End-to-end deadlines over dynamic topologies

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ECRTS'19

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Cloud + IoT + Industry = True?

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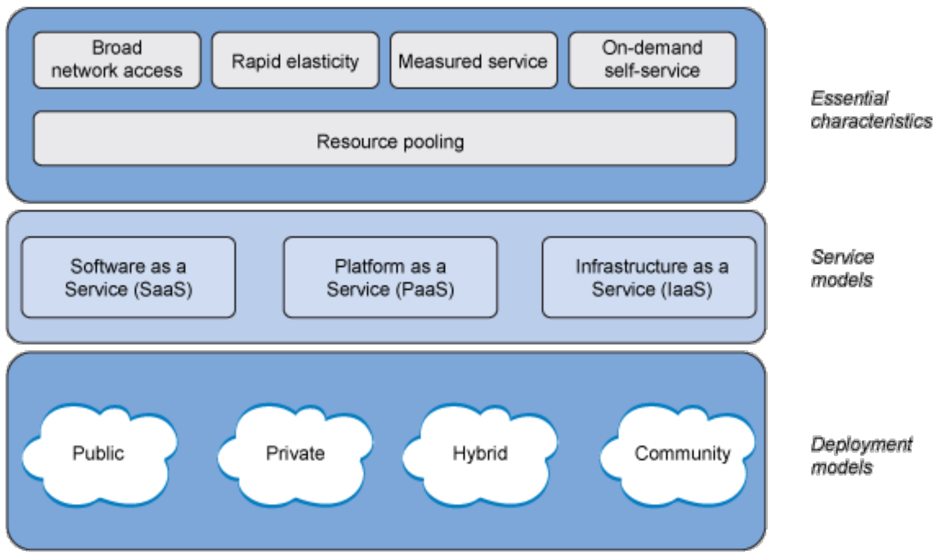

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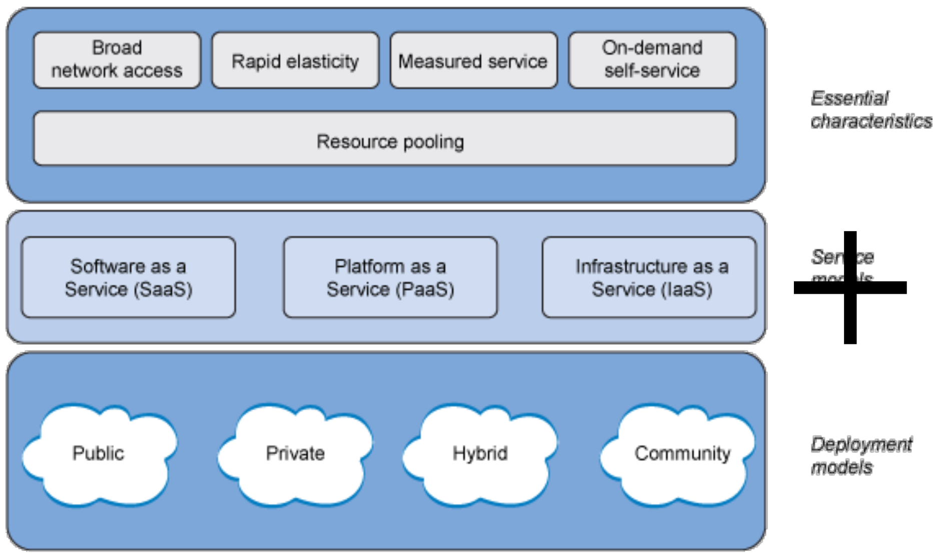

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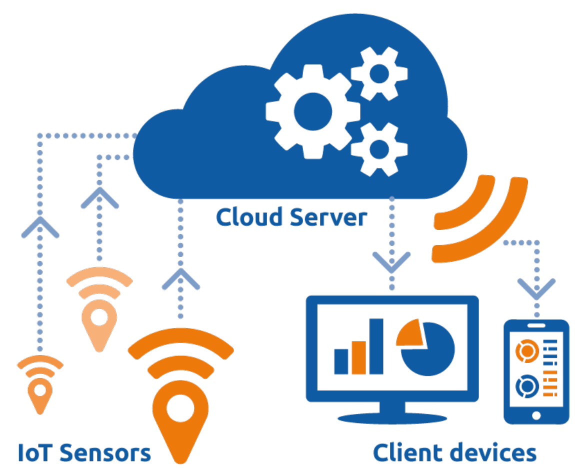

