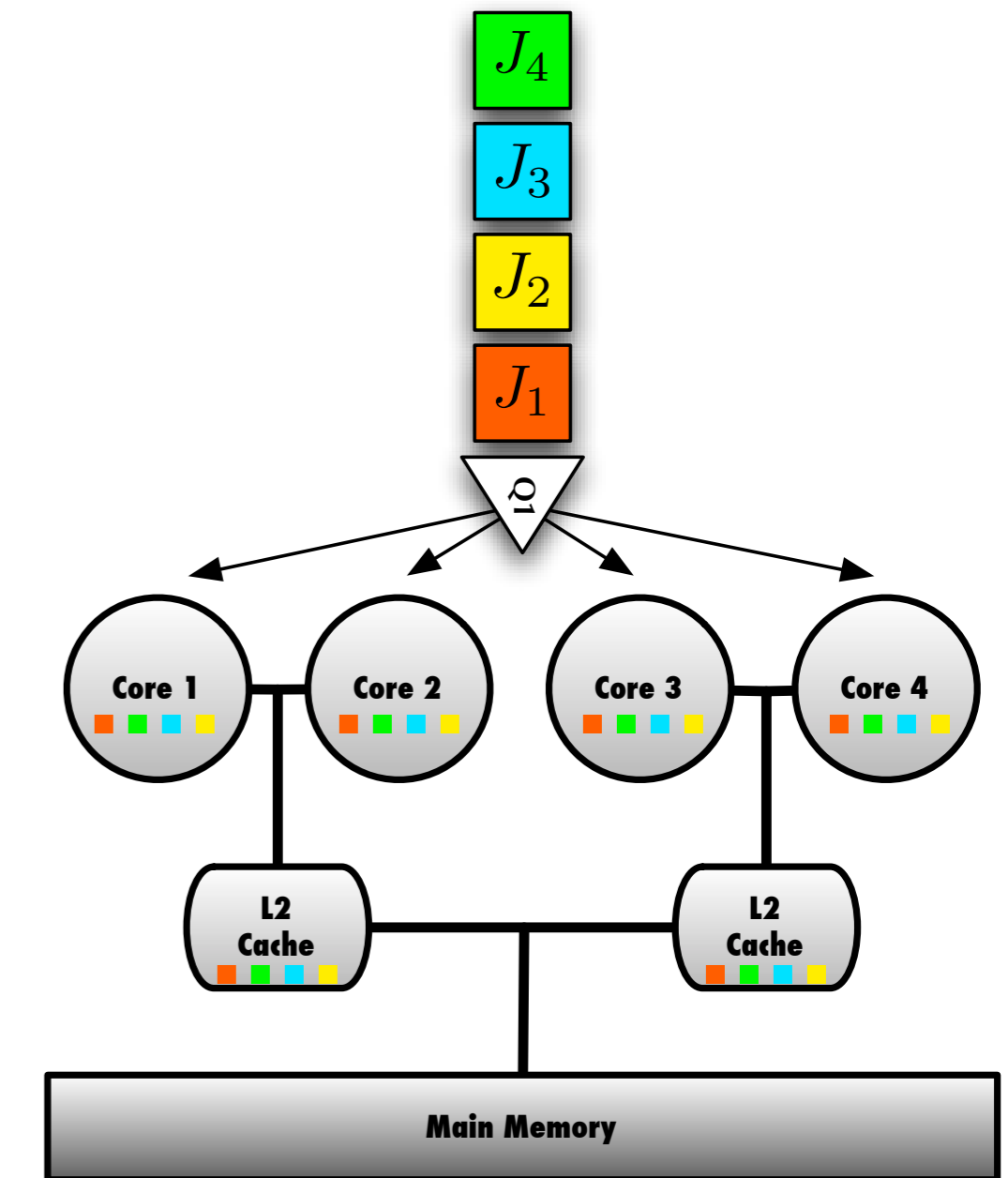
Clustered Scheduling

- ➔ disjoint clusters of processors
 - special cases: partitioned & global
- ➔ *job-level fixed-priority policy* (JLFP)
 - e.g., EDF, static task priorities
- ➔ **cluster size may be non-uniform**

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Sporadic Tasks

- ➔ **arbitrary** deadlines
- ➔ shared resources
 - in the paper: also nested CSs
 - in the talk: only unnested CSs
- ➔ *locking-unrelated self-suspensions*

Definition: S-Aware PI-Blocking

A job J assigned to a cluster with c CPUs incurs s-aware pi-blocking at a time t iff

- (1) J is **not scheduled** at time t , and
- (2) fewer than c **higher-priority jobs** are scheduled.

[— & Anderson, 2010]

Intuition

- ➔ Locking-related delays are **not problematic** iff J would **not** have been scheduled anyway...

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Maximum PI-Blocking

b_i — bound on max. pi-blocking incurred by task T_i
 $\max \{b_i\}$ — **maximum pi-blocking** of any task in task set

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There exist task sets such that under s-aware analysis

$$\max \{b_i\} = \Omega(n)$$

*under **any** suspension-based locking protocol.*

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→ $O(n)$ max. s-aware pi-blocking is asymptotically optimal.

Objective

Define a locking protocol such that

$$\max \{b_i\} = O(n)$$

for **any task set** under **any clustered JLFP scheduler**.

Need to define queue order

→ FIFO works

Need to define progress mechanism

→ To deal with risk of lock-holder preemption




→ Ensure timely completion of critical sections

→ Classic example: priority inheritance

Finding a Suitable Progress Mechanism



Progress Mechanism Choices

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Progress Mechanism Choices

(1) The classic choice: (variants of) **priority inheritance**.

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(2) The partitioned & clustered choice: (variants of) **priority boosting**.

Sub-Optimality of Priority Inheritance

*It is **impossible to construct**
an **asymptotically optimal** locking protocol
(w.r.t. s-aware analysis)
under global JLFP scheduling
based on **priority inheritance**.*

[—, 2011]

(And hence also under clustered JLFP scheduling.)

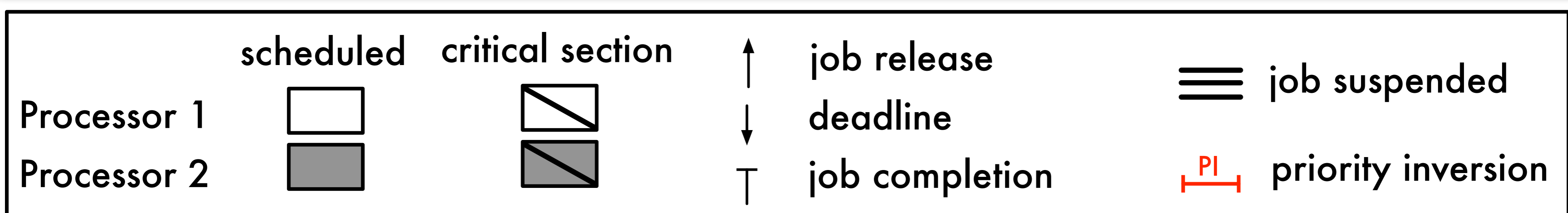
Sub-Optimality of Priority Inheritance

Priority Inheritance Example Schedule

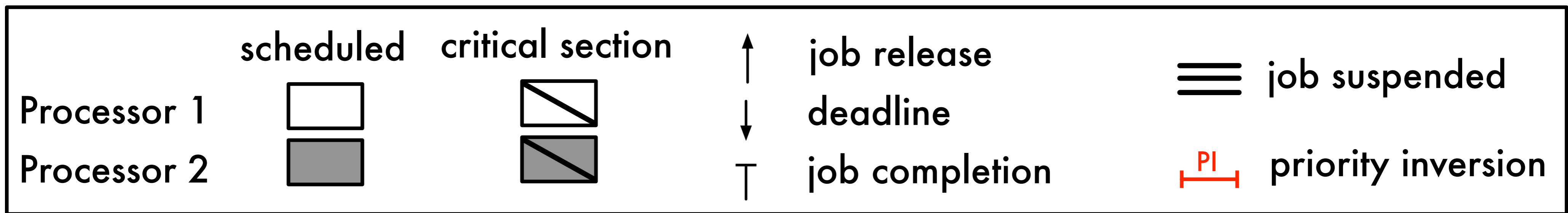
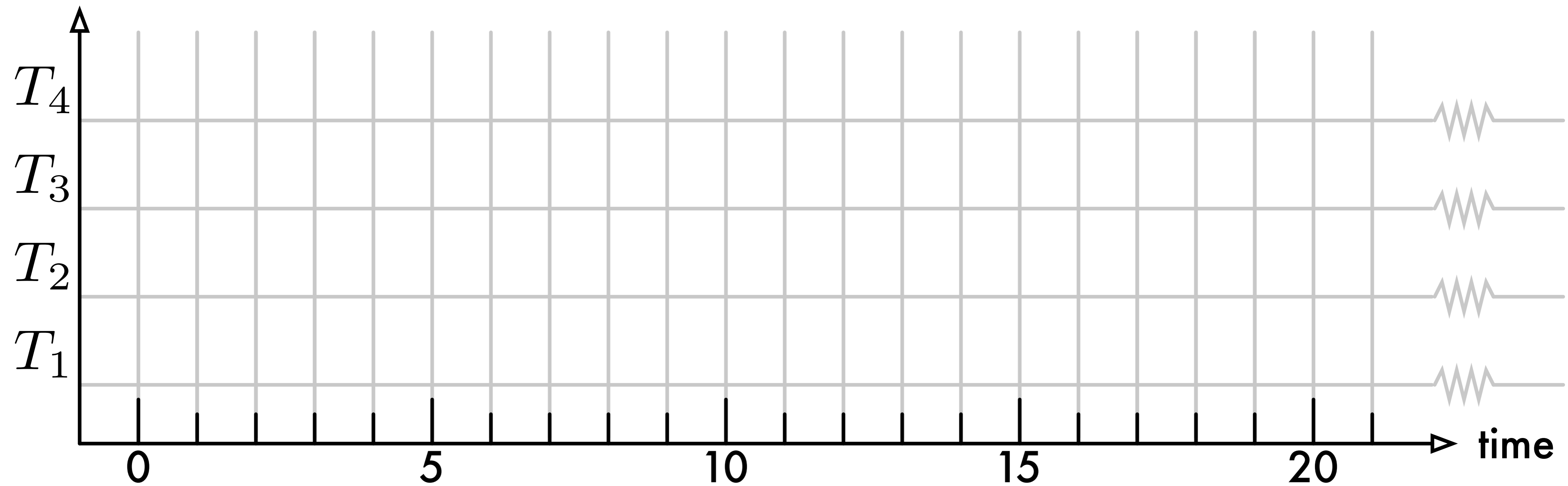
4 tasks on 2 processors

global fixed-priority scheduling

Task priorities (high to low): $T_1 > T_2 > T_3 > T_4$

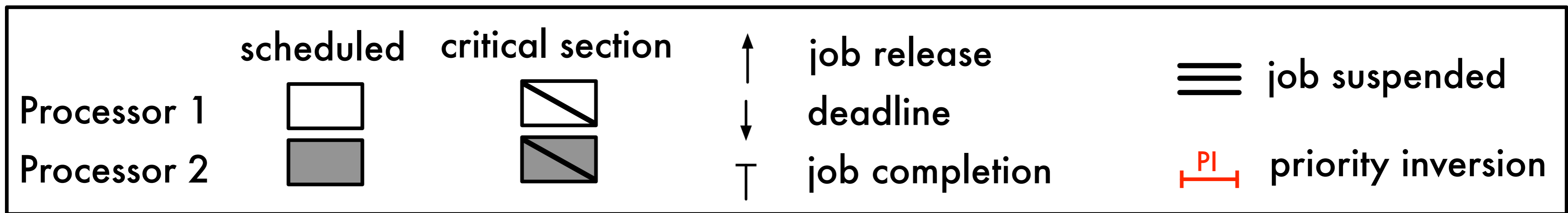
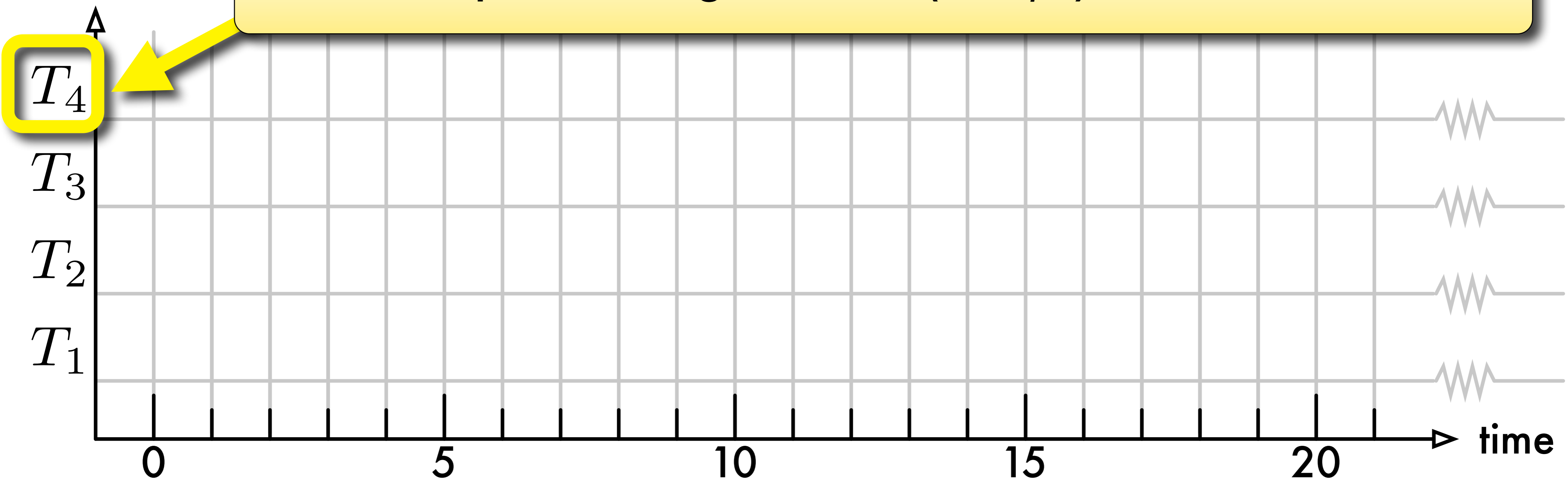


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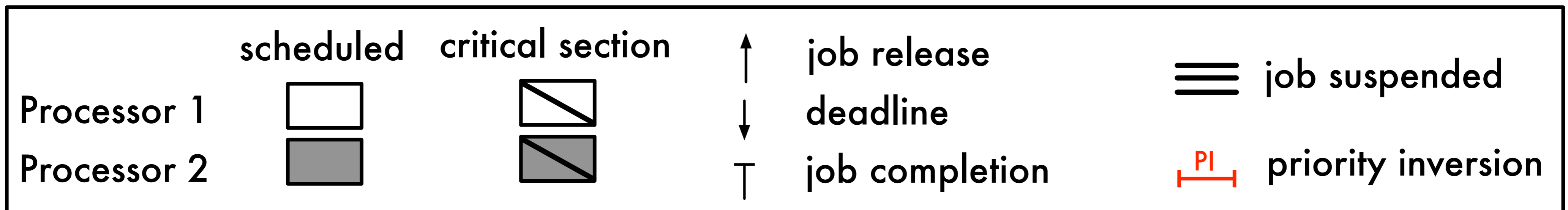
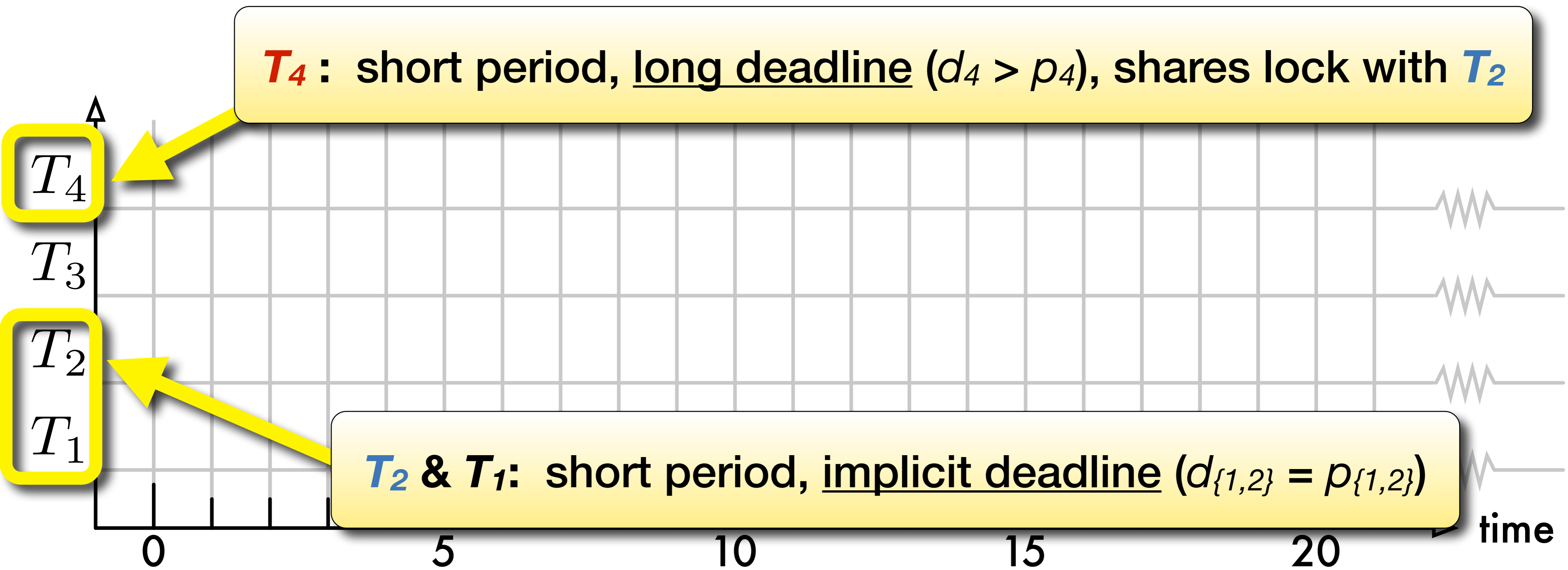


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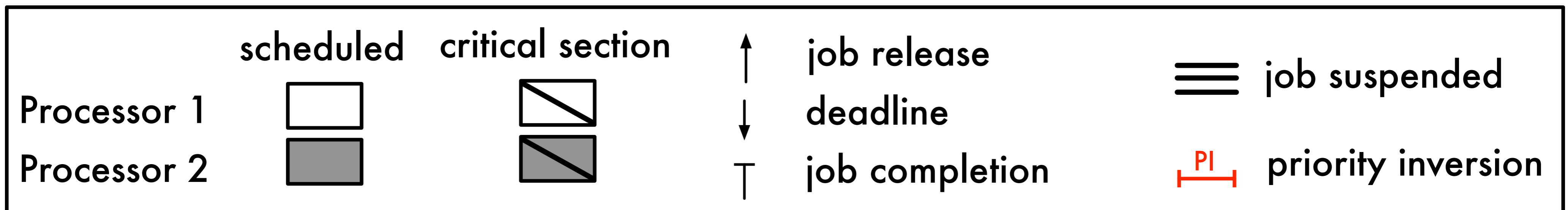
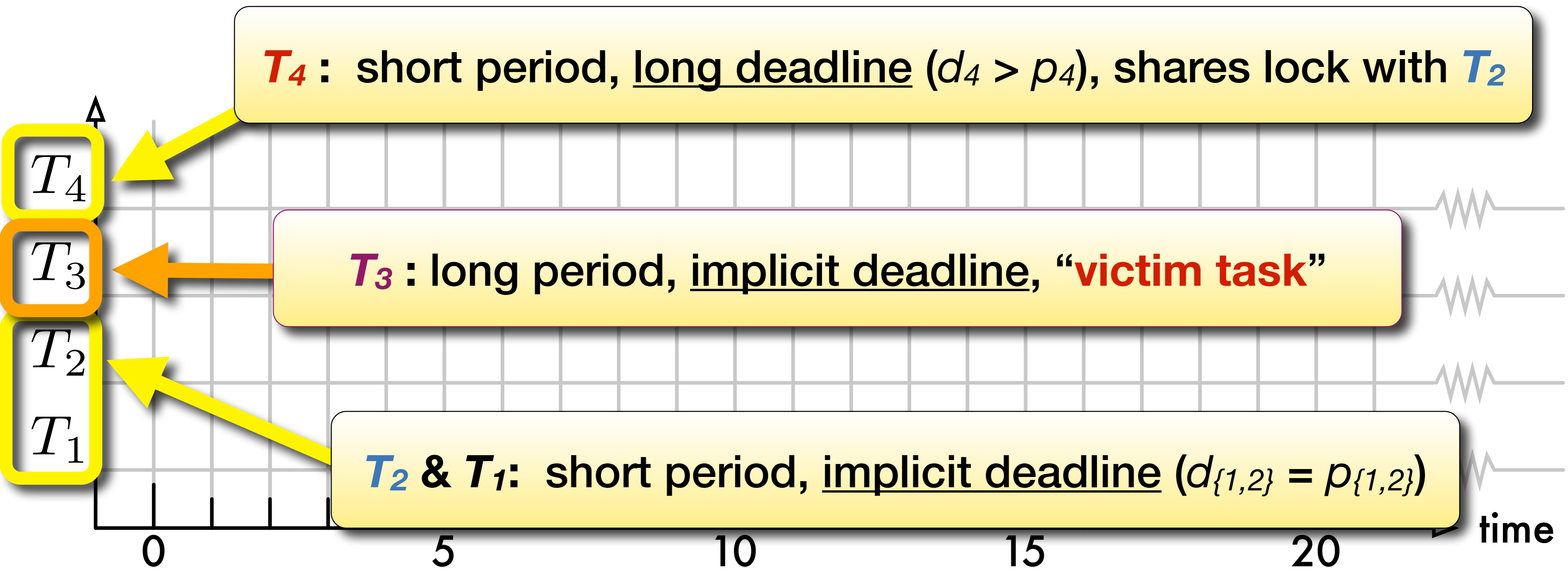
T_4 : short period, long deadline ($d_4 > p_4$), shares lock with T_2



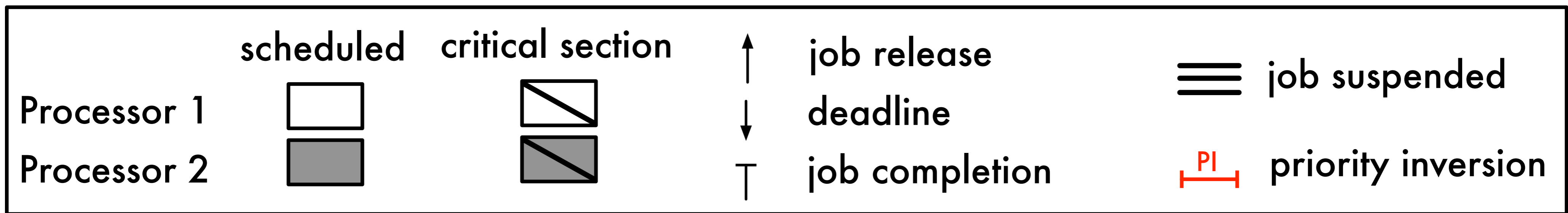
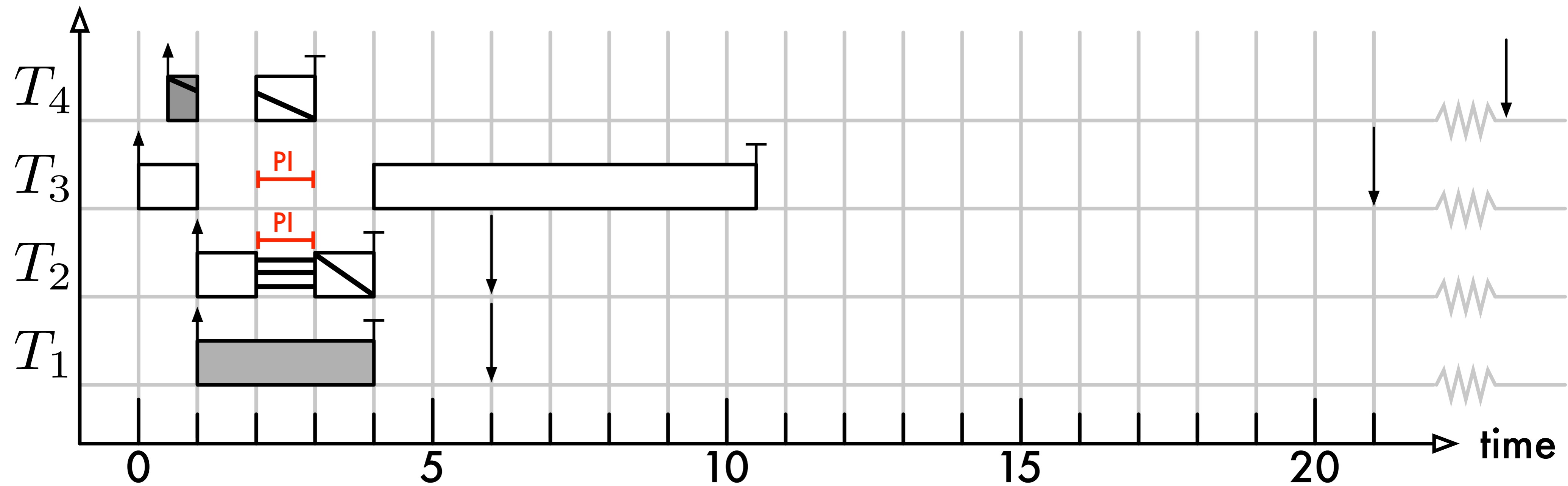
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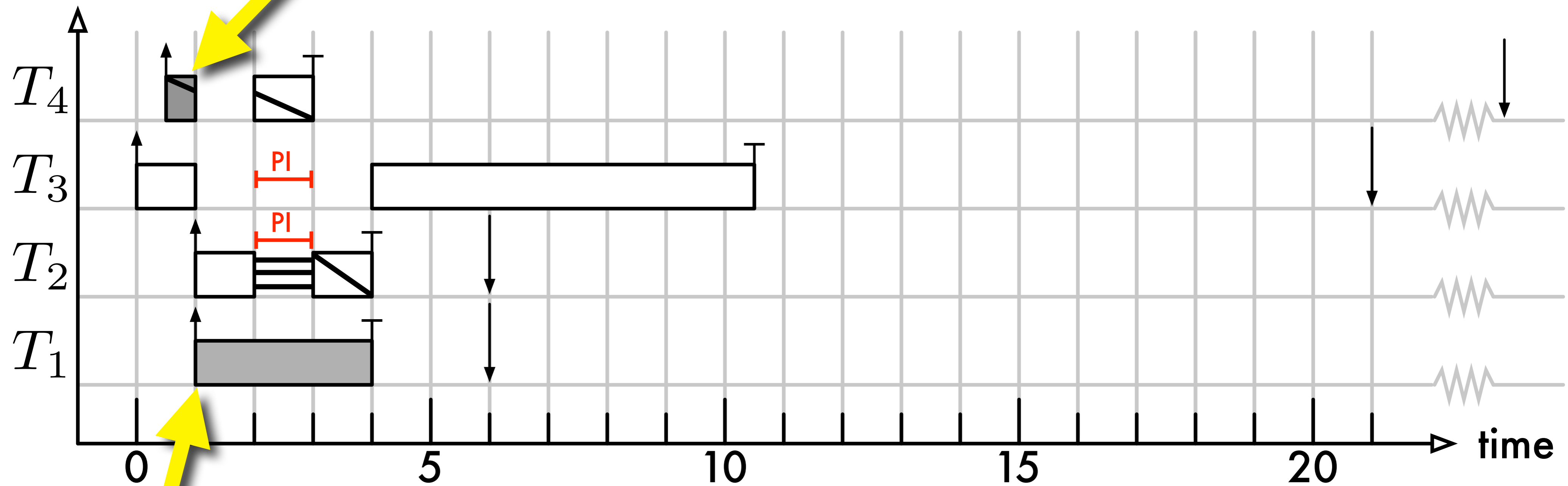


Independent Job Incurs Priority Inversion



Independent Job Incurs Priority Inversion

T_4 acquires lock first...

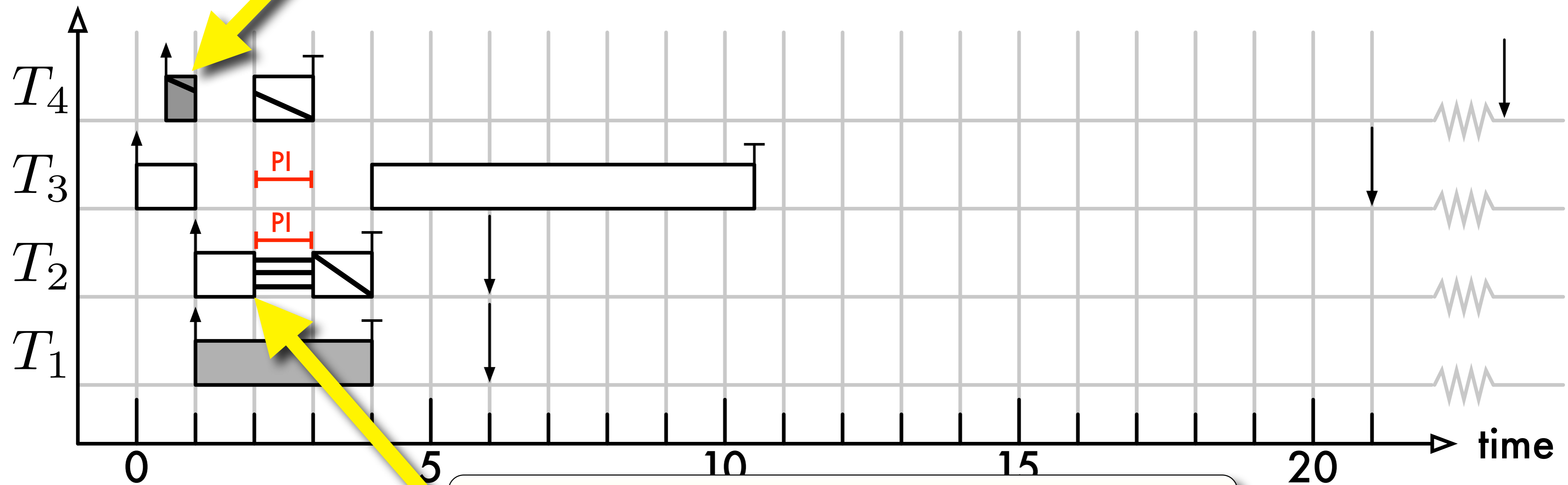


	scheduled	critical section	↑	job release	≡	job suspended
Processor 1			↓	deadline		priority inversion
Processor 2			┐	job completion		








...but is **preempted** by arrival of higher-priority jobs.

Independent Job Incurs Priority Inversion

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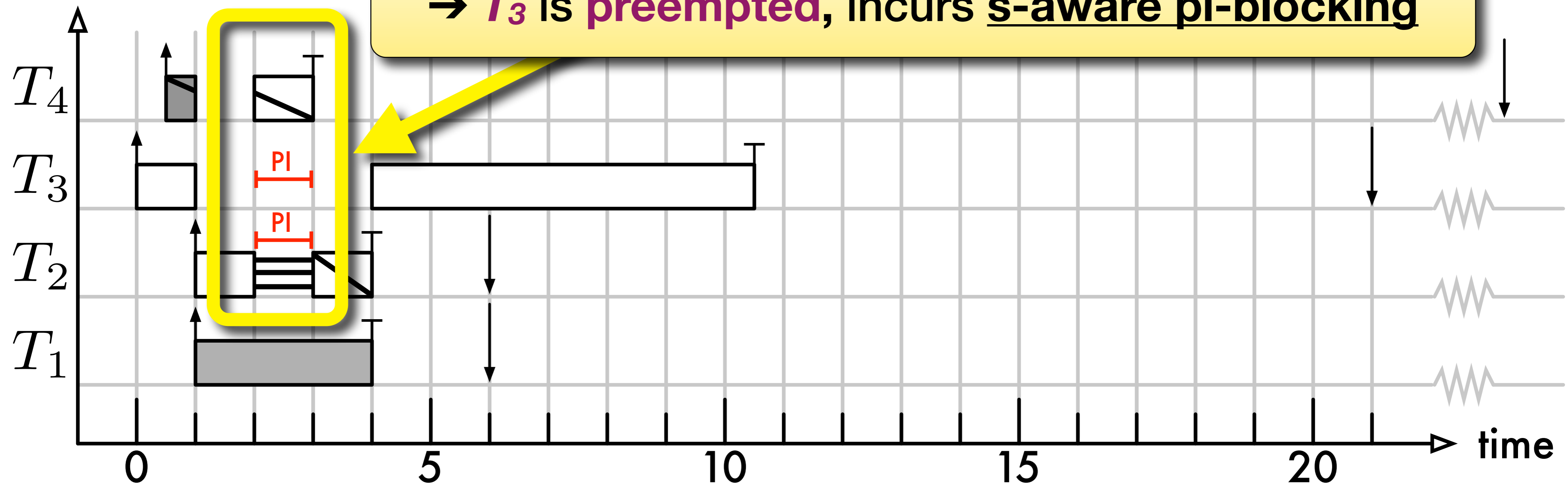


...and T_2 also requires the lock.

Processor 1		scheduled		job suspended
Processor 2				
				deadline
				job completion
				priority inversion

Independent Job Incurs Priority Inversion

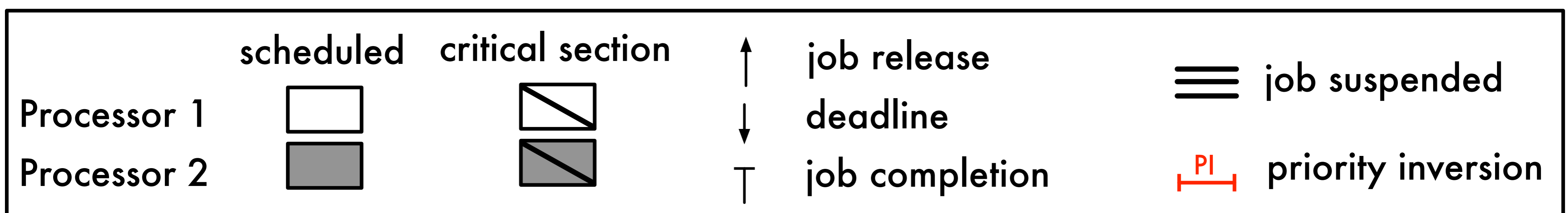
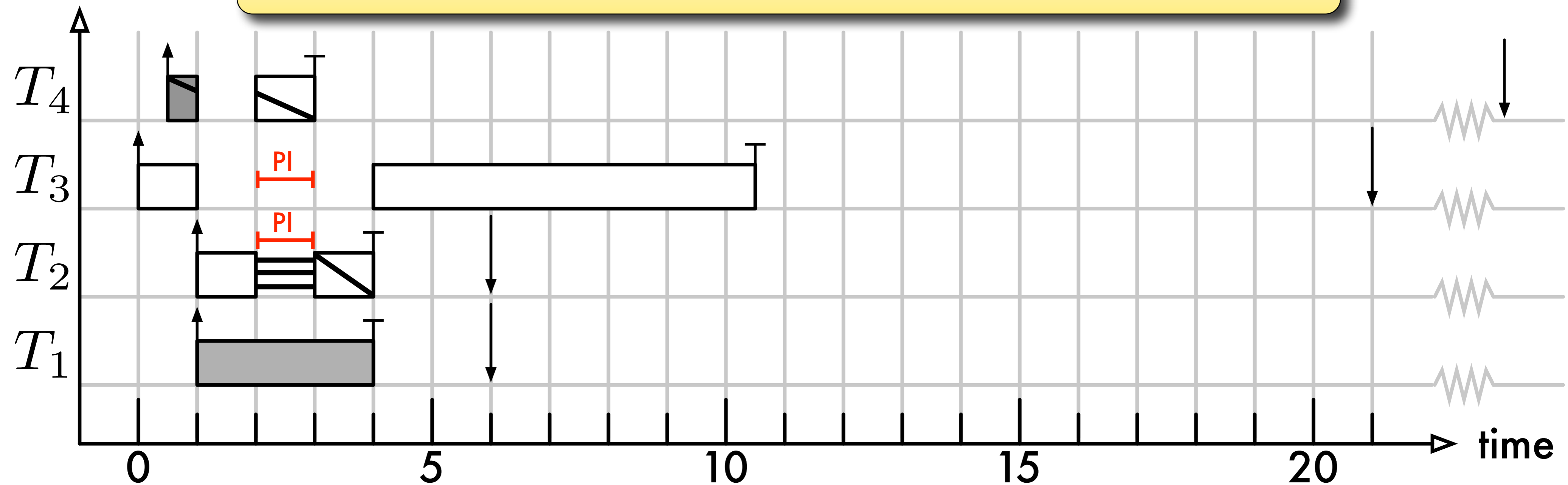
Priority inheritance: T_4 inherits from T_2
→ T_3 is preempted, incurs s-aware pi-blocking



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Processor 1			↓	deadline		
Processor 2			┐	job completion		priority inversion

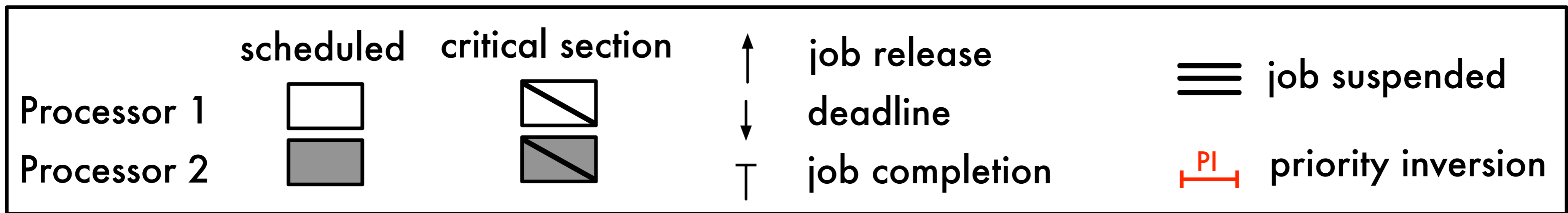
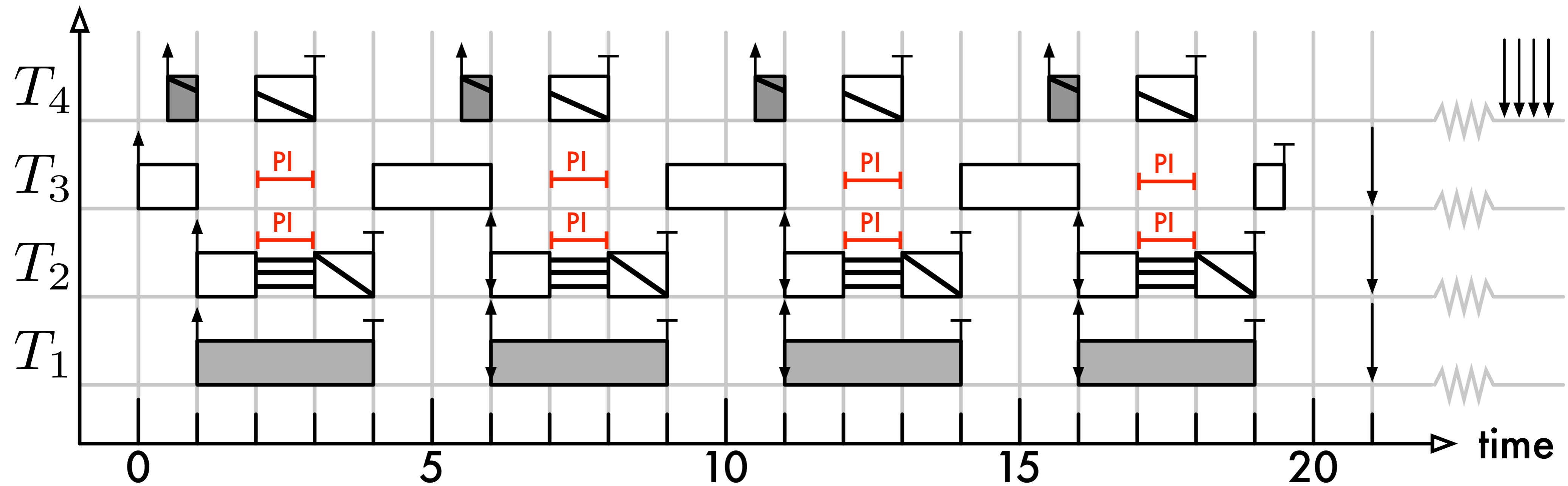
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How often can this scenario repeat?



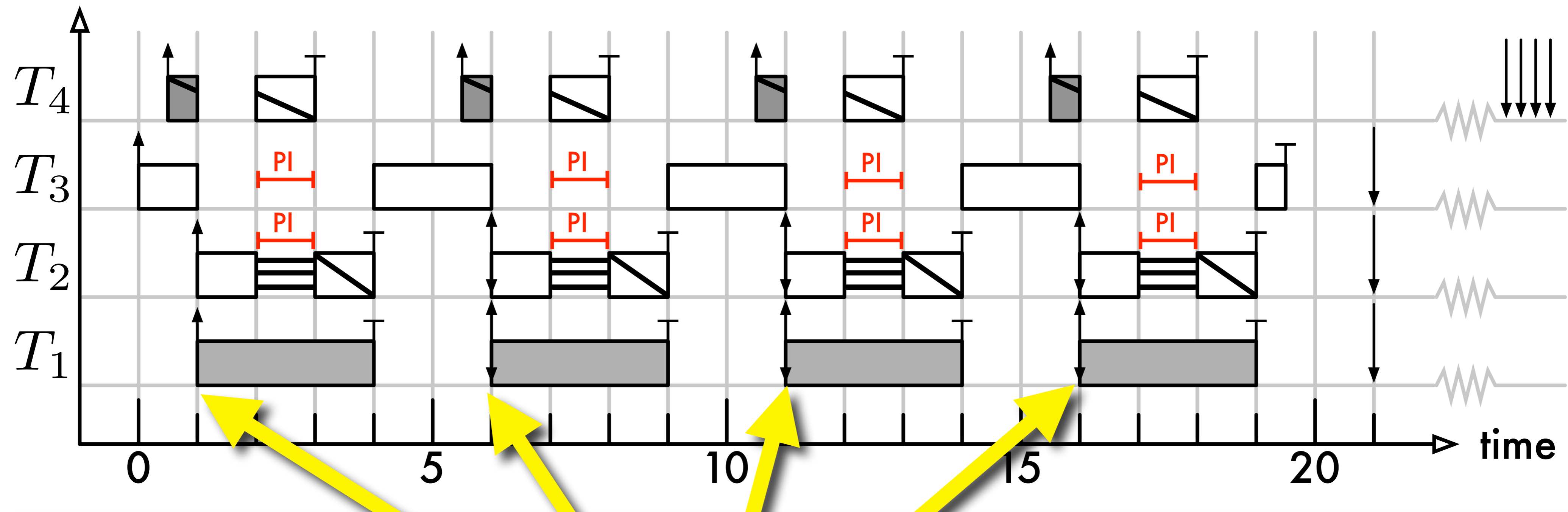
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$\Omega(\phi)$ pi-blocking is possible, where $\phi = \{\text{max response time}\} / \{\text{min period}\}$



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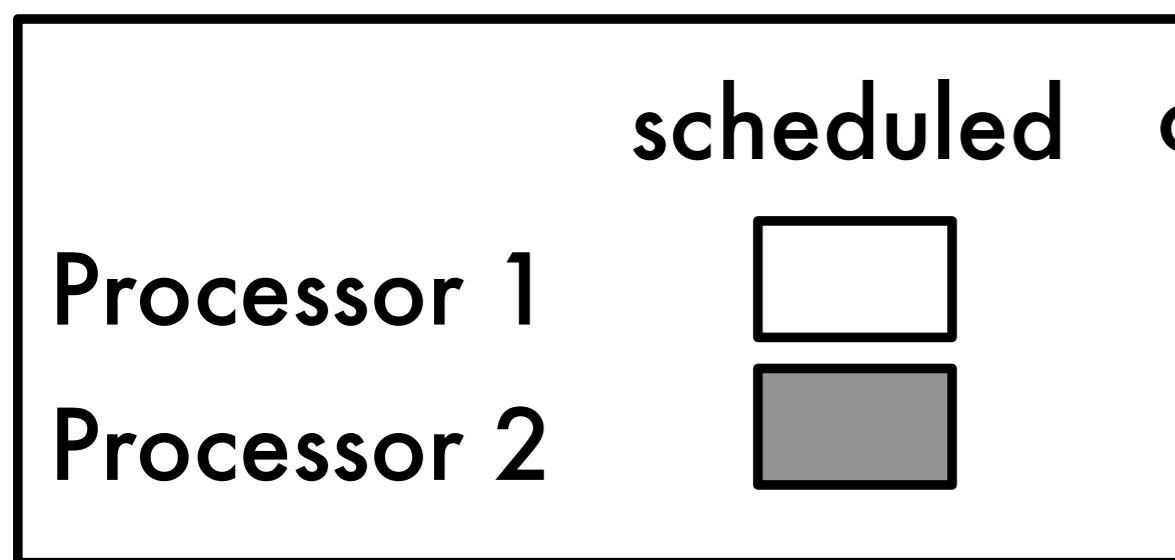
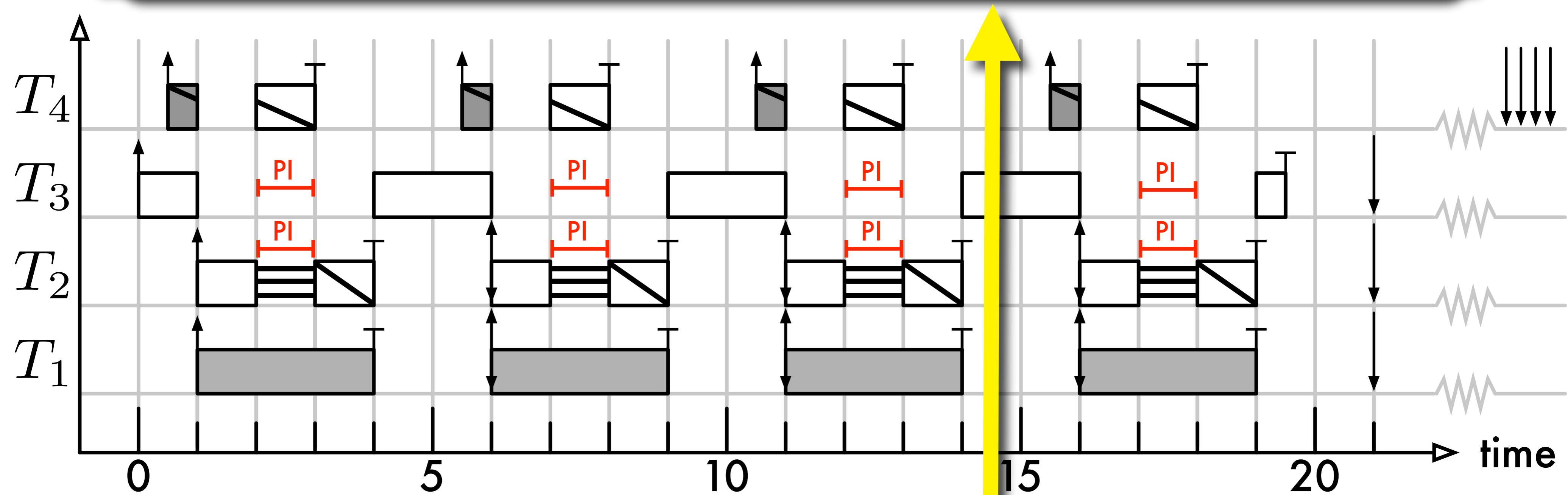


Bounded only by
the number of jobs released by T_1 , T_2 , and T_4 while T_3 is pending.