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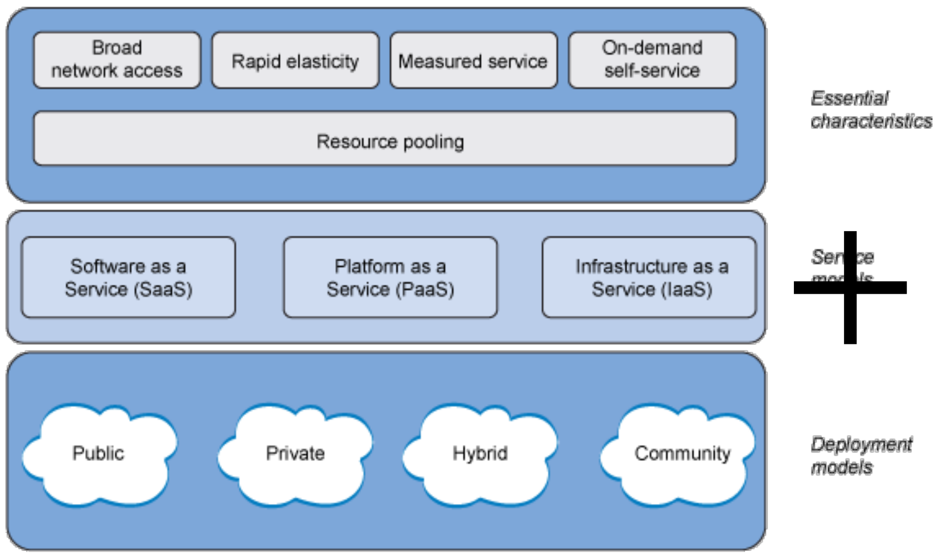

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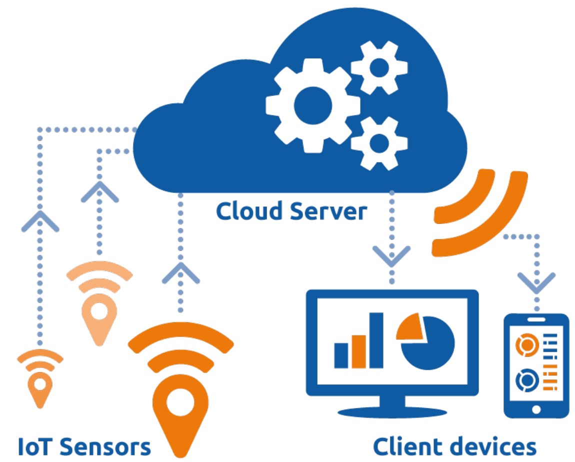

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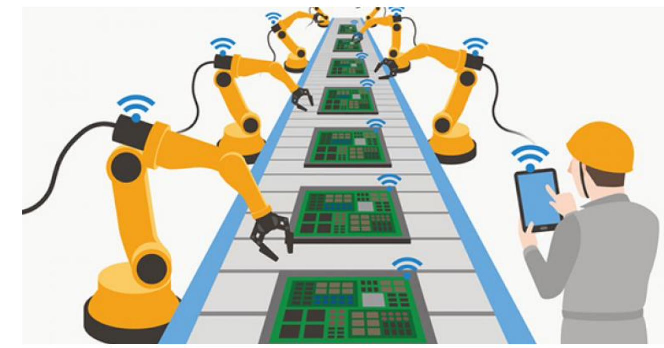

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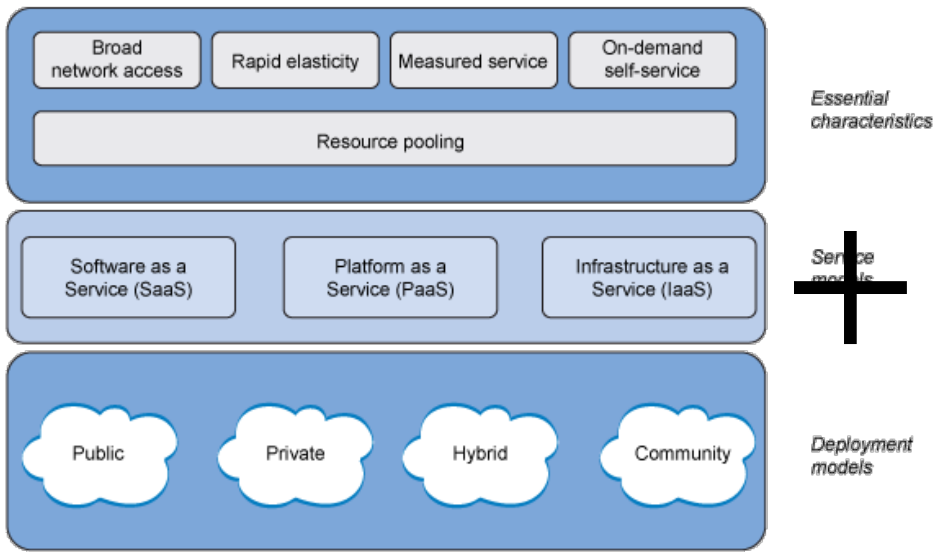