How many jobs? $\rightarrow \phi = \{\text{max response time}\} / \{\text{min period}\}$

Sub-Optimality of Priority Inheritance

$\Omega(\phi)$ pi-blocking is possible, where $\phi = \{\text{max response time}\} / \{\text{min period}\}$



ϕ is *not* bounded by the **number of tasks n** .
 → not asymptotically optimal.

What about Priority Boosting?

(1) The classic choice: (variants of) **priority inheritance**.

	job-level priority	Any DRT scheduler	Deadlines	Any DRT scheduler
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Global (jobs migrate freely)		G-OMLP [— & Anderson, 2010]	FMLP [Block et al., 2007]	?
Clustered (jobs migrate only among subset of processors)		OMIP [— 2012]	?	?
		C-OMLP [— & Anderson, 2011]		

[Block et al., 2007] A Flexible Real-Time Locking Protocol for Multiprocessors, RTCSA 2007.

(2) The partitioned & clustered choice: (variants of) **priority boosting**.

What about Priority Boosting?

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(2) The partitioned & clustered choice: (variants of) **priority boosting**.

Sub-Optimality of Unrestricted Priority Boosting

*lock-holding jobs **always** have higher effective priority than non-lock-holding jobs*



*It is **impossible to construct**
an **asymptotically optimal** locking protocol
(w.r.t. s-aware analysis)
under global JLFP scheduling
based on unrestricted priority boosting.*

[—, 2011]

(And hence also under clustered JLFP scheduling.)

Sub-Optimality of Unrestricted Priority Boosting

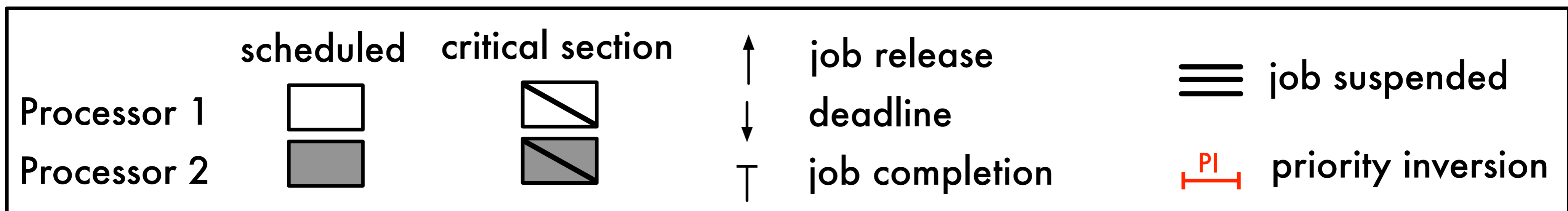
Unrestricted Priority Boosting Example Schedule

[same task set & arrival sequence as before]

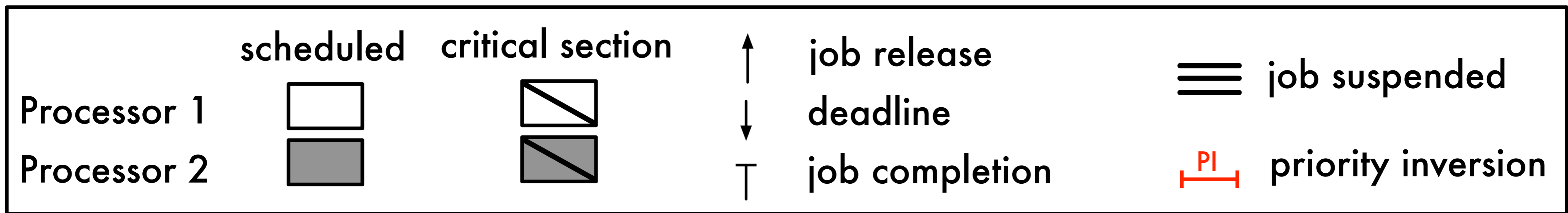
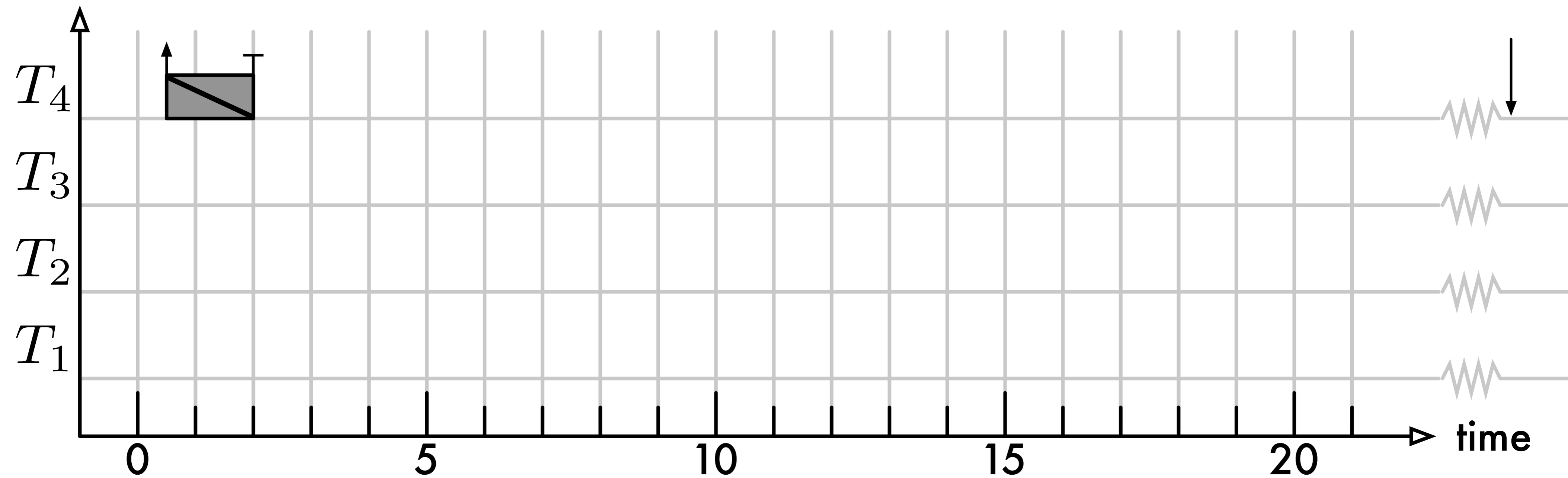
4 tasks on 2 processors

global fixed-priority scheduling

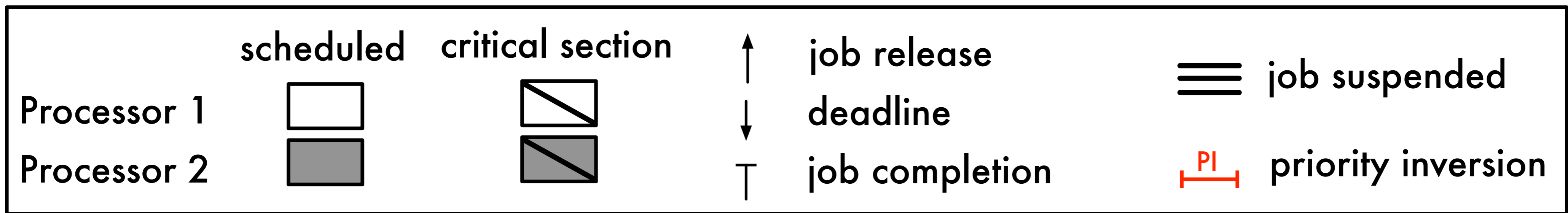
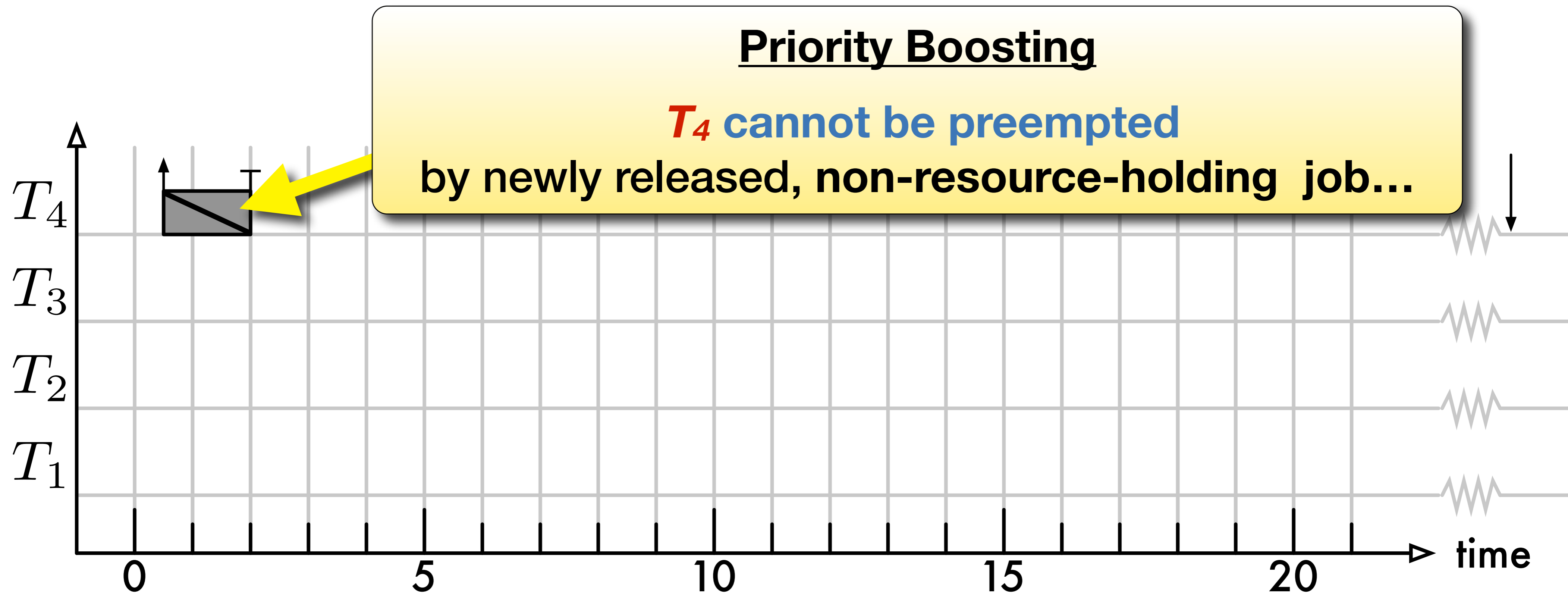
Task priorities (high to low): $T_1 > T_2 > T_3 > T_4$



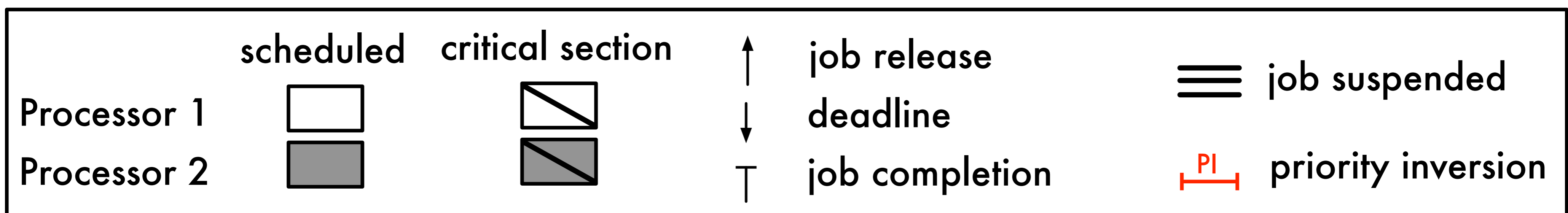
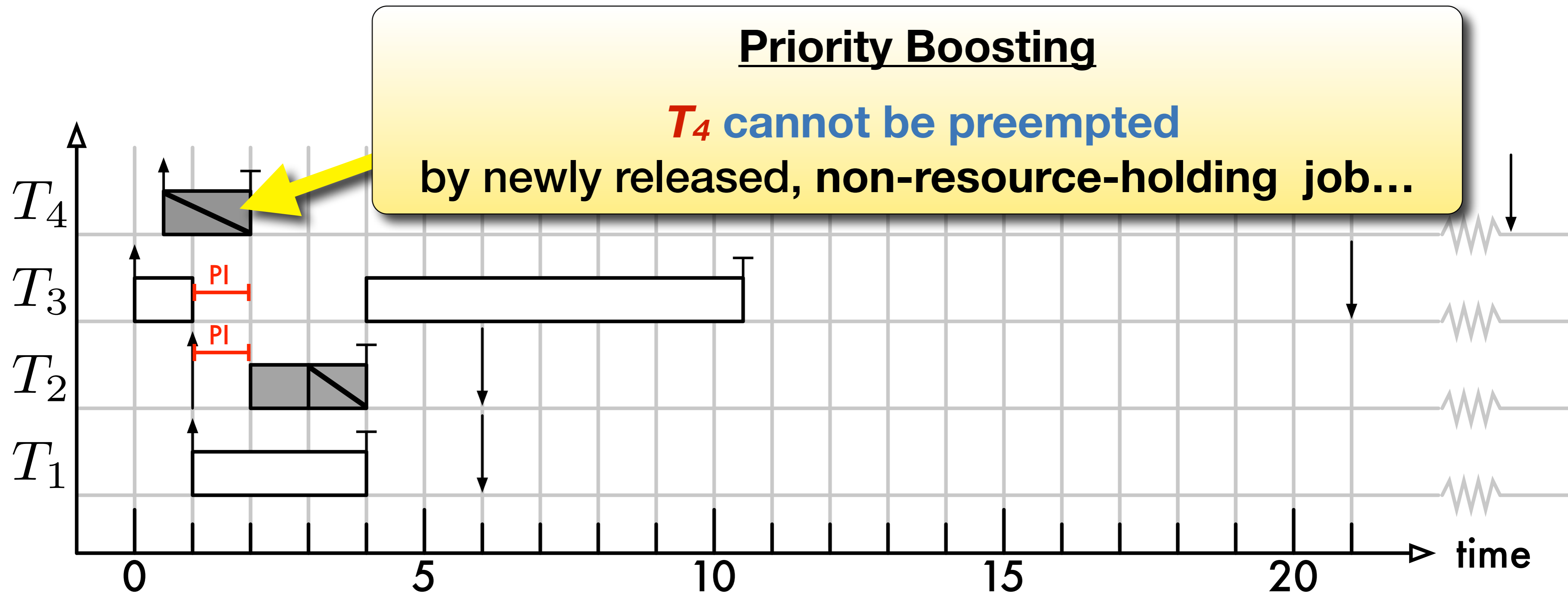
Lock-Holding Jobs Cannot Be Preempted



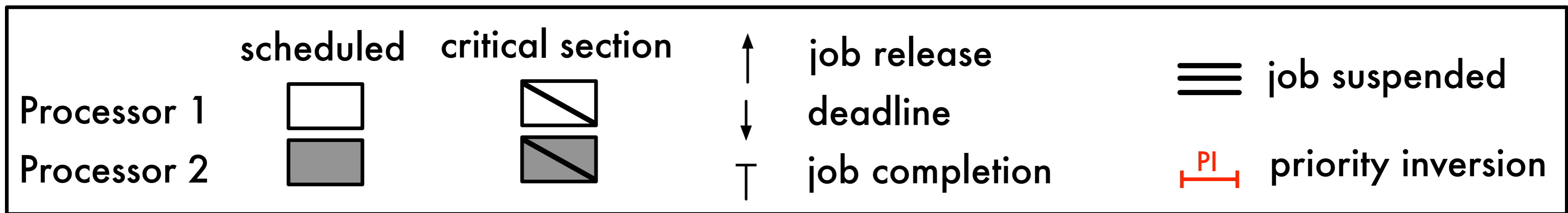
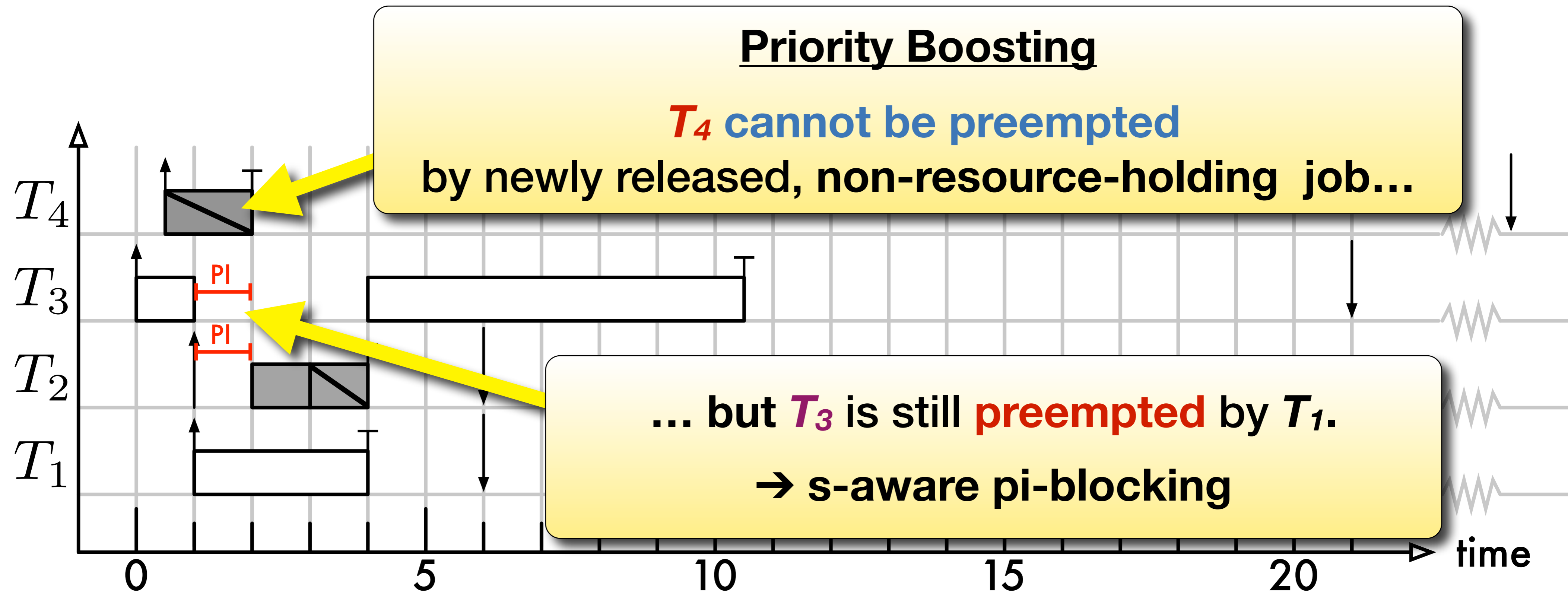
Lock-Holding Jobs Cannot Be Preempted



PI-Blocking Shifted, not Avoided

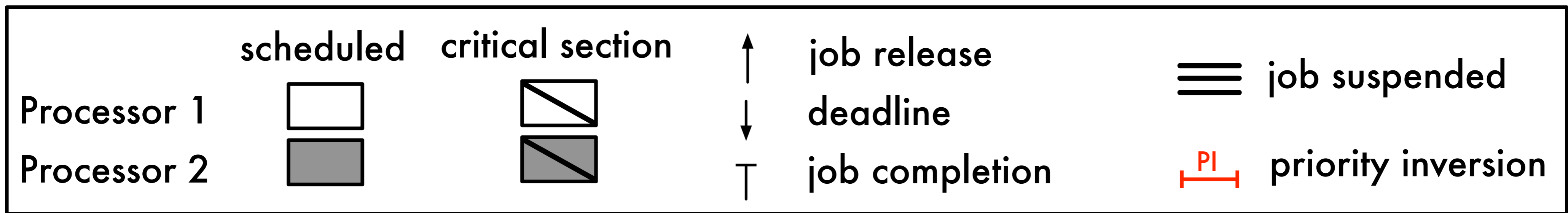
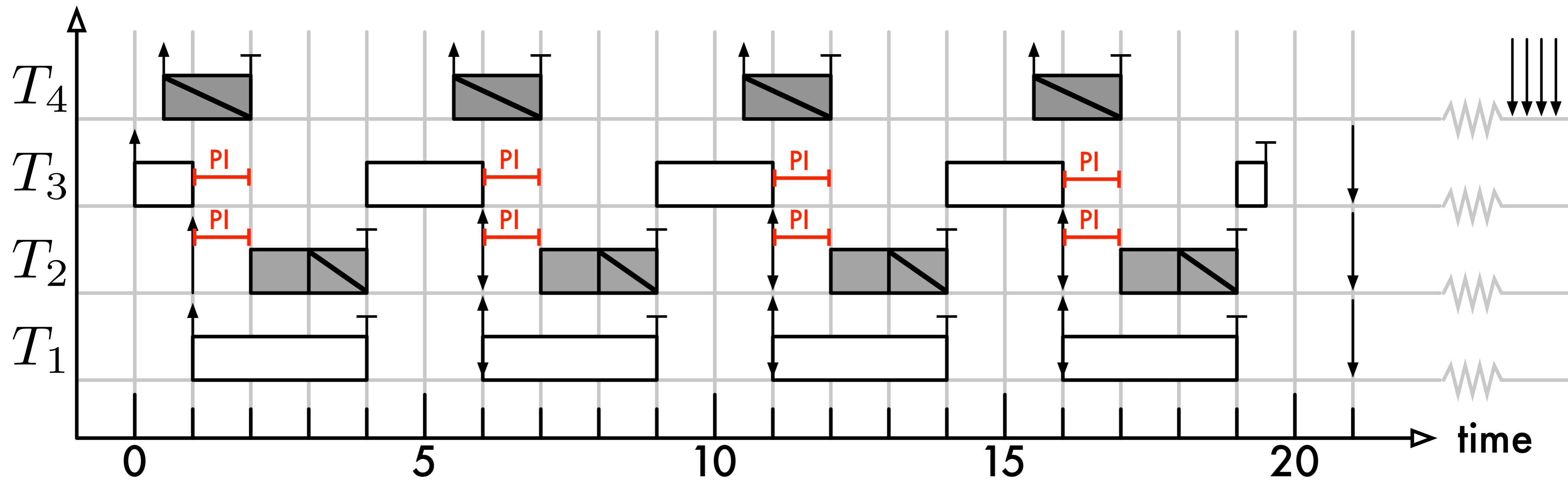


PI-Blocking Shifted, not Avoided



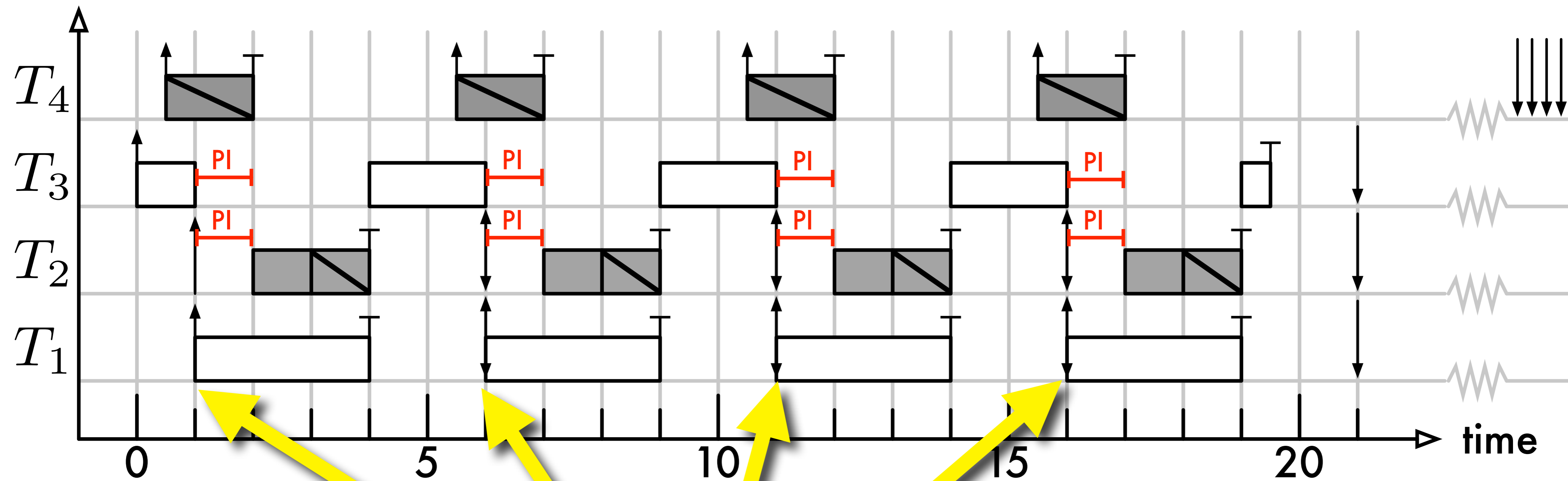
“Victim Task” Accumulates PI-Blocking

$\Omega(\phi)$ pi-blocking is possible, where $\phi = \{\text{max response time}\} / \{\text{min period}\}$



Sub-Optimality of Unrestricted Priority Boosting

$\Omega(\phi)$ *pi*-blocking is possible, where $\phi = \{\text{max response time}\} / \{\text{min period}\}$

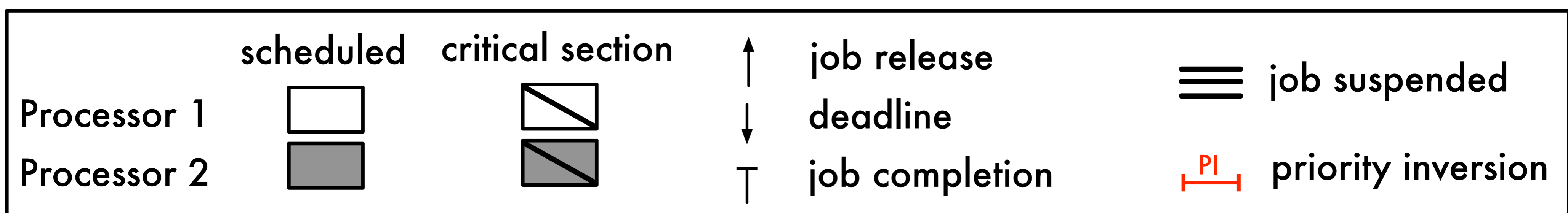
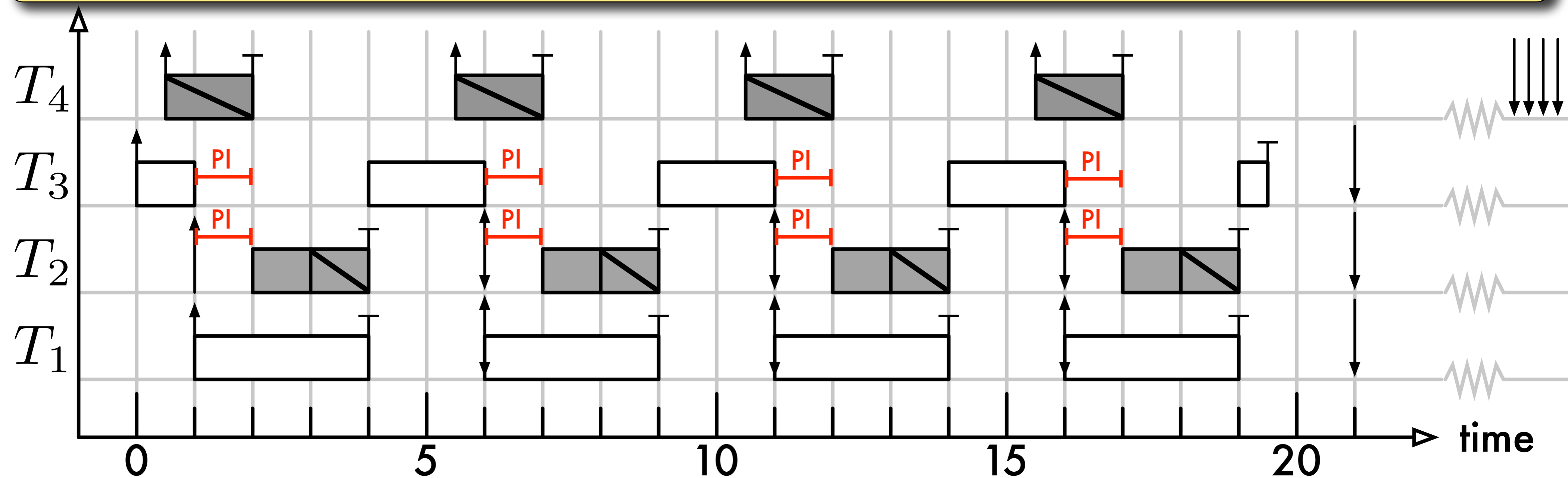


Also repeats $\phi = \{\text{max response time}\} / \{\text{min period}\}$ times...

→ **not asymptotically optimal.**

Remark: examples use **single resource** shared by **only two tasks**

- **queue order irrelevant** (FIFO-ordered, priority-ordered, etc.)
- **cannot simplify problem** with coarser-grained locking



We need something new...

(1) The classic choice: (variants of) **priority inheritance**.

	job-level priority	Any DRT scheduler	Deadlines	Any DRT scheduler
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We need something new...

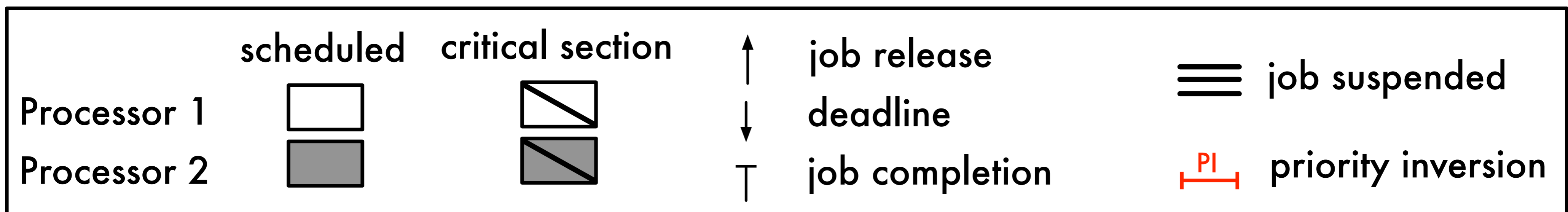
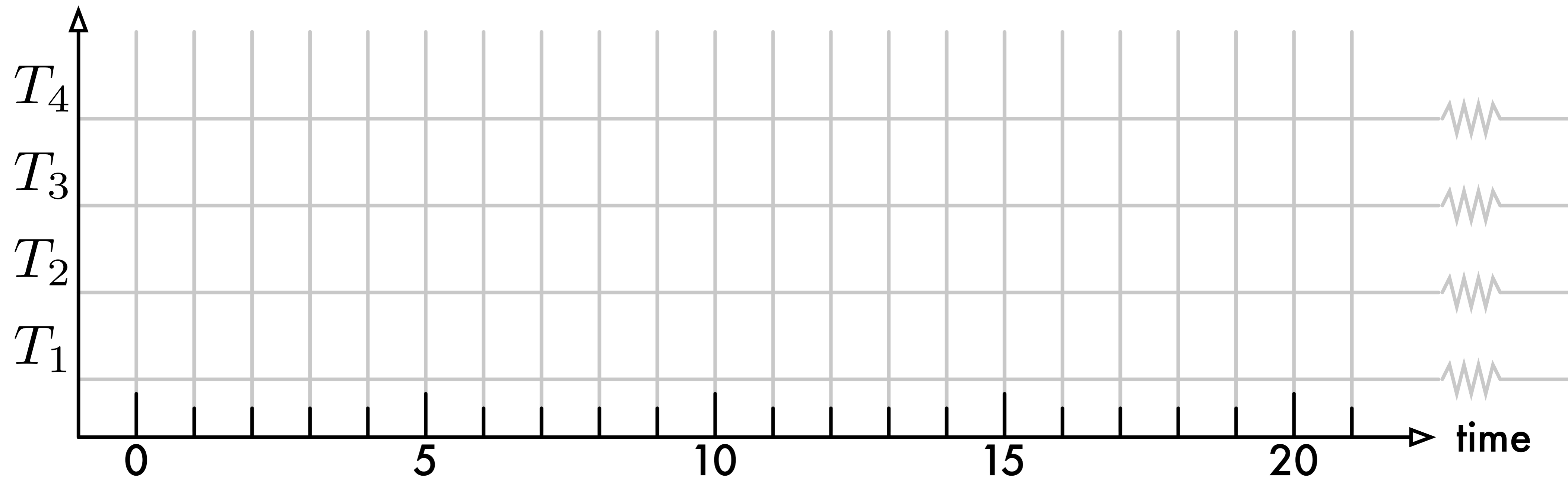
~~(1) The classic choice (variants of) priority inheritance.~~

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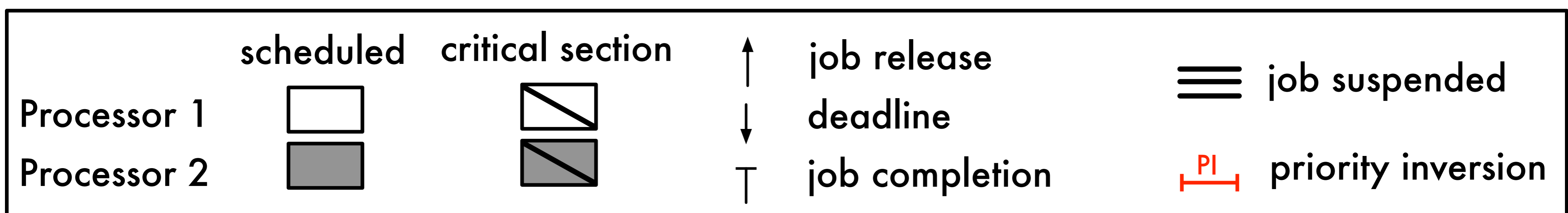
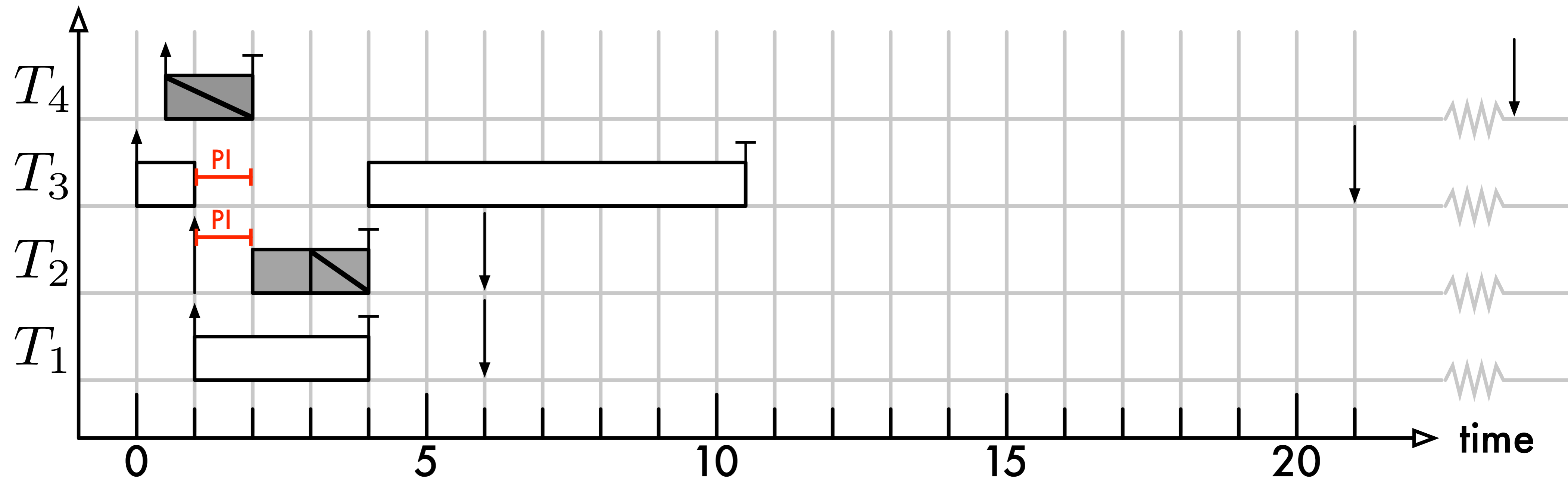
~~(2) The partitioned & clustered choice (variants of) priority boosting.~~

Observation: $O(n)$ PI-Blocking Possible



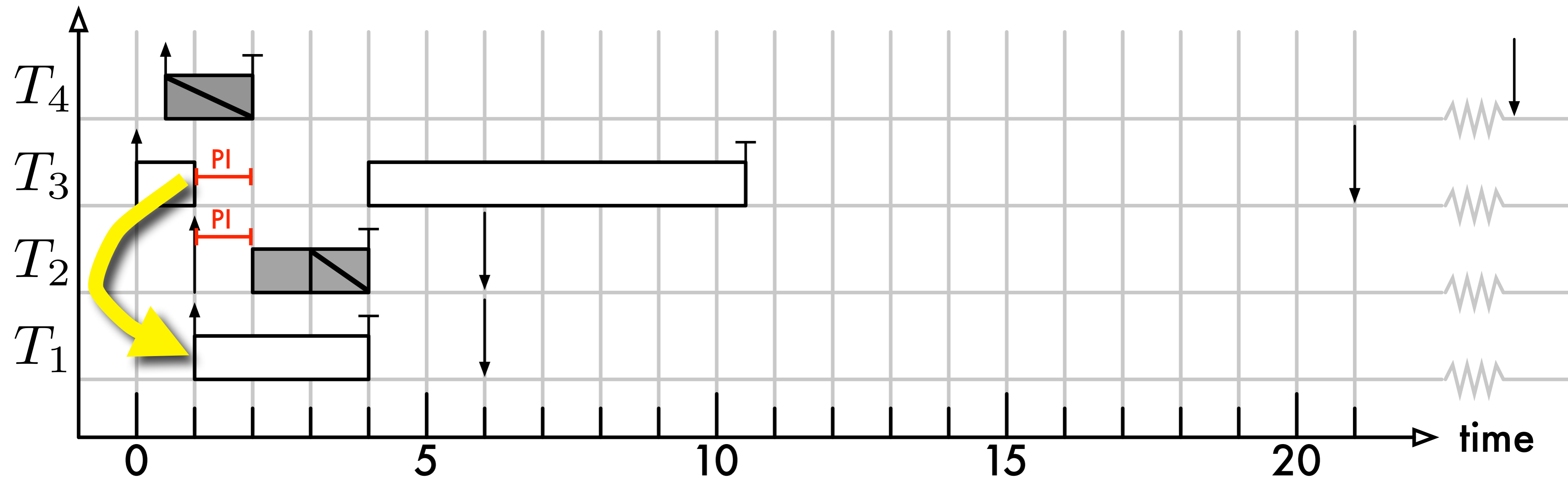
Observation: $O(n)$ PI-Blocking Possible

This schedule: *unrestricted* priority boosting.



Observation: $O(n)$ PI-Blocking Possible

This schedule: **unrestricted** priority boosting.

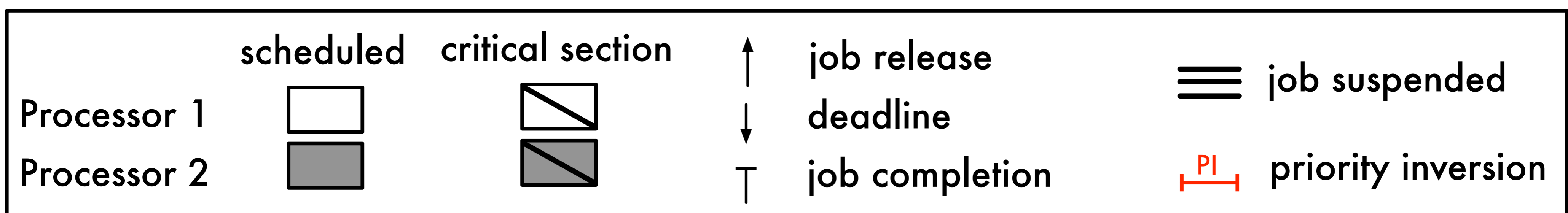
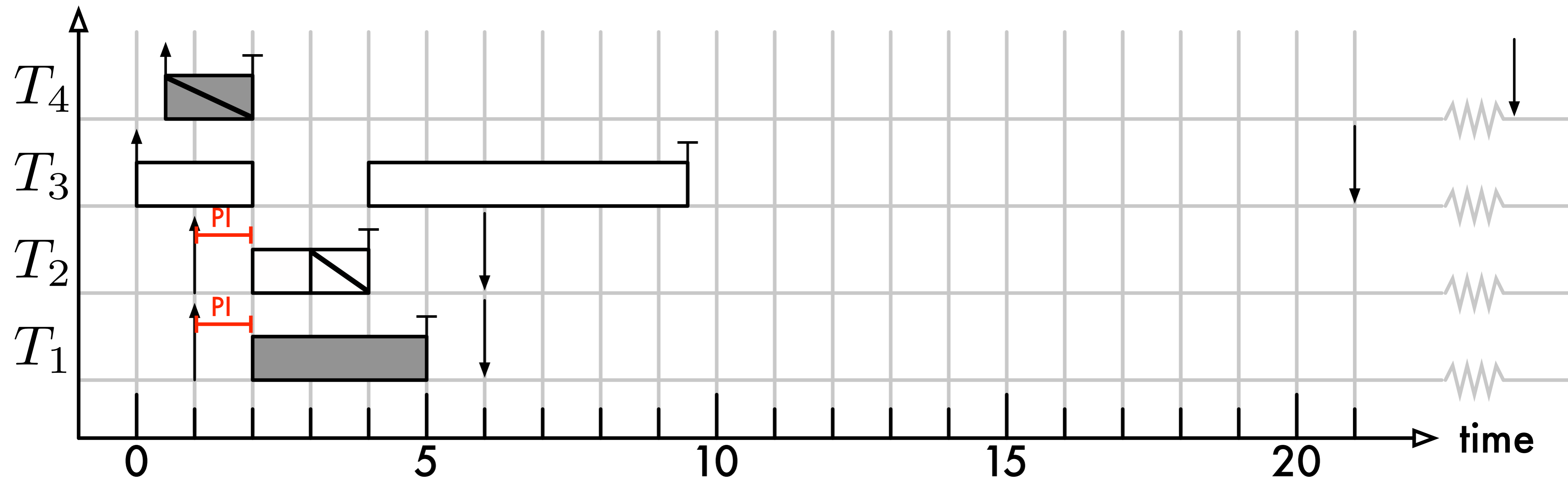


PI-blocking **cannot be avoided**, but it can be **shifted** to **new jobs**.