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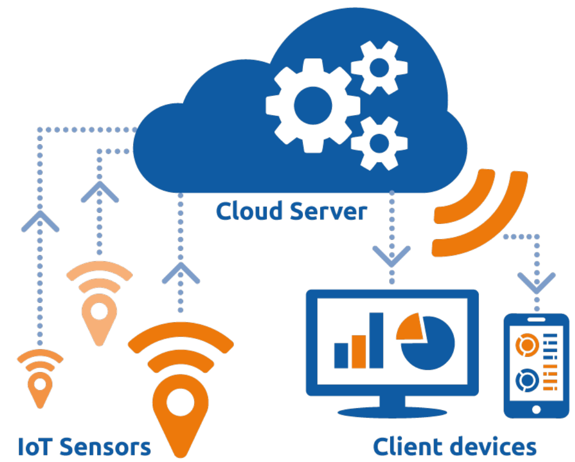

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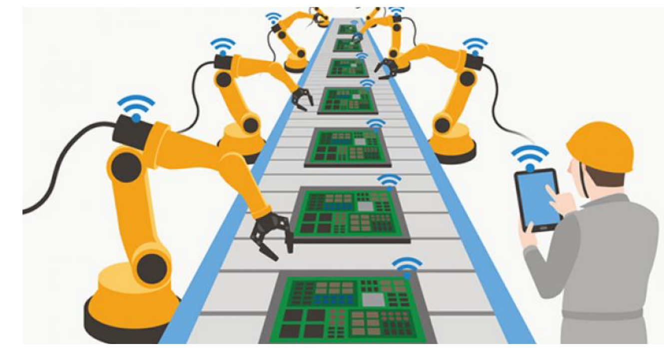

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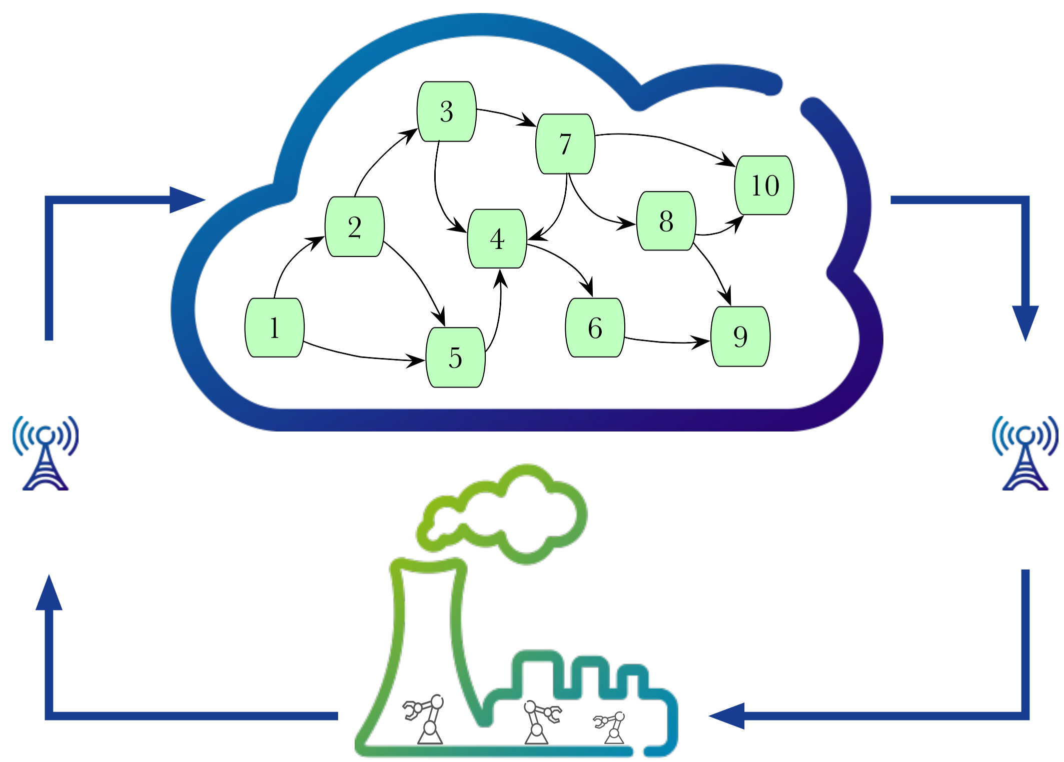

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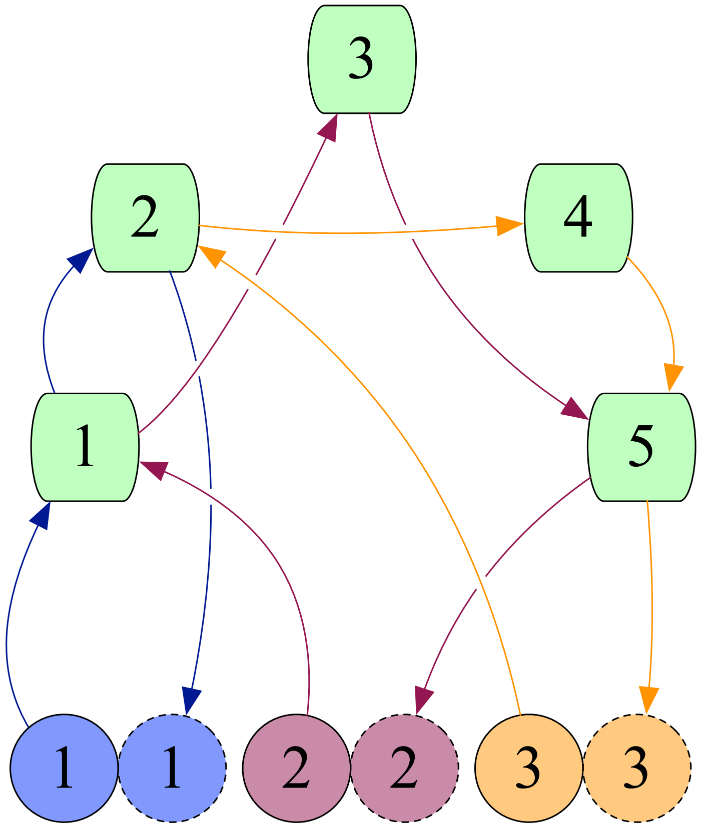
Modeling of the network flows



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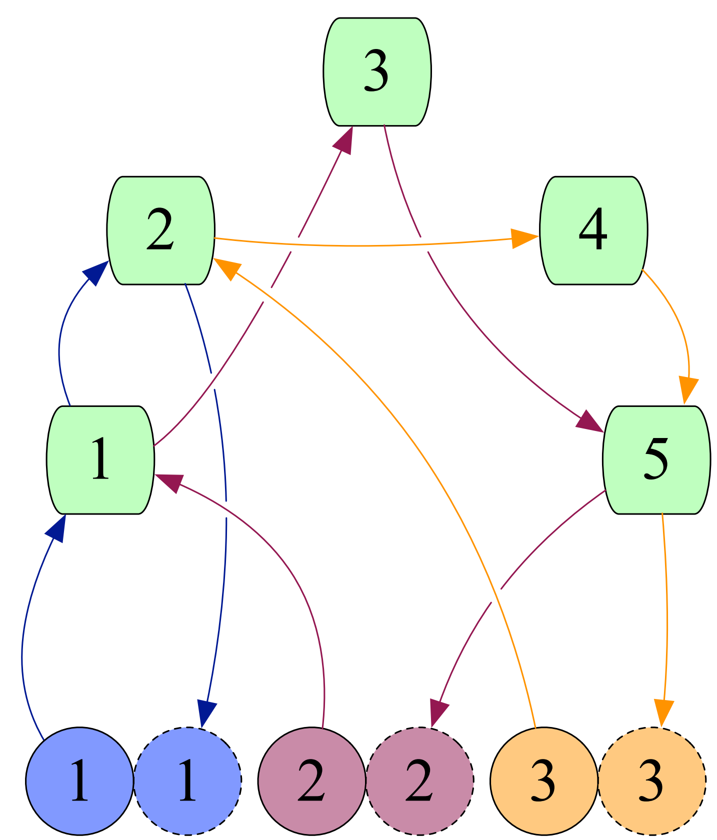
$$p_2 = \{1, 3, 5\}$$

Modeling of the network flows

A network $\mathscr{G} = \{\mathscr{V}, \mathscr{F}\}$



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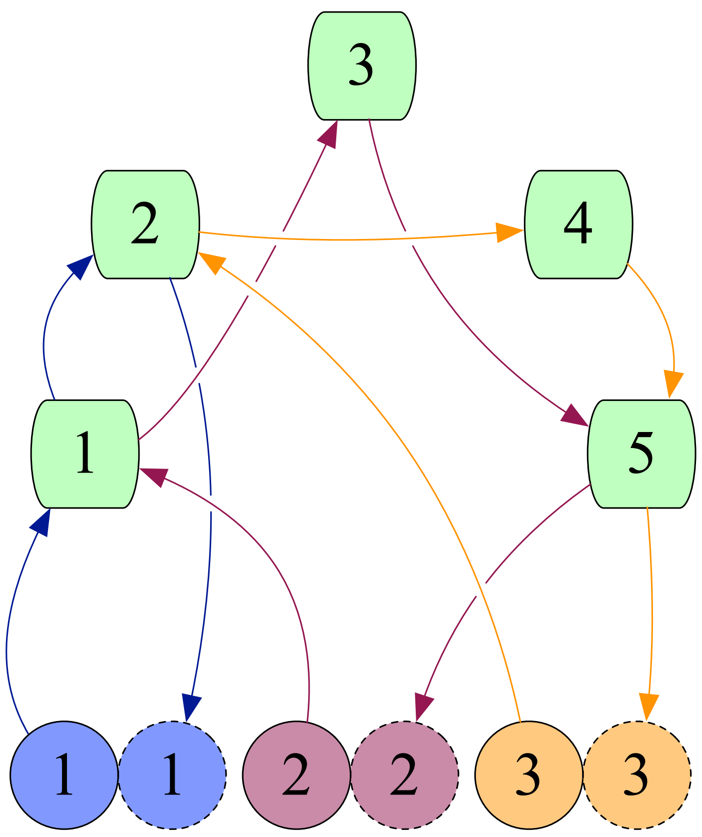