→ **prevent accumulation** of pi-blocking in individual jobs.

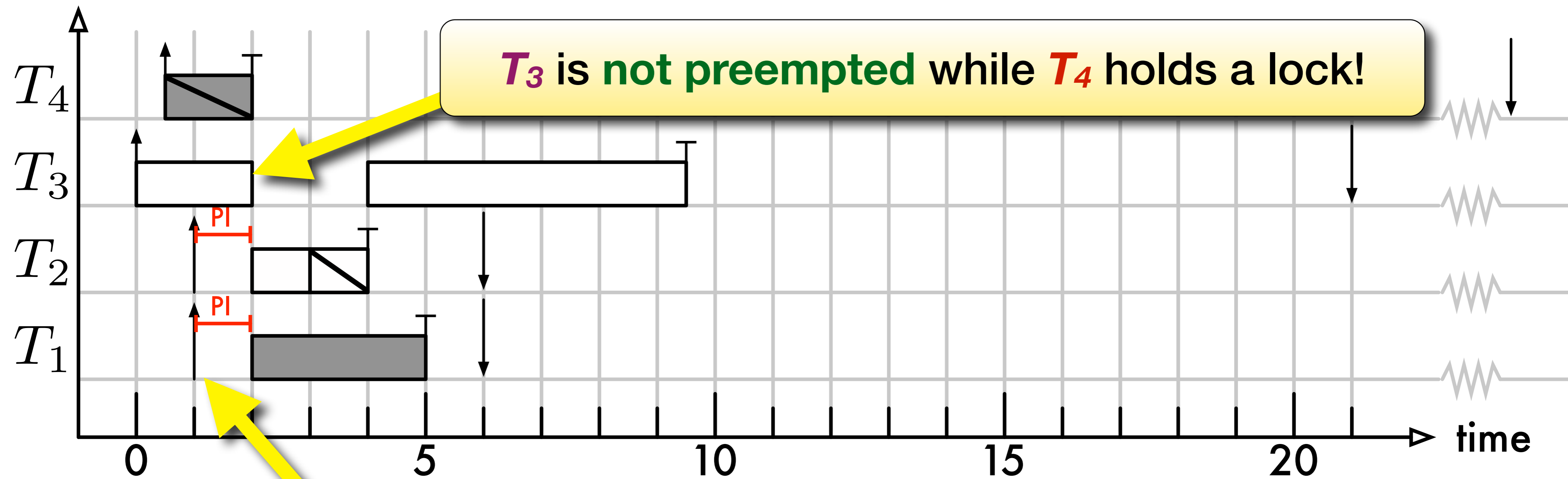
Idea: Protect Existing Independent Jobs

Alternative possible schedule.



Idea: Protect Existing Independent Jobs

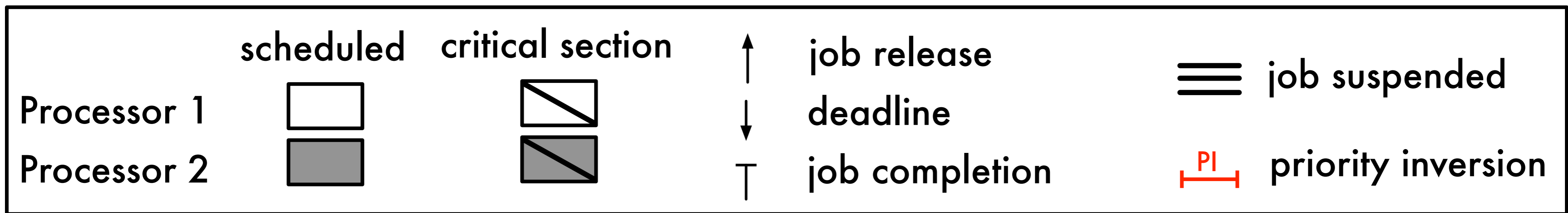
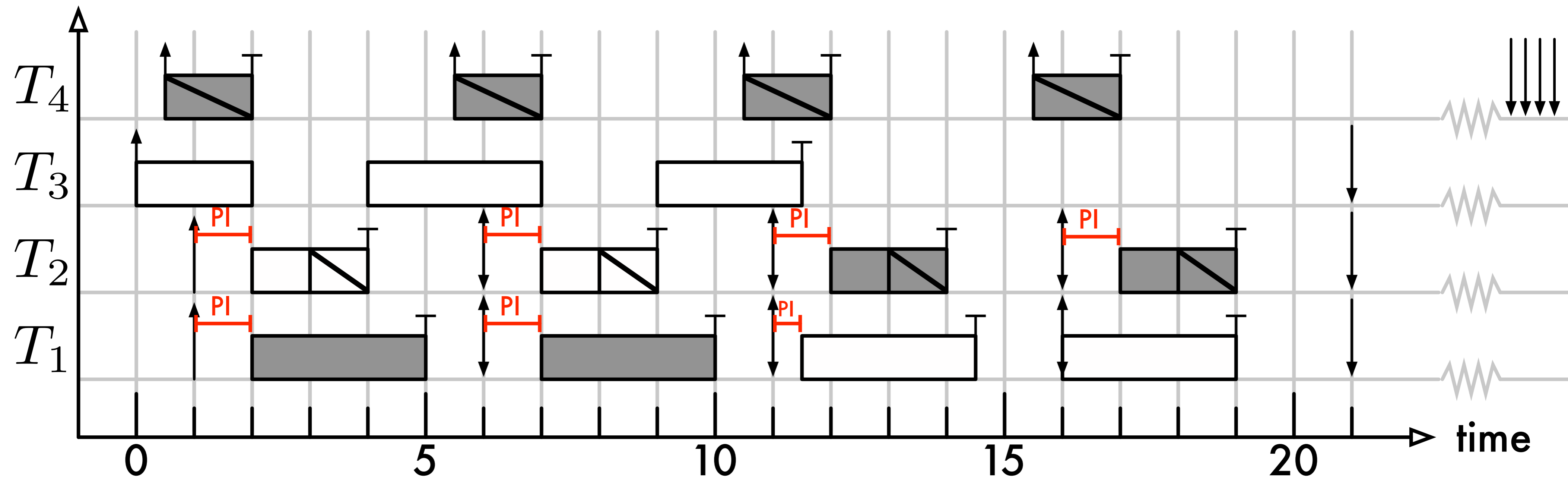
Alternative possible schedule.



Both T_1 and T_2 incur pi-blocking instead.

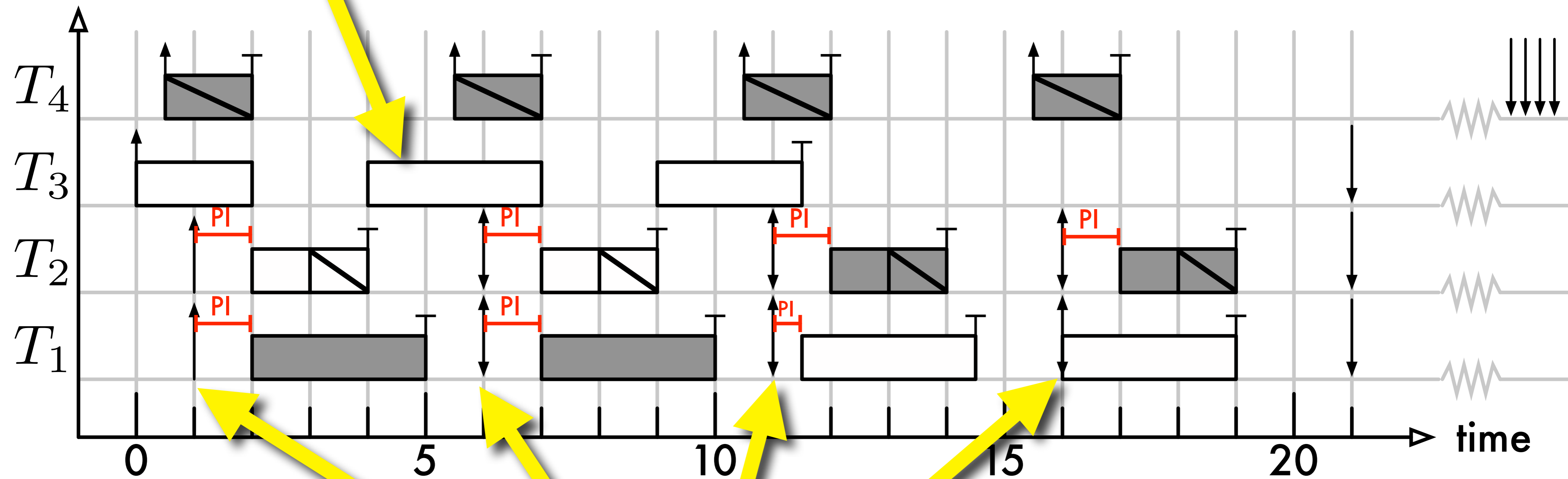
$O(n)$ PI-Blocking Per Job

Alternative possible schedule.



T_3 “protected”: incurs no pi-blocking in this example.

Alternative possible schedule.

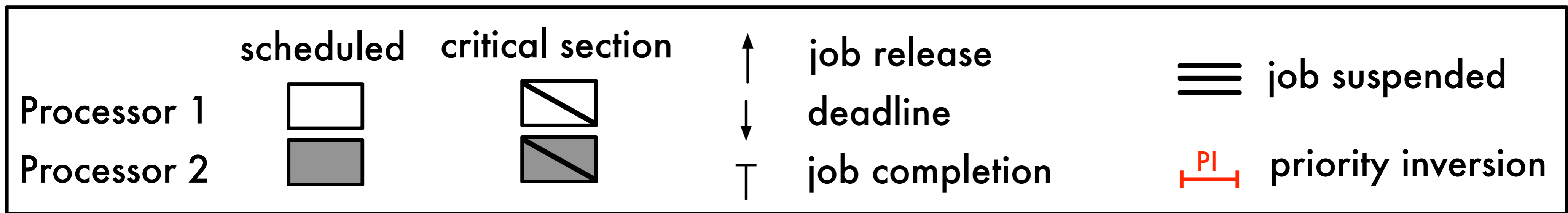
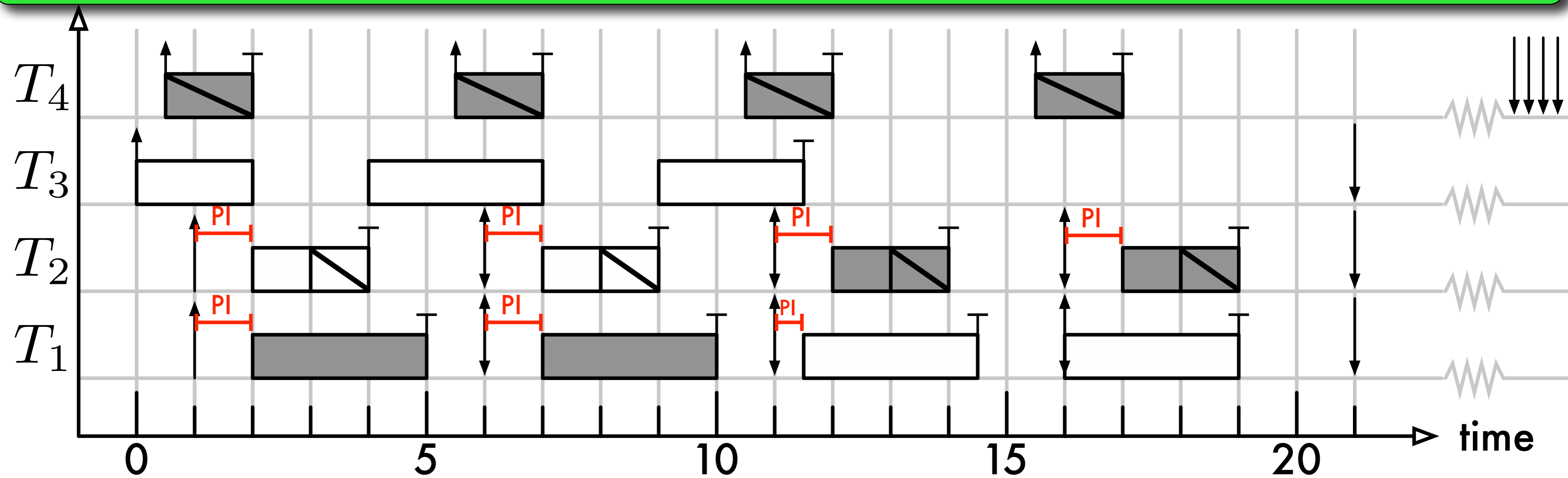


Each job incurs only a limited amount of pi-blocking.

→ asymptotically optimal.

This is actually a **Generalized FMLP⁺** schedule.

*Key question: how to specify that “**T₃ must be protected**”?*



Closing the S-Aware Asymptotic Optimality Gap



Key Problem: Preemptions due to Later-Started Critical Sections

On Uniprocessors

- a job is **blocked only** by critical sections that **are already in progress** when the job is released / resumed.

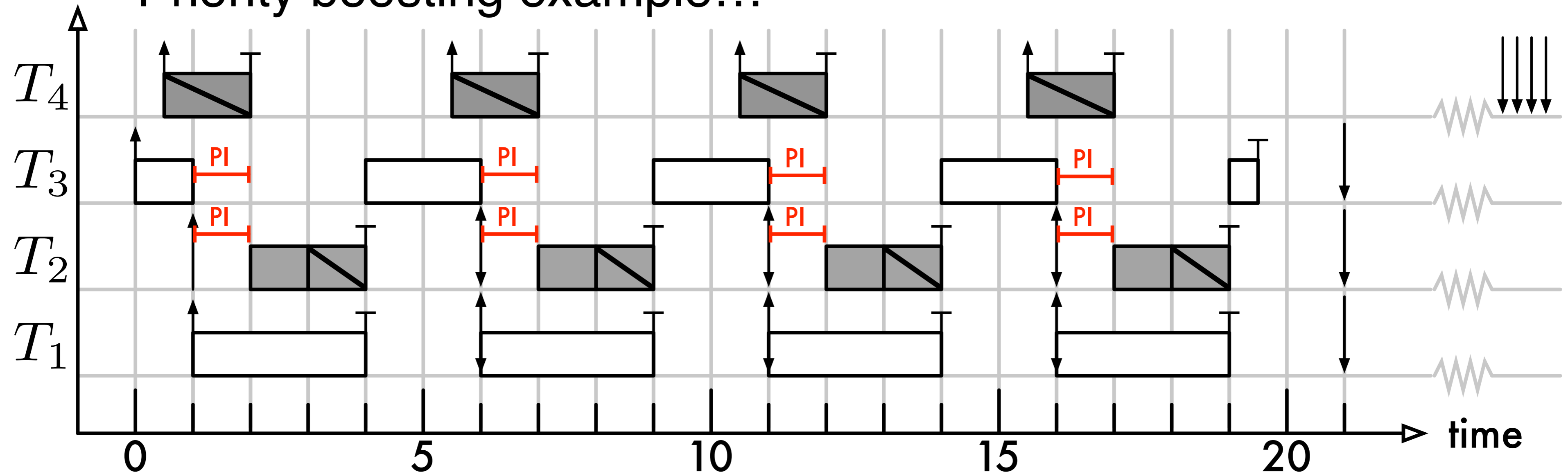
Key Problem: Preemptions due to Later-Started Critical Sections

On Uniprocessors

- a job is **blocked only** by critical sections that **are already in progress** when the job is released / resumed.

On Multiprocessors

- Priority boosting example...



Priority Boosting / Priority Inheritance Examples

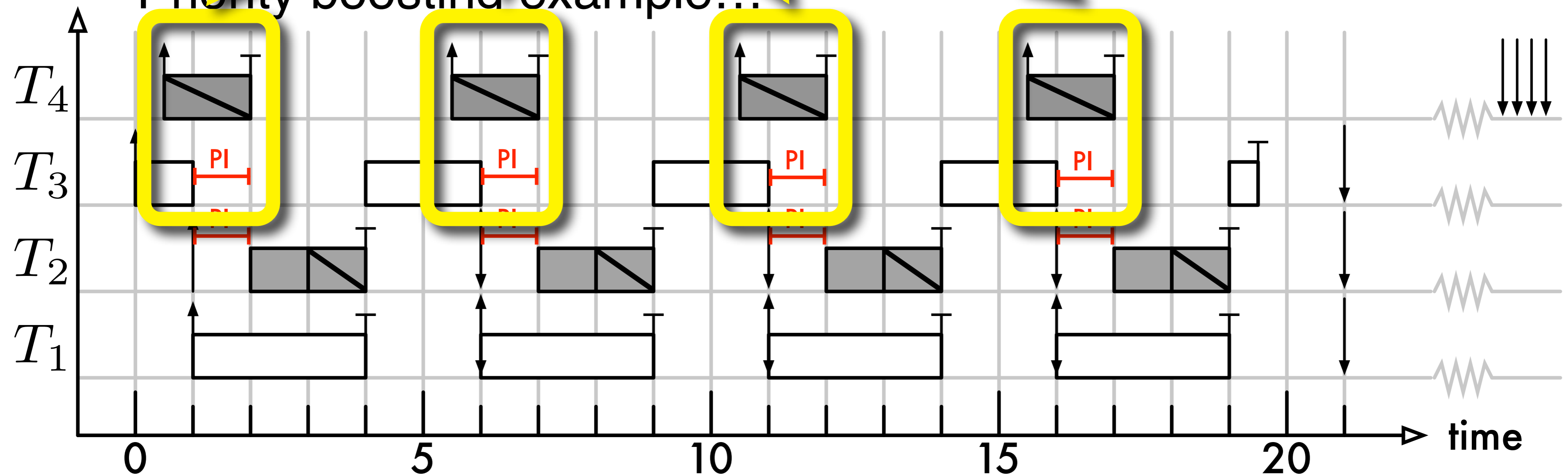
T_3 blocked due to ϕ requests issued **after** T_3 started executing.
 (→ root cause: **parallel scheduling** of lower-priority jobs)

On Uniprocessors

→ a job is **blocked only** by critical sections that **are already in progress** when the job is released / resumed.

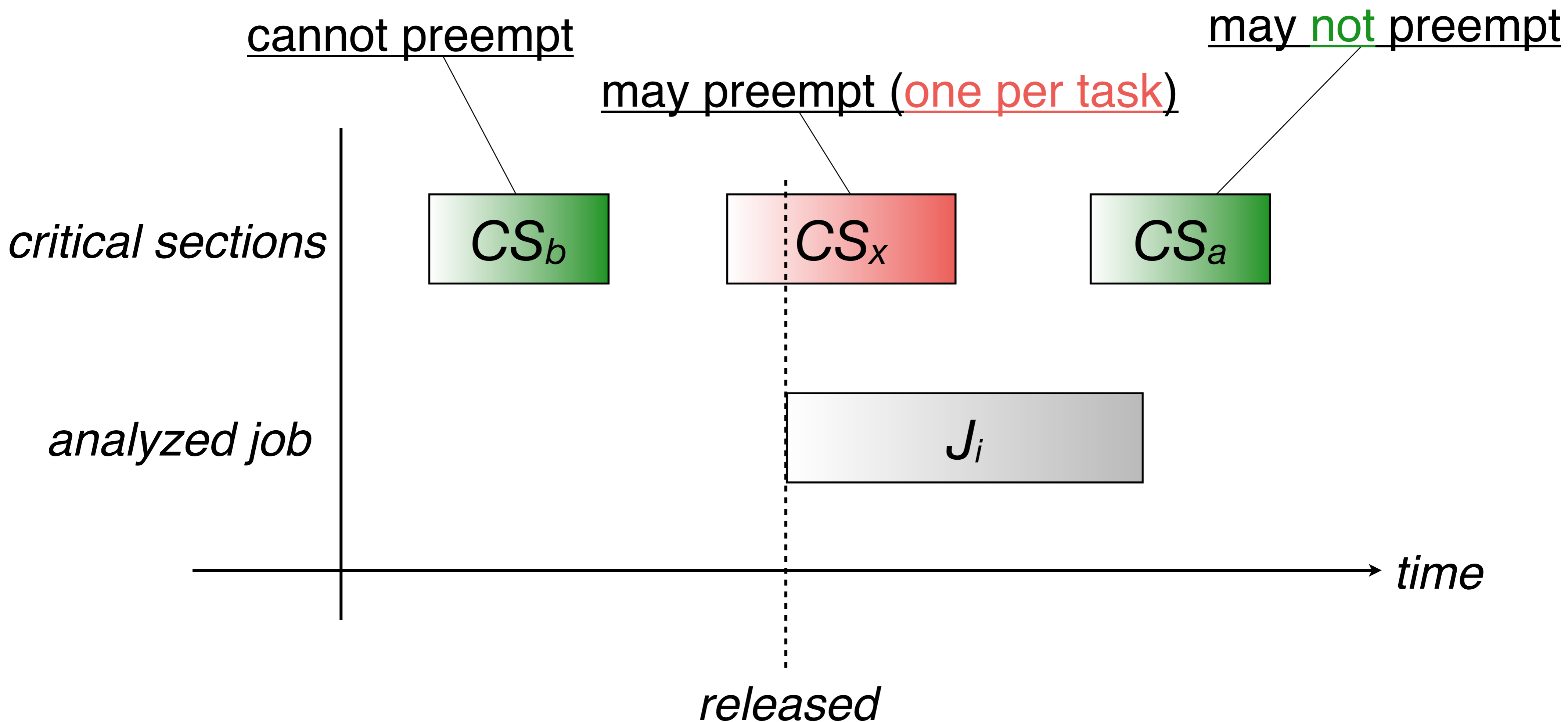
On Multiprocessors

→ Priority boosting example...



What if this is disallowed...?