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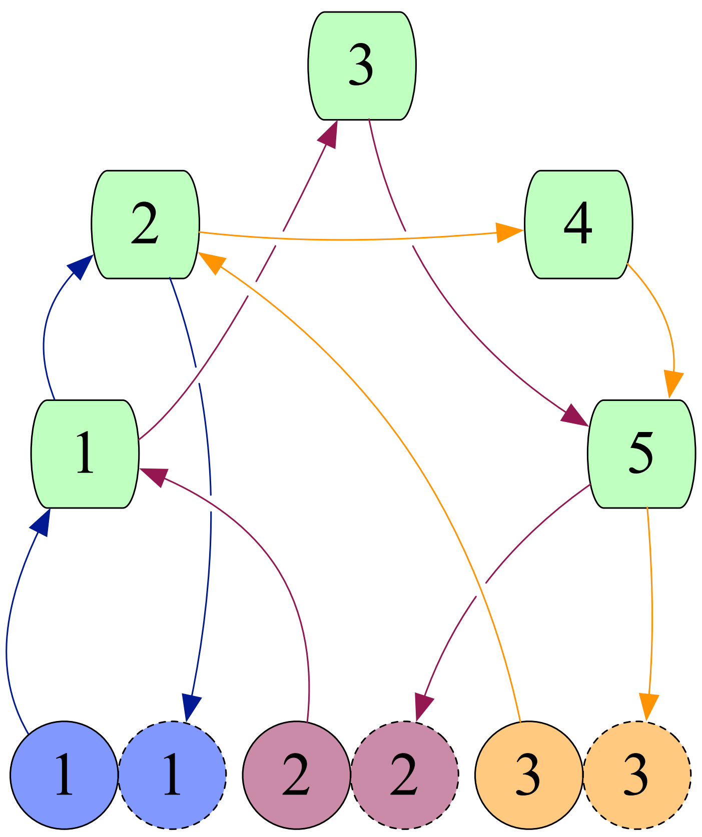

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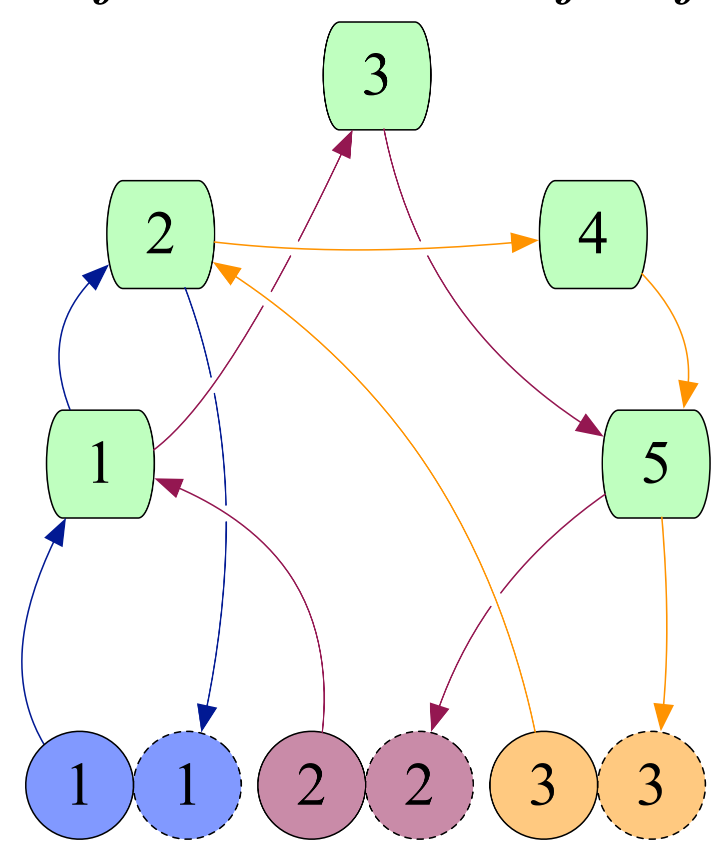

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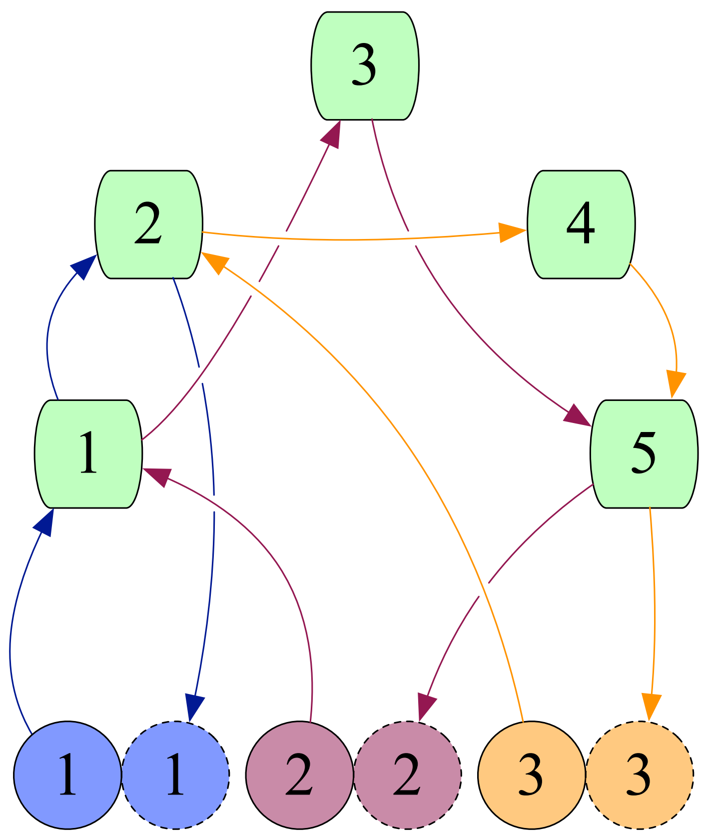

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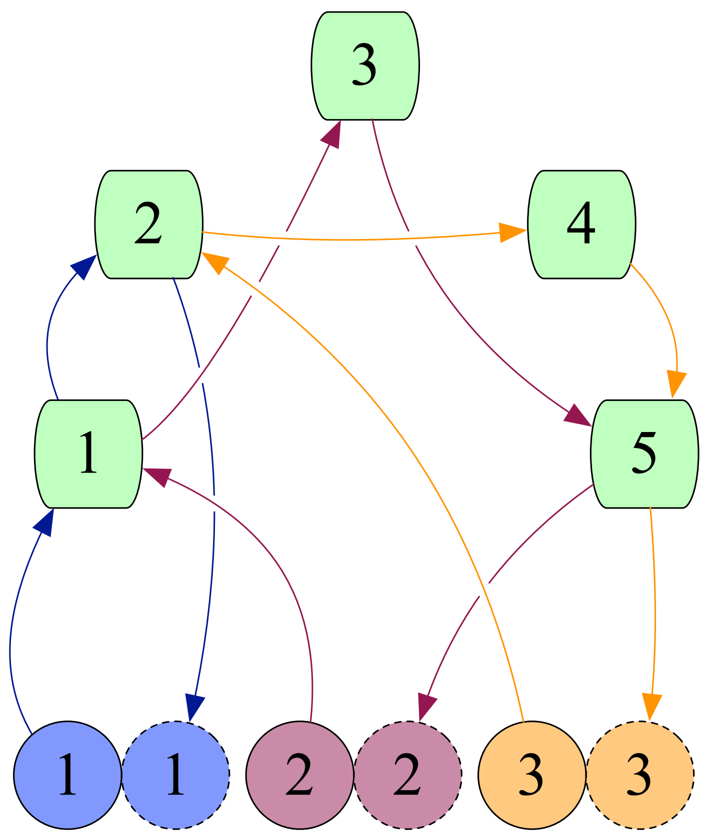