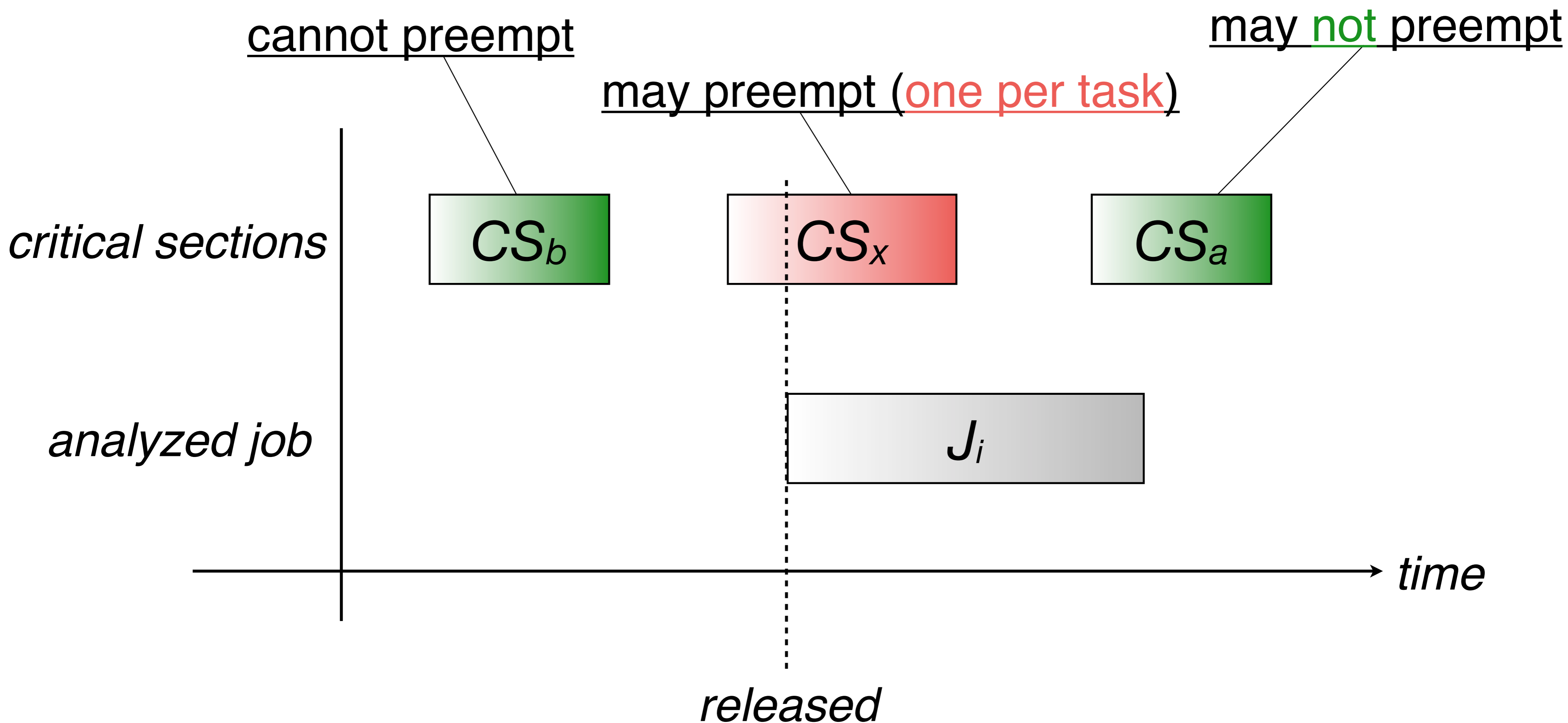
strawman rule: jobs cannot be preempted due to later-issued requests



→ $O(n)$ preemptions

This is the **desired effect**, but the **simple rule fails in corner cases**.

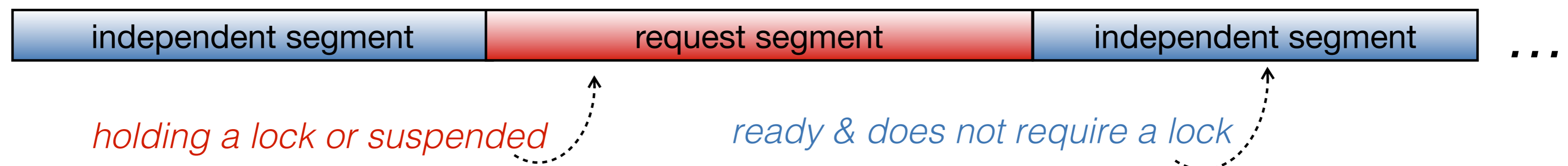
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Independent & Request Segments

a job at runtime:



Note: exact segments known only at runtime

- ➔ potentially complex, non-linear **control flow** determines which resources are required and in which order
- ➔ approach **not** limited to linear, branch-free tasks

Key Concept: Segment Start Time

Simply the start time of a job's current segment.

a job at runtime:

**Note: exact segments known only at runtime**

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- ➔ approach **not** limited to linear, branch-free tasks

A Lock Holder's Co-Boosting Set

Key idea underlying the Generalized FMLP⁺

*If a job is **priority-boosted**,
then **certain other jobs** must also be **co-boosted**.*

A Lock Holder's Co-Boosting Set

If a job J_b holds a lock at **time t** , then its co-boosting set is defined as:

$$\left\{ J_y \mid \begin{array}{l} J_y \text{ executes an } \underline{\text{independent segment}} \text{ at } \mathbf{time } t \text{ and} \\ J_y \text{ has } \underline{\text{higher priority}} \text{ than } J_b \text{ and} \\ J_y \text{'s current segment } \underline{\text{started before}} J_b \text{'s segment.} \end{array} \right\}$$

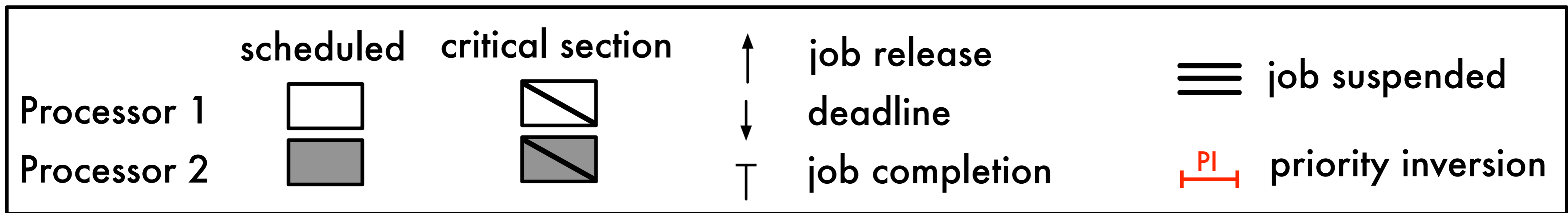
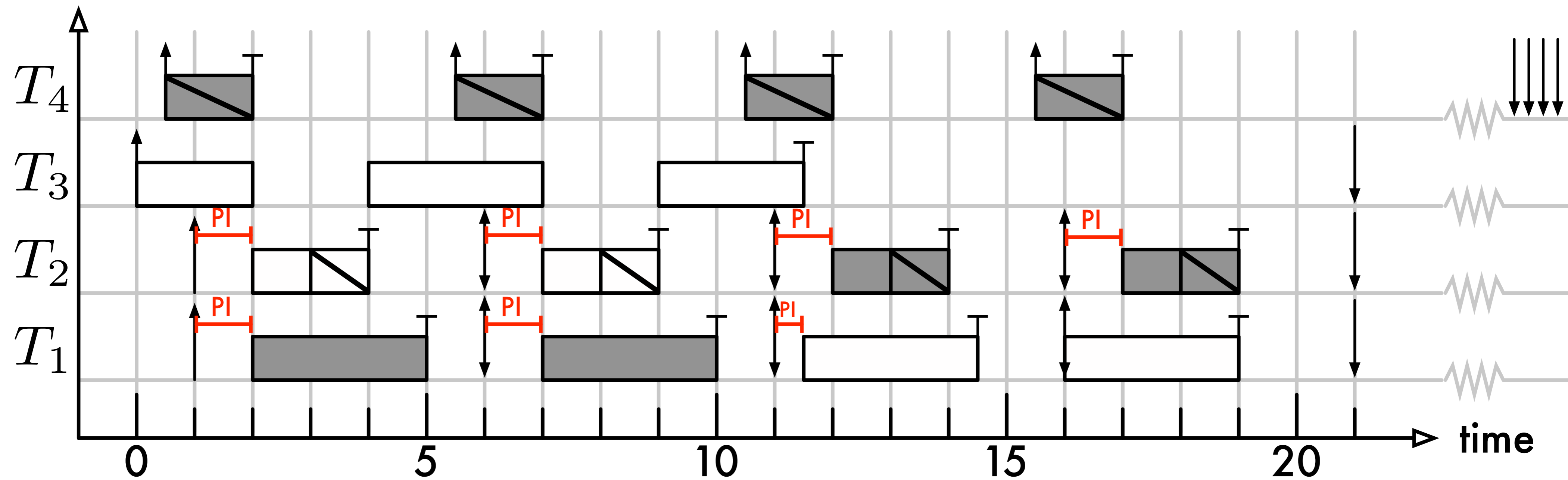
(Note: in this talk, I'll use "task" and "job" interchangeably.)

Intuition

→ The set of jobs **at risk** of **accumulating pi-blocking** due to J_b .

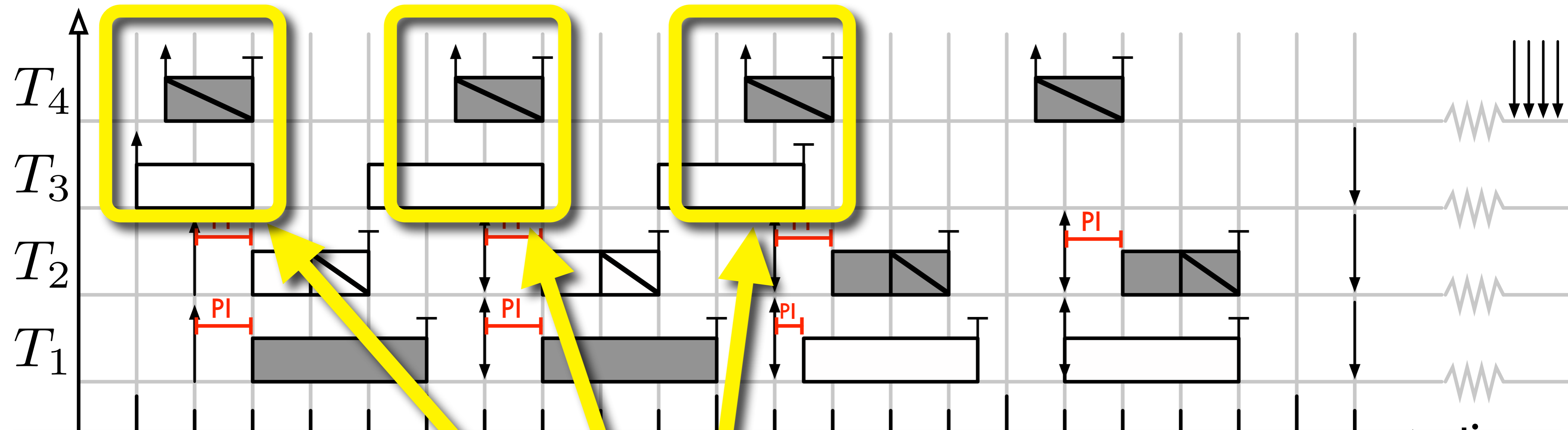
Example: $\{ T_3 \}$ is T_4 's Co-Boosting Set

Generalized FMLP⁺ schedule.



Example: $\{ T_3 \}$ is T_4 's Co-Boosting Set

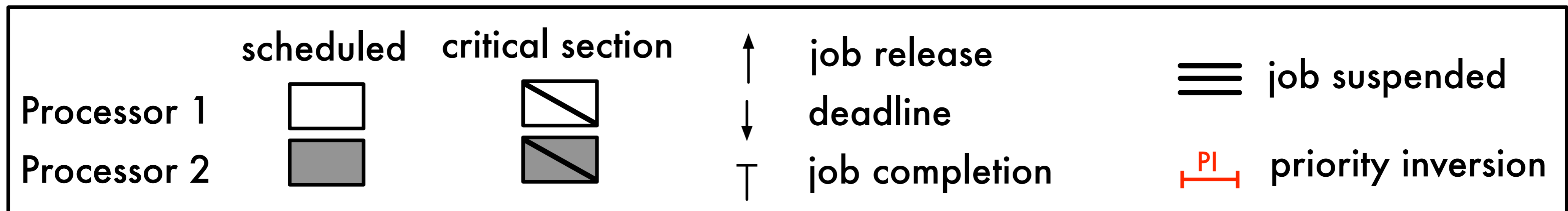
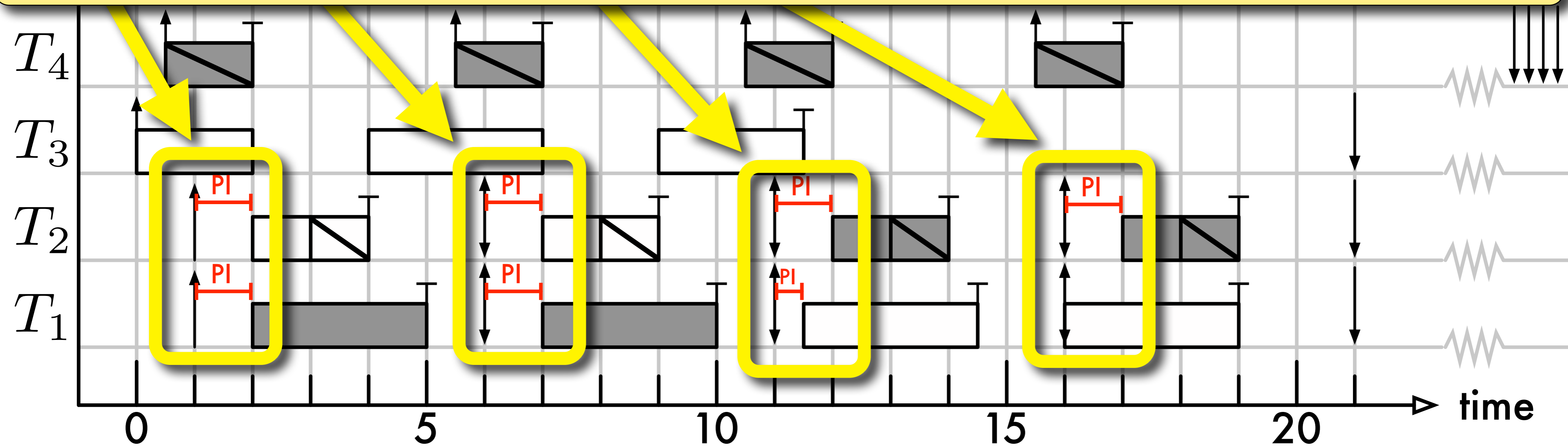
Generalized FMLP⁺ schedule.



$\left\{ T_3 \right\}$

- T_3 executes an independent segment at **time t** and
- T_3 has higher priority than T_4 and
- T_3 's current segment started before T_4 's segment.

T_1 and T_2 execute independent segments at **time t** and
 T_1 and T_2 have higher priority than T_4 but
 T_1 and T_2 's current segments did NOT start before T_4 's segment.



Restricted Segment Boosting

In a cluster with c CPUs, at any point in time t , schedule the following jobs:

Restricted Segment Boosting

In a cluster with c CPUs, at any point in time t , schedule the following jobs:

A **Single Boosted Job** J_b

The **lock-holding ready job** (if any) with the **earliest segment start time**.

(any ties broken arbitrarily but consistently)

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The **lock-holding ready job** (if any) with the **earliest segment start time**.

Up to $c - 1$ jobs from J_b 's **Co-Boosting Set**

Select the (up to) $c - 1$ jobs with the **earliest segment start times**.

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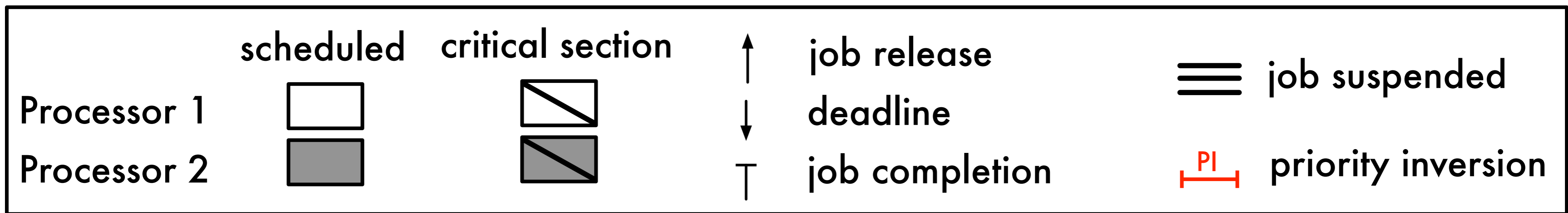
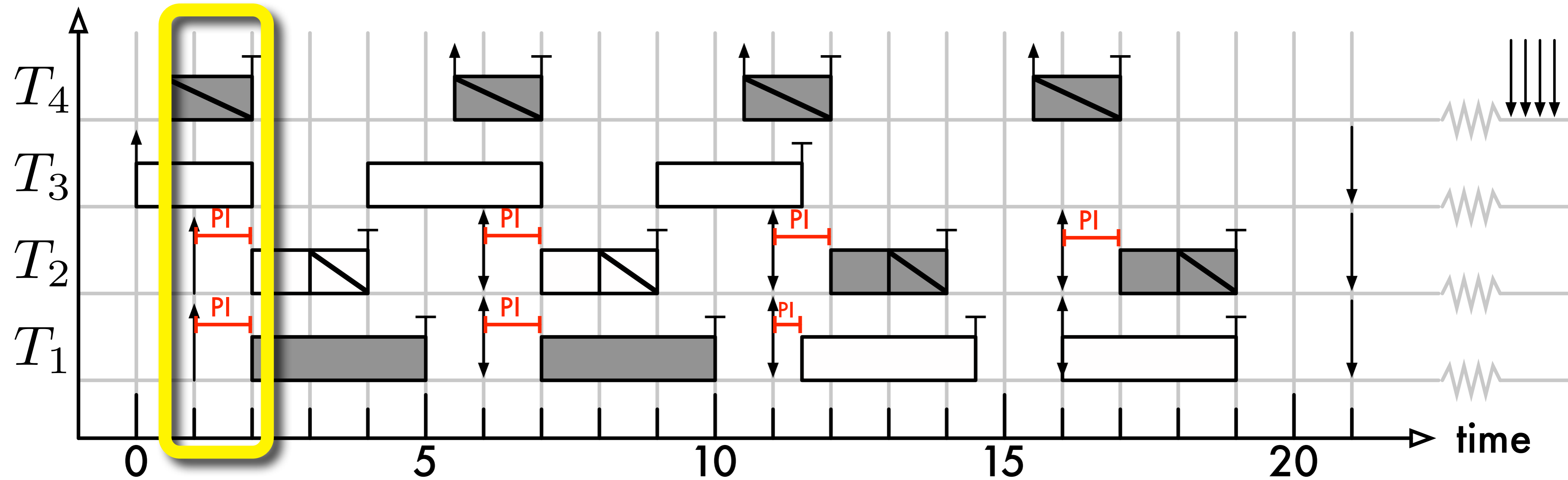
If less than c jobs scheduled so far: **any other ready jobs**

Select the **highest-priority ready jobs** not yet scheduled (may hold locks).

(any ties broken arbitrarily but consistently)

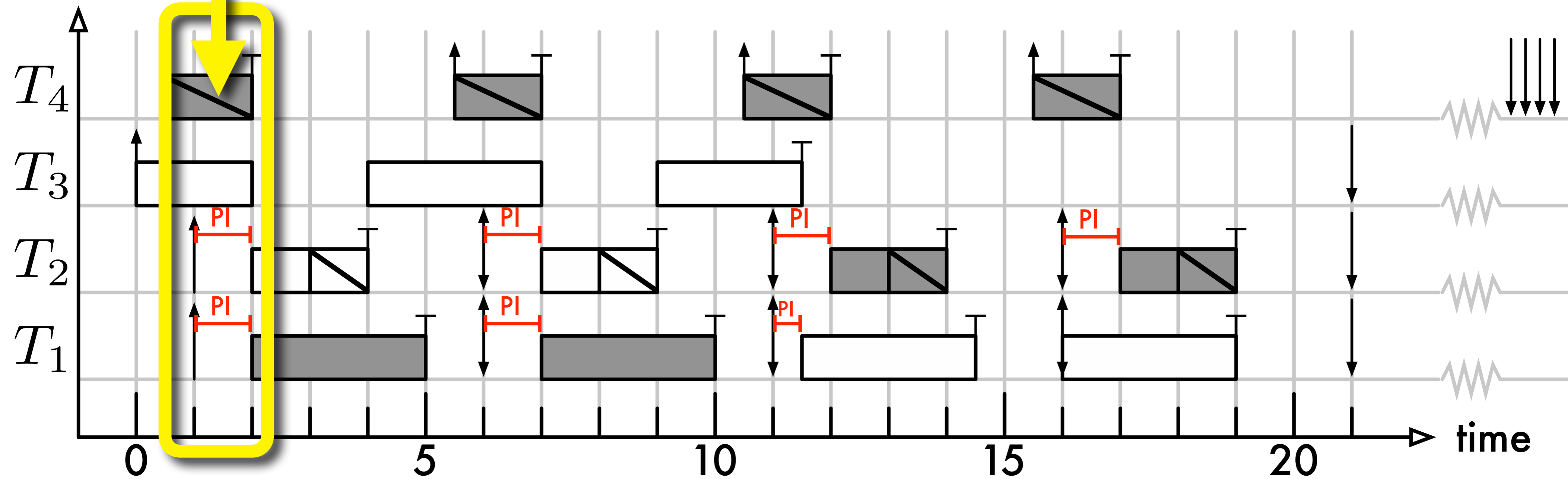
Restricted Segment Boosting at Time 1

Generalized FMLP⁺ schedule.



(1) The lock-holding ready job (if any) with the **earliest segment start time**.