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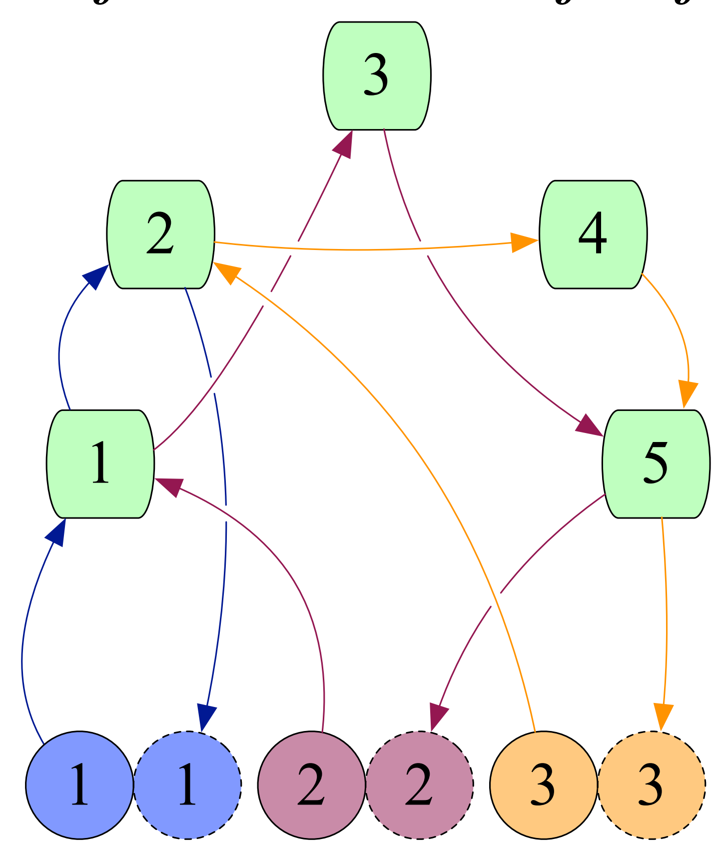

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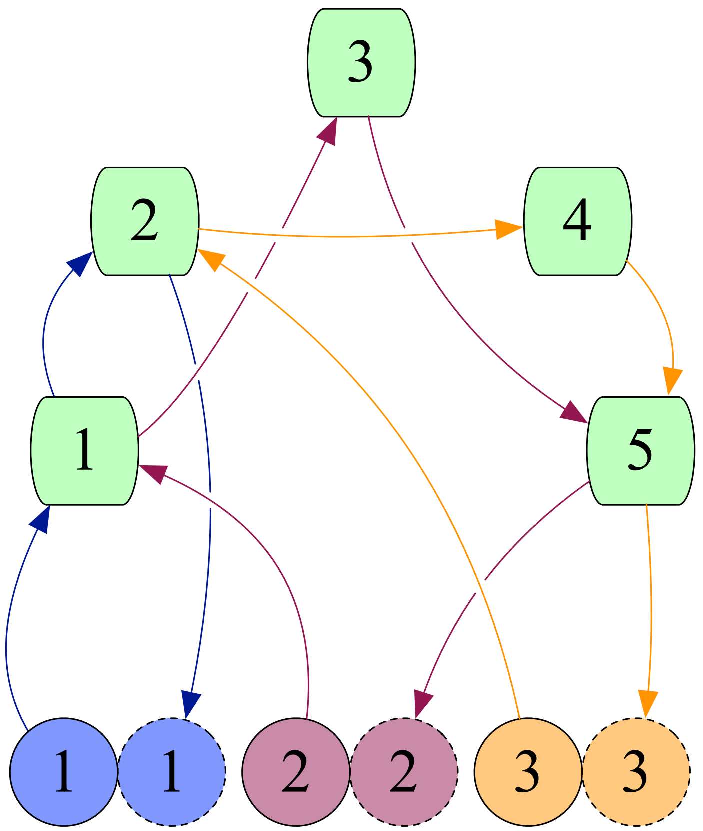
Goal:

The goal of the work is to ensure that the end-to-end response-time of every flow is less then their end-to-end deadline:

$$\forall t \geq 0, \forall j \in \mathscr{F}, \quad \mathscr{R}_j(t) \leq \mathscr{D}_j \, .$$

Modeling of the nodes



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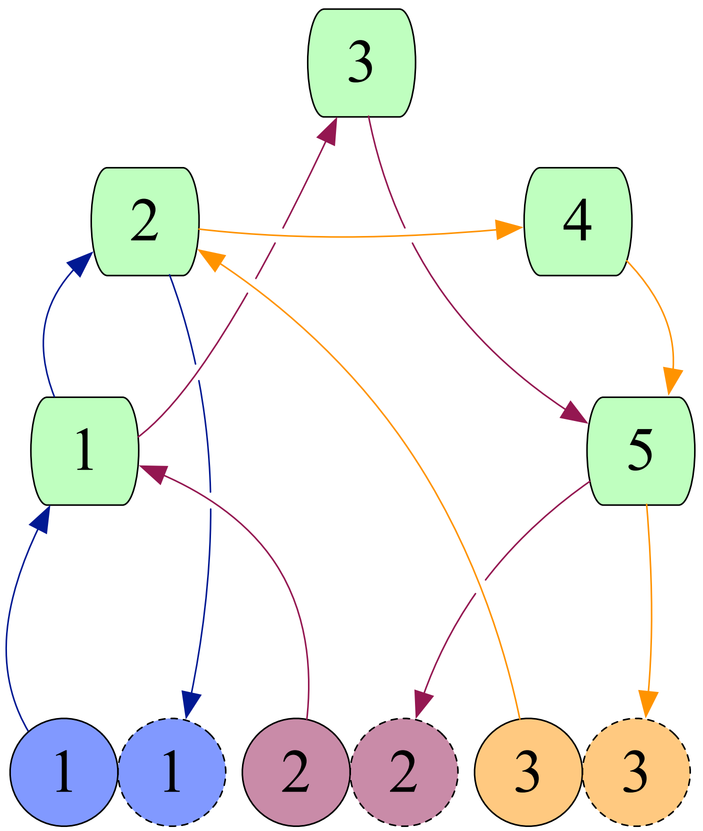