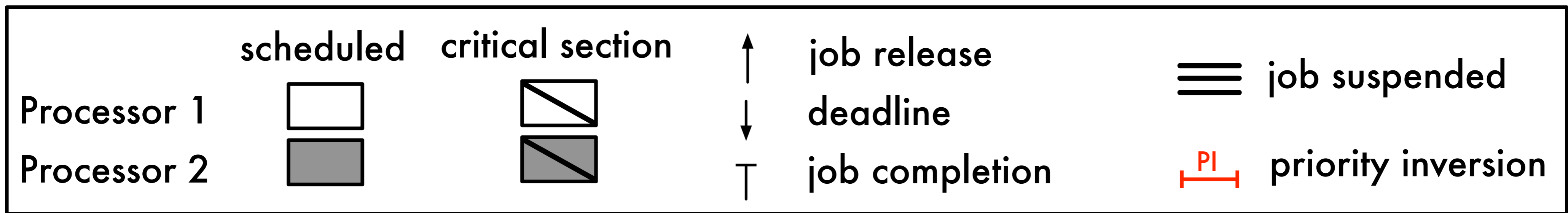
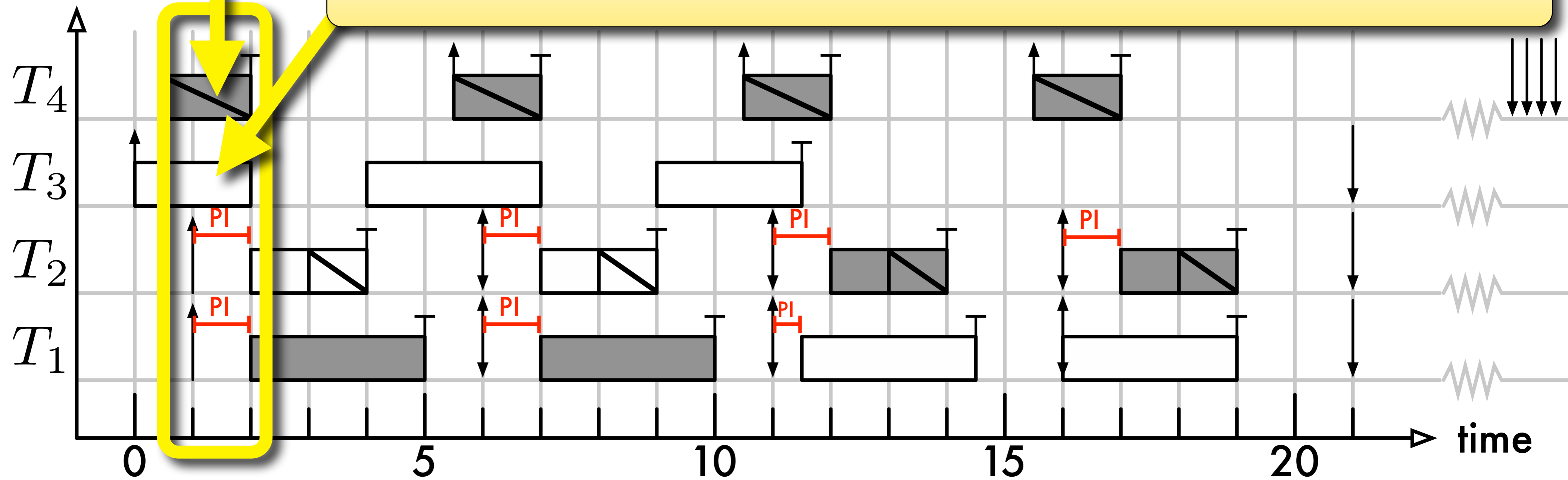
Generalized FMLP⁺ schedule.



	scheduled	critical section	↑	job release	≡	job suspended
Processor 1			↓	deadline		priority inversion
Processor 2			⊥	job completion		

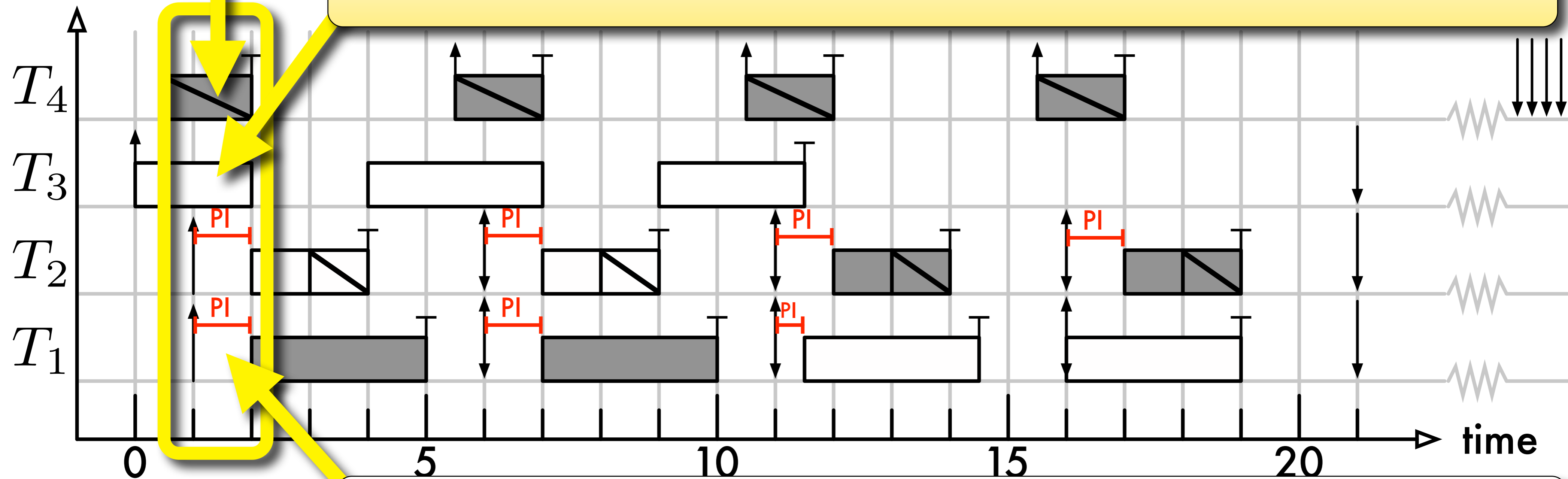
(1) The lock-holding ready job (if any) with the **earliest segment start time**.

(2) Up to $c - 1 = 1$ jobs from T_4 's **co-boosting set** = $\{T_3\}$.



(1) The lock-holding ready job (if any) with the **earliest segment start time**.

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(3) If less than $c = 2$ jobs scheduled so far: any other ready jobs.

At time 1: no more CPUs available after steps 1 & 2.

sched

Processor 1



Processor 2



┆ job completion



priority inversion

The Generalized FIFO Multiprocessor Locking Protocol (FMLP⁺)

Restricted Segment Boosting + ***Per-Resource FIFO Queues***

The Generalized FIFO Multiprocessor Locking Protocol (FMLP⁺)

Restricted Segment Boosting + **Per-Resource FIFO Queues**

S-Aware PI-Blocking per Segment

- ➔ ...during request segment: $O(n)$.
 - Proof: rather straightforward → see paper.
- ➔ ...during independent segment: $O(n)$.
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- Constant **#requests** and **#self-suspensions** per job
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Overall Max. S-Aware PI-Blocking

- $O(n)$ under clustered JLFP scheduling.

Multiprocessor Real-Time Locking Optimality Results

<p><u>JLFP</u> <i>job-level fixed-priority</i></p>	<p><u>Suspension Oblivious</u> Any JLFP Scheduler</p>	<p><u>Suspension Aware</u> Any JLFP Scheduler</p>
<p>Partitioned (no migrations)</p>	<p>P-OMLP ✓ [— & Anderson, 2010] + RNLP [Ward & Anderson, 2012]</p>	<p>The Generalized FMLP⁺ (restricted segment boosting)</p>  <p>+ RNLP [Ward & Anderson, 2012] for nested critical sections</p>
<p>Global (jobs migrate freely)</p>	<p>G-OMLP ✓ [— & Anderson, 2010] + RNLP [Ward & Anderson, 2012]</p>	
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[Block et al., 2007]

A Flexible Real-Time Locking Protocol for Multiprocessors, RTCSA 2007.

[— & Anderson, 2010]

Optimality Results for Multiprocessor Real-Time Locking, RTSS 2010.

[— & Anderson, 2011]

Real-Time Resource-Sharing under Clustered Scheduling: Mutex, Reader-Writer, and k-Exclusion Locks, EMSOFT 2011.

[—, 2011]

Scheduling and Locking in Multiprocessor Real-Time Operating Systems, PhD thesis, UNC, 2011.

[Ward & Anderson, 2012]


Supporting Nested Locking in Multiprocessor Real-Time Systems, ECRTS 2012.

[—, 2013]

A Fully Preemptive Multiprocessor Semaphore Protocol for Latency-Sensitive Real-Time Applications, ECRTS 2013.

The Generalized FMLP⁺ based on Restricted Segment Boosting closes the s-aware asymptotic optimality gap.

See paper & online appendix for **large-scale empirical evaluation**.
 (Summary: **the FMLP⁺ works well if the schedulability analysis is accurate enough.**)

<u>JLFP</u> <i>job-level fixed-priority</i>	<u>Suspension Oblivious</u> Any JLFP Scheduler	<u>Suspension Aware</u> Any JLFP Scheduler
Partitioned (no migrations)	P-OMLP ✓ [— & Anderson, 2010] + RNLP [Ward & Anderson, 2012]	<div style="border: 2px solid orange; padding: 10px; text-align: center;"> <h2>The Generalized FMLP⁺ (restricted segment boosting)</h2>  <p>+ RNLP [Ward & Anderson, 2012] for nested critical sections</p> </div>
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[—, 2013]

A Fully Preemptive Multiprocessor Semaphore Protocol for Latency-Sensitive Real-Time Applications, ECRTS 2013.

Conclusion



Summary

The Generalized FMLP⁺

- ➔ priority boosting & inheritance **unsuitable**
- ➔ based instead on **restricted segment boosting**
- Key idea: **co-boosting of independent jobs**

In the Paper

- ➔ Empirical evaluation.
- ➔ How to integrate with **locking-unrelated self-suspensions...**
- ... also **within critical sections.**
- ➔ How to integrate with Ward & Anderson's RNLP [2012] for asymptotically optimal pi-blocking given **nested critical sections.**

Online Appendix

- ➔ **Fine-grained blocking analysis** based on linear programming framework [—, 2013].
- ➔ Complete evaluation results (5760 graphs).

The FMLP⁺: An Asymptotically Optimal Real-Time Locking Protocol for Suspension-Aware Analysis

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Abstract—Multiprocessor real-time locking protocols that are asymptotically optimal under suspension-oblivious schedulability analysis (where suspensions are pessimistically modeled as processor demand) are known for partitioned, global, and clustered job-level fixed priority (JLFP) scheduling. However, for the case of more accurate suspension-aware schedulability analysis (where suspensions are accounted for explicitly), asymptotically optimal protocols are known only for partitioned JLFP scheduling. In this paper, the gap is closed with the introduction of the first semaphore protocol for suspension-aware analysis that is asymptotically optimal under global and clustered JLFP scheduling. To this end, a new progress mechanism that avoids repeated priority inversions is developed and analyzed, based on the key observation that if lock-holding, low-priority jobs are priority-boosted, then certain other non-lock-holding, higher-priority jobs must be co-boosted.

I. INTRODUCTION

The purpose of suspension-based real-time locking protocols is to limit *priority inversions* [22], which, intuitively, occur when a high-priority task that should be scheduled is instead delayed by a remote or lower-priority task. Such locking-related delay, also called *priority inversion blocking* (*pi-blocking*), is problematic because it can result in deadline misses. However, some pi-blocking is unavoidable when using locks and thus must be bounded and accounted for during schedulability analysis.

Clearly, an “optimal” locking protocol should minimize pi-blocking to the extent possible. Formally, a locking protocol is asymptotically optimal if it ensures that, for any task set, maximum pi-blocking is bounded within a constant factor of the pi-blocking unavoidable in some task set [11]. Interestingly, there exist two classes of schedulability analysis that yield different lower bounds: under *suspension-oblivious* (*s-oblivious*) analysis, $\Omega(m)$ pi-blocking is fundamental, whereas under *suspension-aware* (*s-aware*) analysis, $\Omega(n)$ pi-blocking is unavoidable in the general case [7, 11], where m and n denote the number of processors and tasks, respectively. As the names imply, the key difference is that suspensions are accounted for explicitly under s-aware analysis, whereas they are (pessimistically) modeled as processor demand in the s-oblivious case. In principle, s-aware schedulability analysis is preferable, but s-oblivious analysis is easier to derive and permits simpler pi-blocking bounds.

And indeed, for the simpler s-oblivious case, asymptotically optimal locking protocols are known for partitioned, global, and clustered *job-level fixed-priority*¹ (JLFP) scheduling [9, 11, 12]. In contrast, the s-aware case is analytically much more challenging and less understood: asymptotically optimal protocols are known so far only for partitioned JLFP scheduling [7, 11]. The

¹See Sec. II for definitions and a review of essential background.

general problem of optimal s-aware locking under global and clustered JLFP scheduling, however, has remained unsolved.

A. Contributions

We answer this fundamental question by introducing the generalized *FIFO Multiprocessor Locking Protocol* (FMLP⁺), the first semaphore protocol for clustered scheduling that ensures $O(n)$ maximum s-aware pi-blocking under any JLFP policy.

While it was initially assumed [11] that a variant of Block *et al.*'s *Flexible Multiprocessor Locking Protocol* (FMLP) [6]—which uses $O(n)$ FIFO queues together with *priority inheritance* [22]—is asymptotically optimal under global scheduling, we show in Sec. III that this holds only under some, but not all global JLFP schedulers. In fact, we show that both priority inheritance and (unrestricted) *priority boosting* [22], which are the two mechanisms used in all prior locking protocols for s-aware analysis to avoid unbounded pi-blocking, can give rise to non-optimal $\Omega(\Phi)$ pi-blocking, where Φ is the ratio of the longest and the shortest period (and not bounded by m or n).

To overcome this lower bound, we introduce in Sec. IV-A a new progress mechanism called “restricted segment boosting,” which boosts at most one carefully chosen lock-holding job in each cluster while simultaneously “co-boosting” certain other, non-lock-holding jobs to interfere with the underlying JLFP schedule as little as possible. Together with simple FIFO queues, this ensures $O(n)$ maximum s-aware pi-blocking (within about a factor of two of the lower bound, see Sec. IV-C). Notably, our analysis permits non-uniform cluster sizes, allows each cluster to use a different JLFP policy, supports self-suspensions within critical sections (Sec. IV-F), and can be easily combined with prior work [25] to support nested critical sections (Sec. IV-G).

Finally, while answering the s-aware blocking optimality question in the general case is the main contribution of this paper, Sec. V presents a schedulability study that shows the FMLP⁺ to outperform s-oblivious approaches if the underlying s-aware schedulability analysis is sufficiently accurate.

B. Related Work

On uniprocessors, the blocking optimality problem has long been solved: both the classic *Stack Resource Policy* [3] and the *Priority Ceiling Protocol* [22, 24] limit pi-blocking to at most one (outermost) critical section, which is obviously optimal.

On multiprocessors, there are two major lock types: *spin locks*, wherein blocked jobs busy-wait, and suspension-based *semaphores*. Spin locks are well understood and it is not difficult to see that non-preemptable FIFO spin locks, which ensure $O(m)$

[Ward & Anderson, 2012] *Supporting Nested Locking in Multiprocessor Real-Time Systems*, ECRTS 2012.

[—, 2013] *Improved Analysis and Evaluation of Real-Time Semaphore Protocols for P-FP Scheduling*, RTAS 2013.

Future Work & Open Questions



Apply this technique to reader-writer locks?

→ Lower bounds on s-aware pi-blocking?



Apply this technique to k-exclusion locks?

→ GPUs & other co-processors

→ Lower bounds on s-aware pi-blocking?




Overheads of restricted segment boosting?

→ Tracking **segment start times** is simple and cheap.

→ But... additional preemptions?

Multiprocessor Real-Time Locking Optimality Results

<p><u>JLFP</u> <i>job-level fixed-priority</i></p>	<p><u>Suspension Oblivious</u> Any JLFP Scheduler</p>	<p><u>Suspension Aware</u> Any JLFP Scheduler</p>
<p>Partitioned (no migrations)</p>	<p>P-OMLP ✓ [— & Anderson, 2010] + RNLP [Ward & Anderson, 2012]</p>	<p style="text-align: center;">The Generalized FMLP⁺ (restricted segment boosting)</p> <div style="text-align: center;">  </div> <p style="text-align: center;">+ RNLP [Ward & Anderson, 2012] for nested critical sections</p>
<p>Global (jobs migrate freely)</p>	<p>G-OMLP ✓ [— & Anderson, 2010] + RNLP [Ward & Anderson, 2012]</p>	
<p>Clustered (jobs migrate only among subset of processors)</p>	<p>OMIP ✓ [—, 2013] C-OMLP ✓ [— & Anderson, 2011] + RNLP [Ward & Anderson, 2012]</p>	

[Block et al., 2007]

A Flexible Real-Time Locking Protocol for Multiprocessors, RTCSA 2007.

[— & Anderson, 2010]

Optimality Results for Multiprocessor Real-Time Locking, RTSS 2010.

[— & Anderson, 2011]

Real-Time Resource-Sharing under Clustered Scheduling: Mutex, Reader-Writer, and k-Exclusion Locks, EMSOFT 2011.

[—, 2011]

Scheduling and Locking in Multiprocessor Real-Time Operating Systems, PhD thesis, UNC, 2011.

[Ward & Anderson, 2012]

Supporting Nested Locking in Multiprocessor Real-Time Systems, ECRTS 2012.

[—, 2013]

A Fully Preemptive Multiprocessor Semaphore Protocol for Latency-Sensitive Real-Time Applications, ECRTS 2013.

Appendix



Multiprocessor Real-Time Locking Optimality Classes

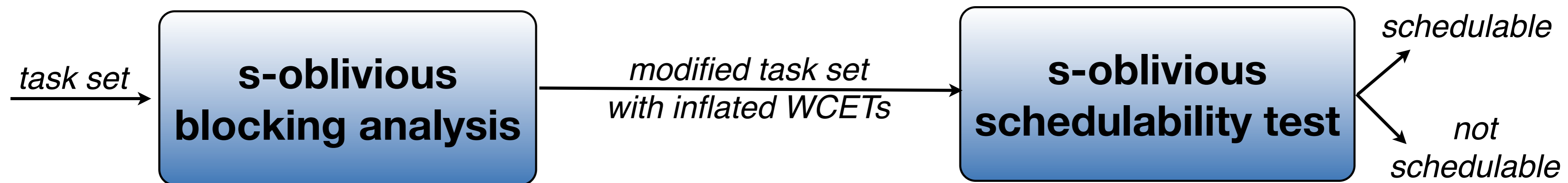
<p>Blocking Optimality [— & Anderson, 2010]</p>	<p>suspension oblivious</p>	<p>suspension aware</p>
<p>How are suspensions analyzed?</p>	<p>CPU demand is over-approximated</p>	<p>CPU demand is modeled accurately</p>
<p>Lower bound on <u>maximum priority inversion blocking</u> $\max_i\{b_i\}$</p>	<p>$\Omega(m)$ $m = \#CPUs$</p>	<p>$\Omega(n)$ $n = \#tasks$</p>

[— & Anderson, 2010] *Optimality Results for Multiprocessor Real-Time Locking*, RTSS 2010.

S-Aware vs. S-Oblivious Analysis

S-Aware vs. S-Oblivious Analysis

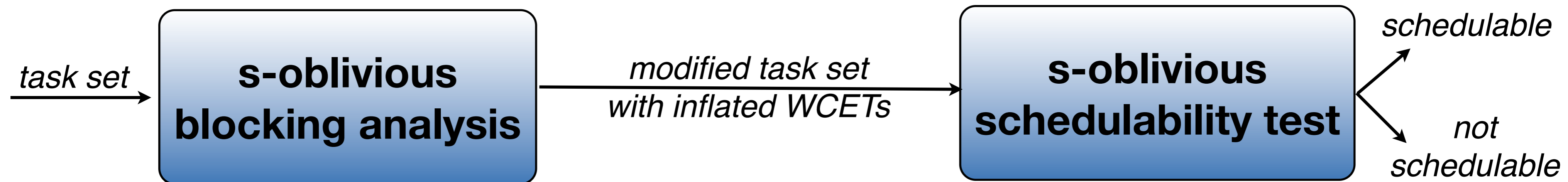
Suspension-oblivious (*s-oblivious*) Analysis



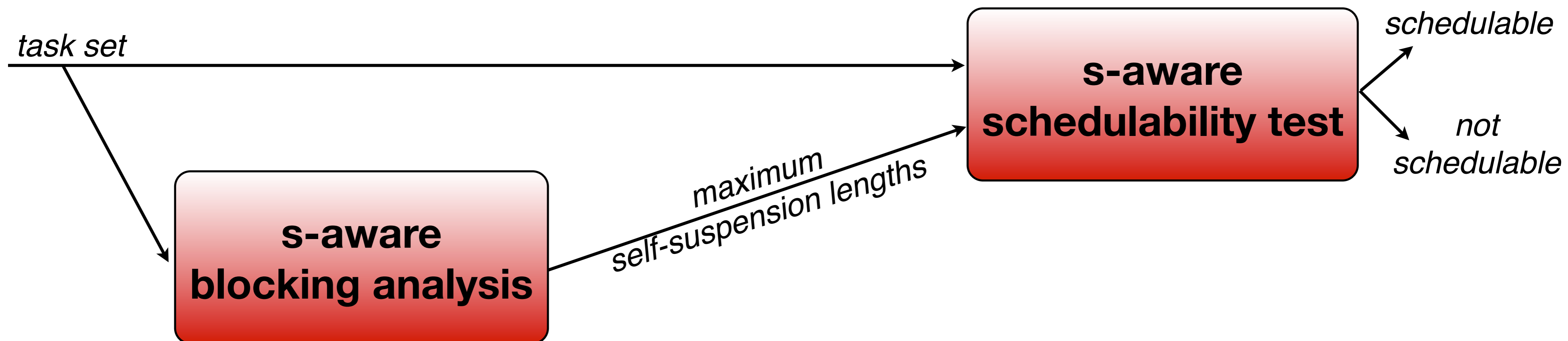
*Motivation: reuse existing schedulability analyses that assume **independent**, **always ready** tasks.*

S-Aware vs. S-Oblivious Analysis

Suspension-oblivious (*s-oblivious*) Analysis



Suspension-aware (*s-aware*) Analysis

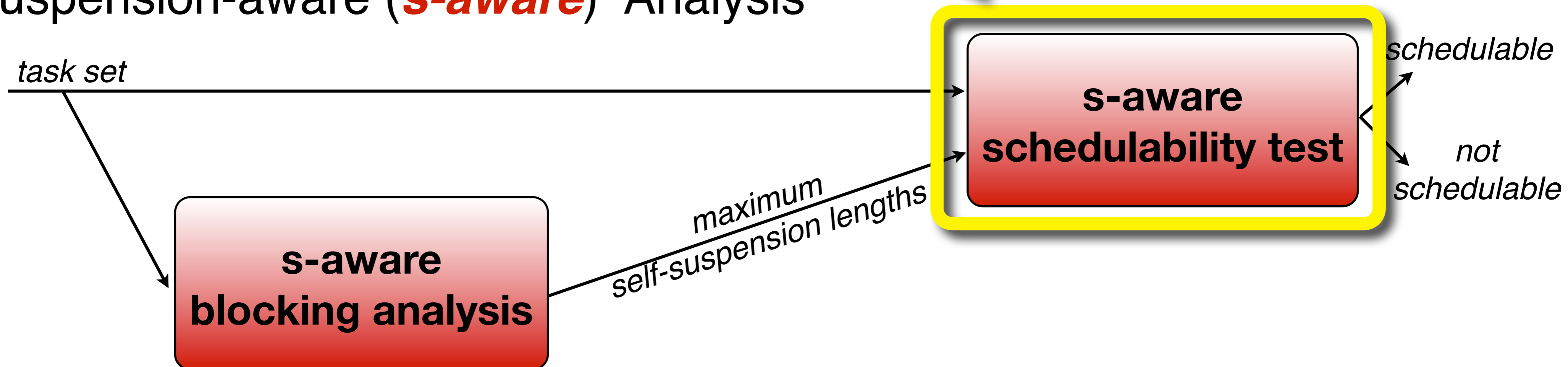


Requires availability of a schedulability test that accounts (reasonably accurately) for **self-suspensions.**
 (...which can be tricky to derive)

Suspension-oblivious (*s-oblivious*) Analysis



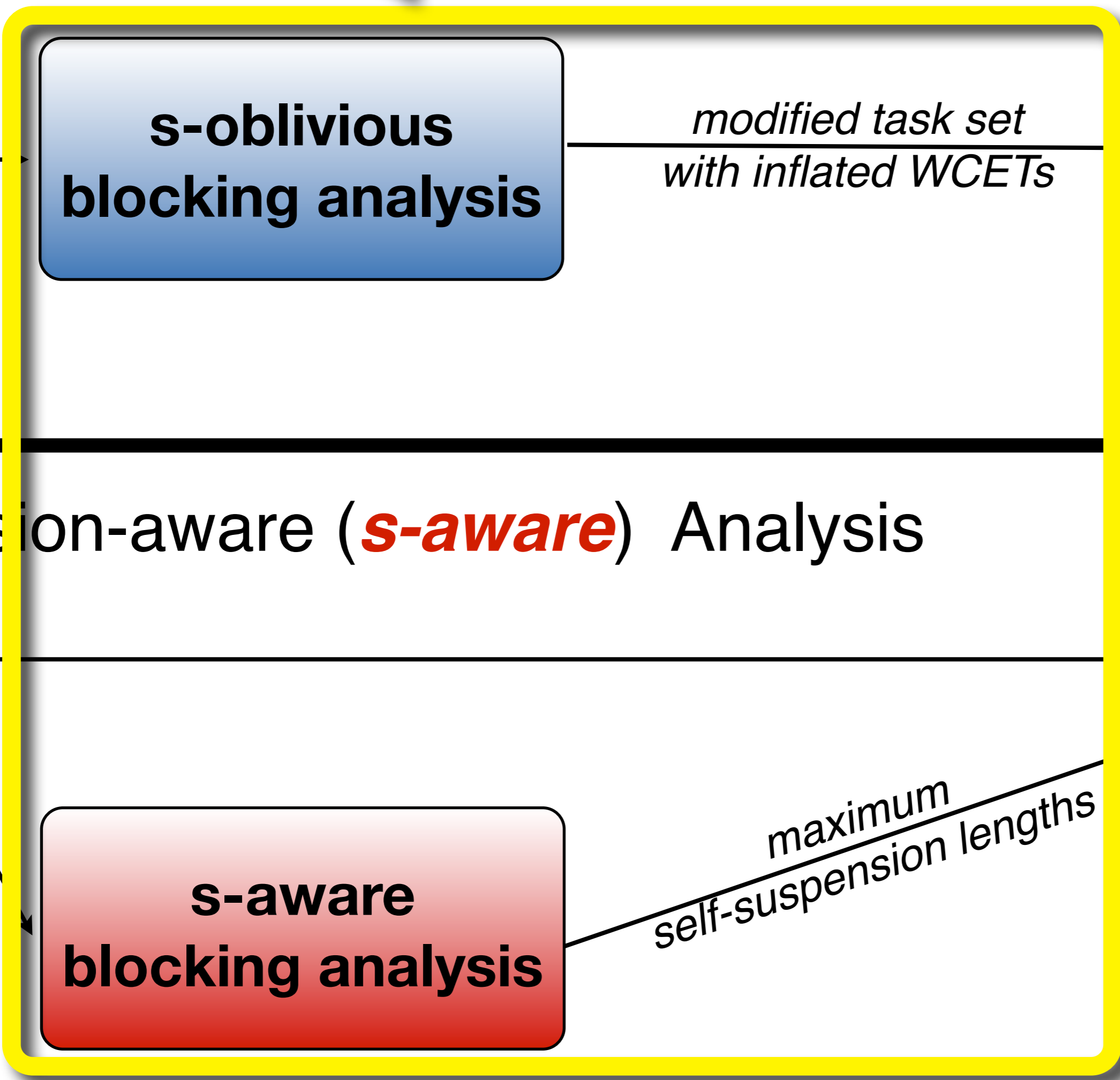
Suspension-aware (*s-aware*) Analysis



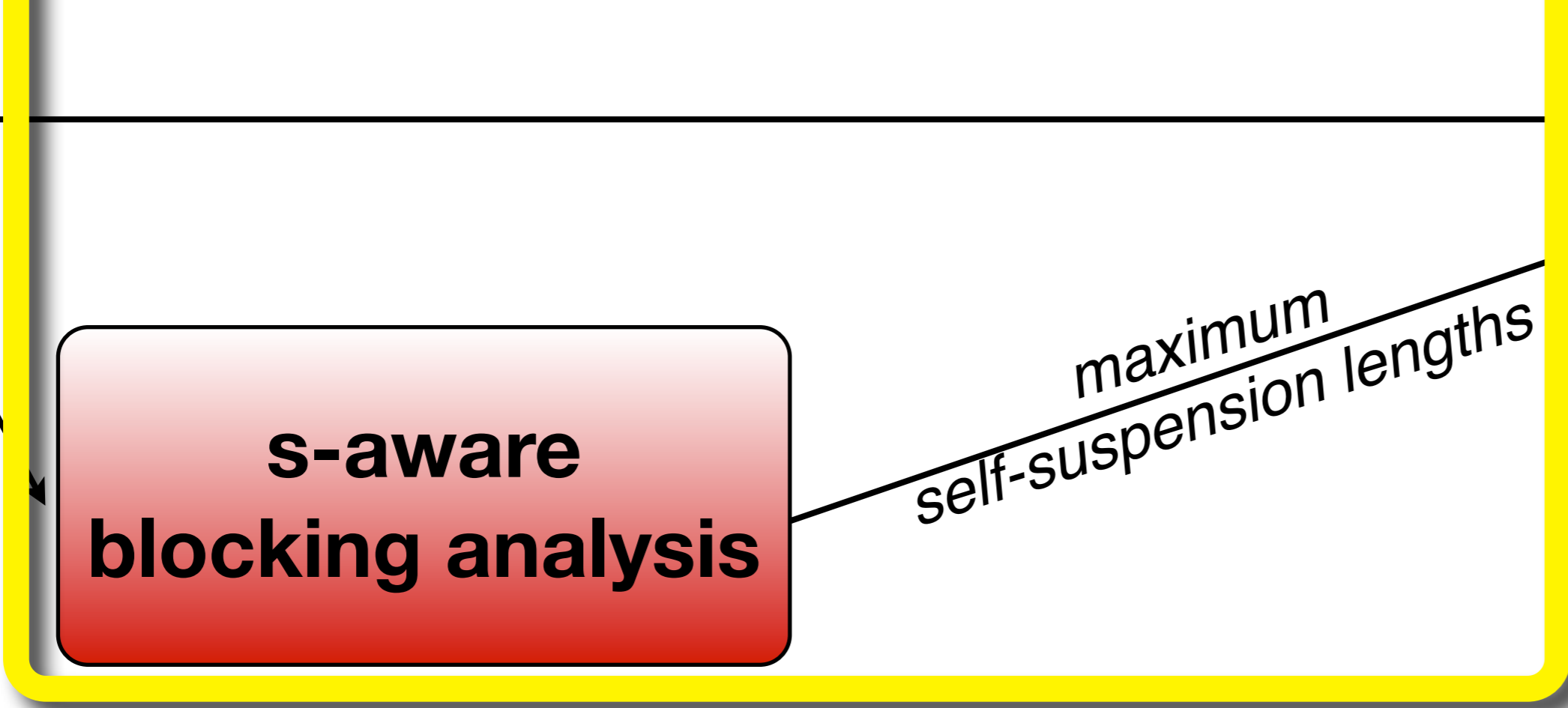
S Analysis

Different notions of “processor demand”
 → different definitions of “**priority inversion**”.

Suspension-oblivious (*s-oblivious*) Analysis

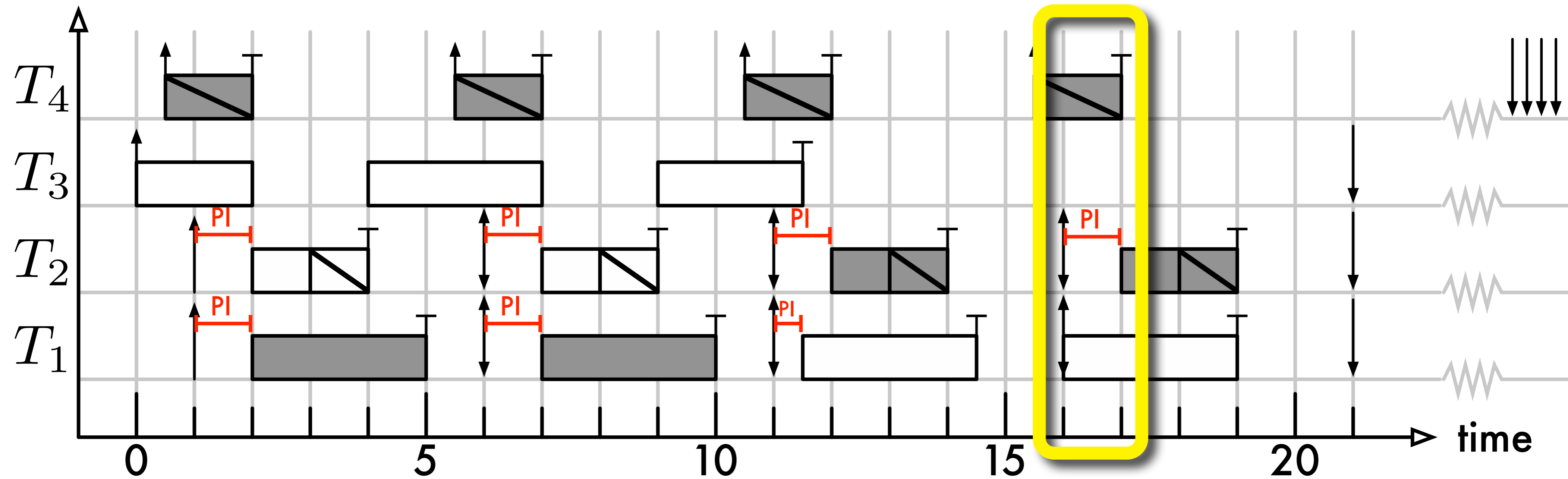


Suspension-aware (*s-aware*) Analysis



Restricted Segment Boosting at Time 16

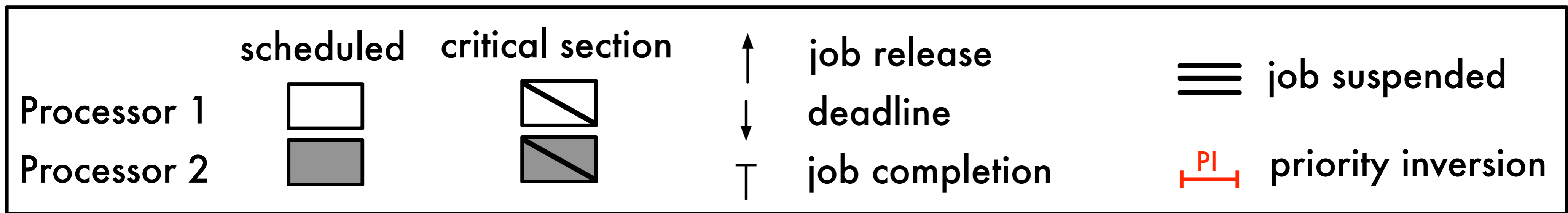
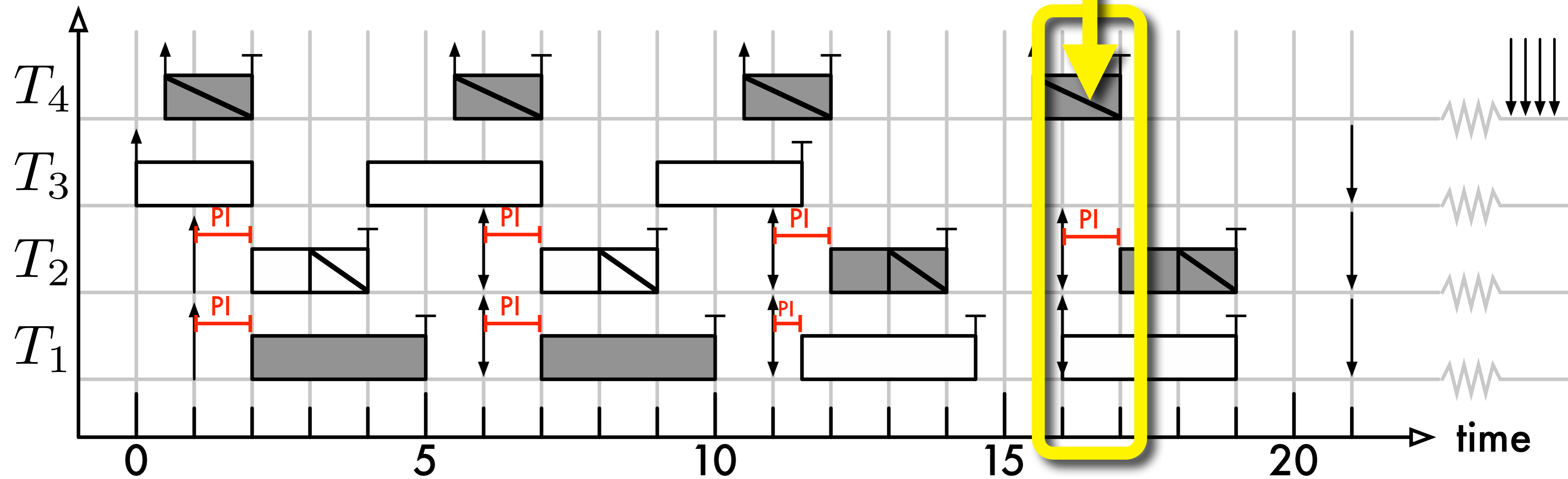
Generalized FMLP⁺ schedule.



	scheduled	critical section	↑	job release	≡	job suspended
Processor 1			↓	deadline		priority inversion
Processor 2			┘	job completion		

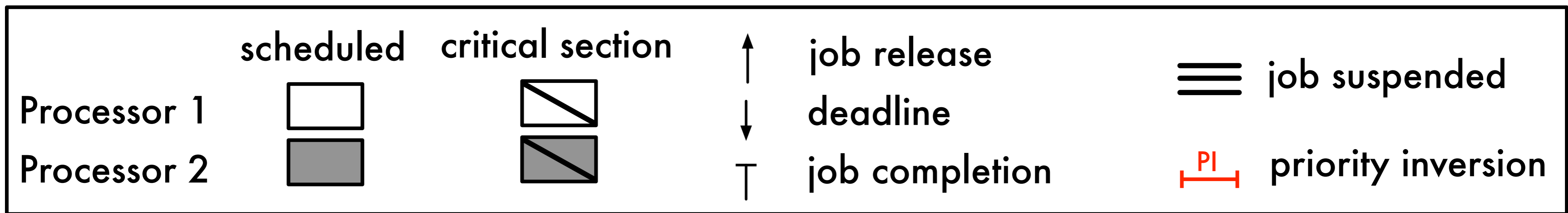
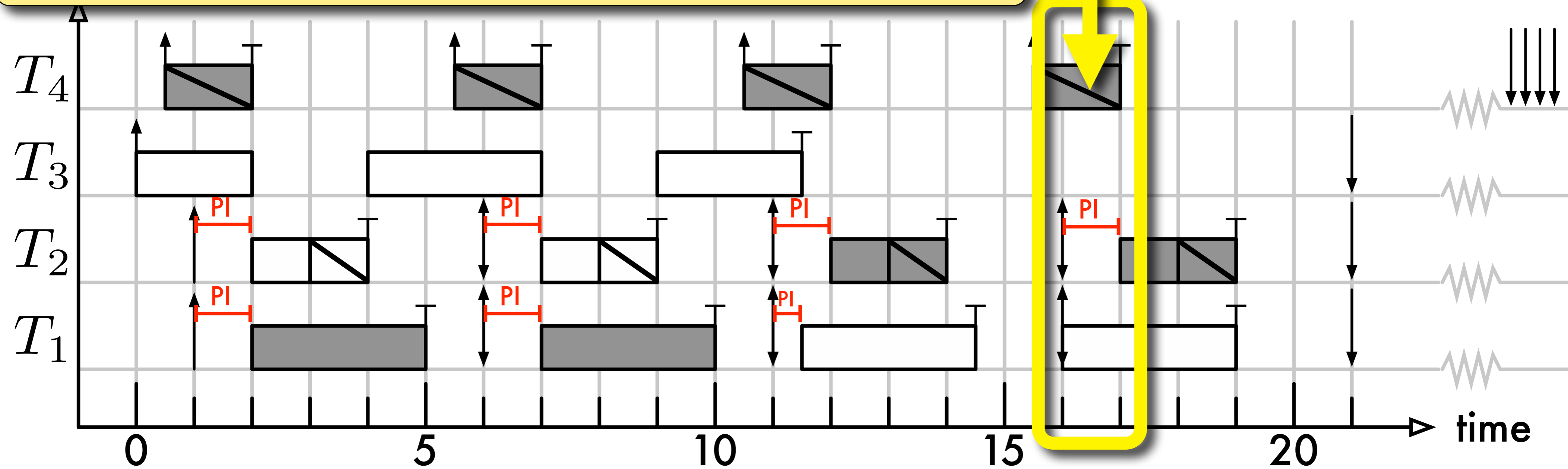
(1) The lock-holding ready job (if any) with the **earliest segment start time**.

Generalized FMLP⁺ schedule.



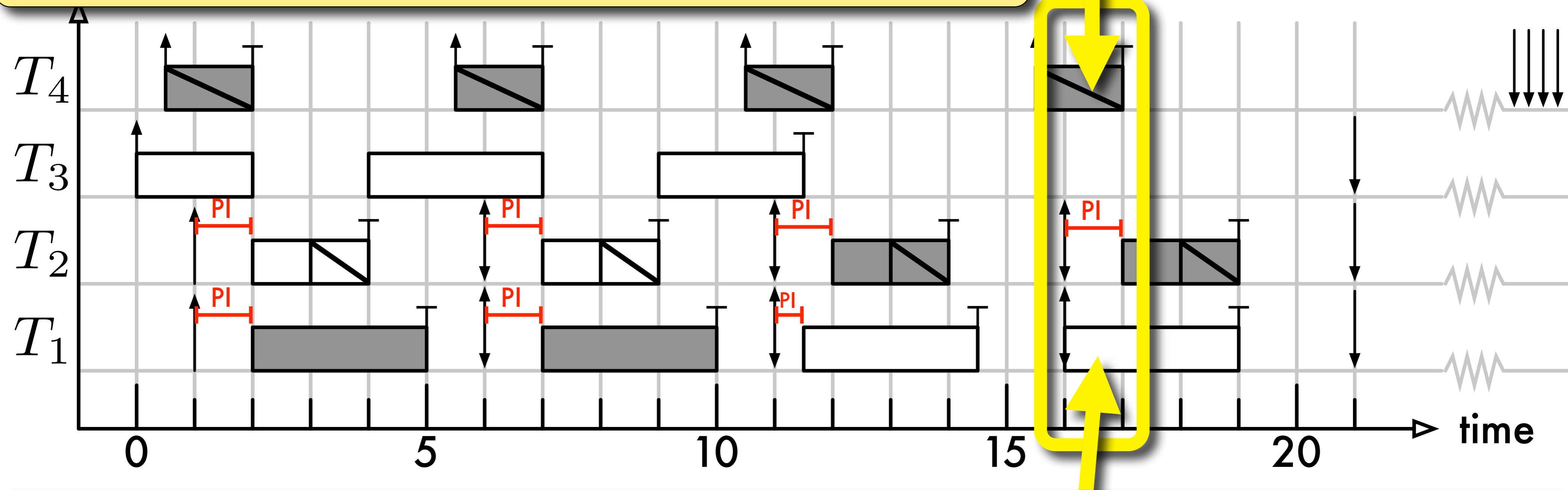
(1) The lock-holding ready job (if any) with the **earliest segment start time**.

(2) Up to $c - 1 = 1$ jobs from T_4 's **co-boosting set** = \emptyset .



(1) The lock-holding ready job (if any) with the **earliest segment start time**.

(2) Up to $c - 1 = 1$ jobs from T_4 's **co-boosting set** = \emptyset .

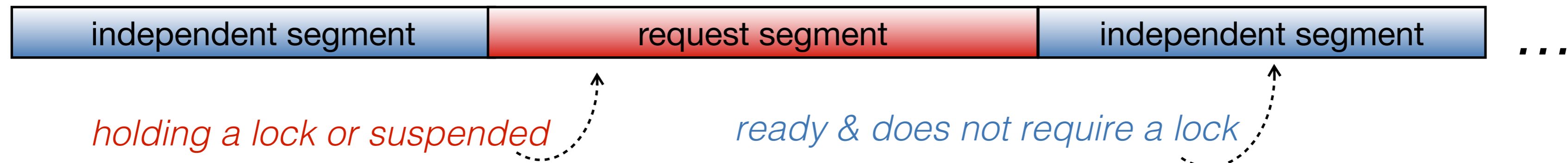


(3) If less than $c = 2$ jobs scheduled so far: **any other ready jobs**.

At time 16: one **CPU** available after steps 1 & 2
 → schedule **highest-priority task** T_1 .

Definition: Job Segments

a job at runtime:



Independent segment

- starts when a job is **released** or **resumed**, or when it **unlocks a resource**
- ends when job **completes**, **suspends**, or **requests a lock**

Request segment

- starts when a job **requests a lock**
- ends when it **unlocks the resource**