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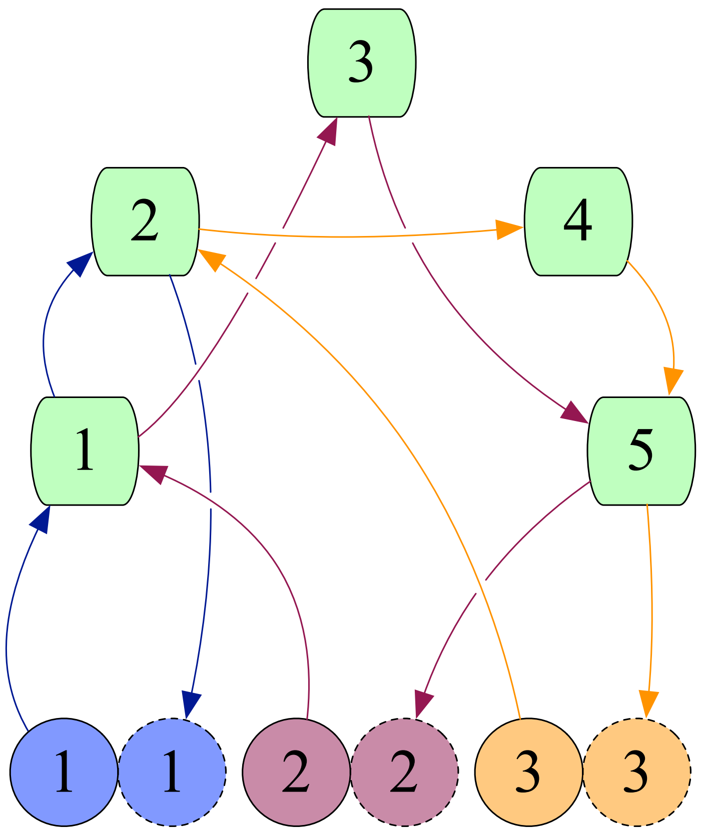

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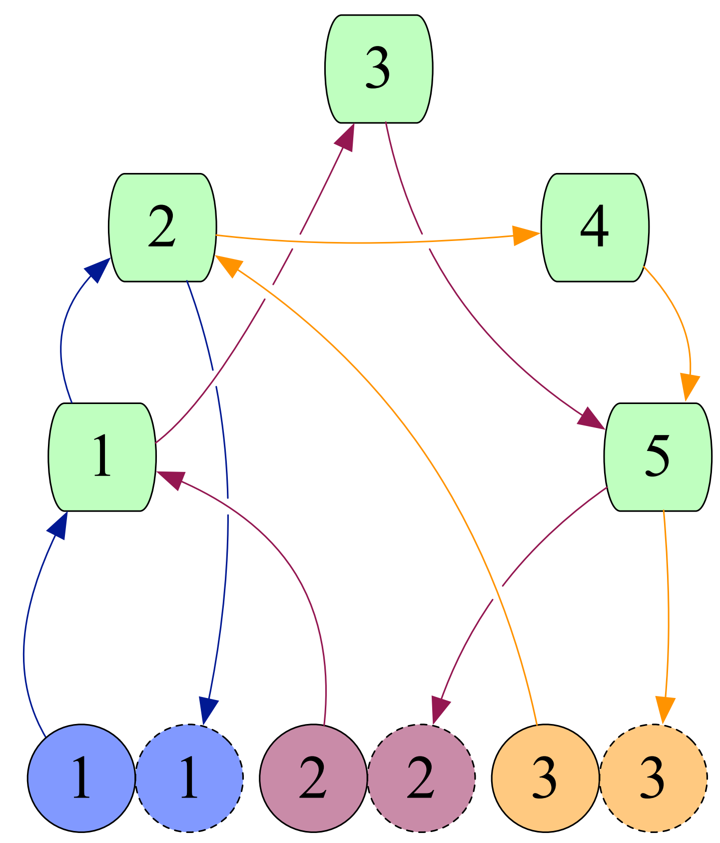

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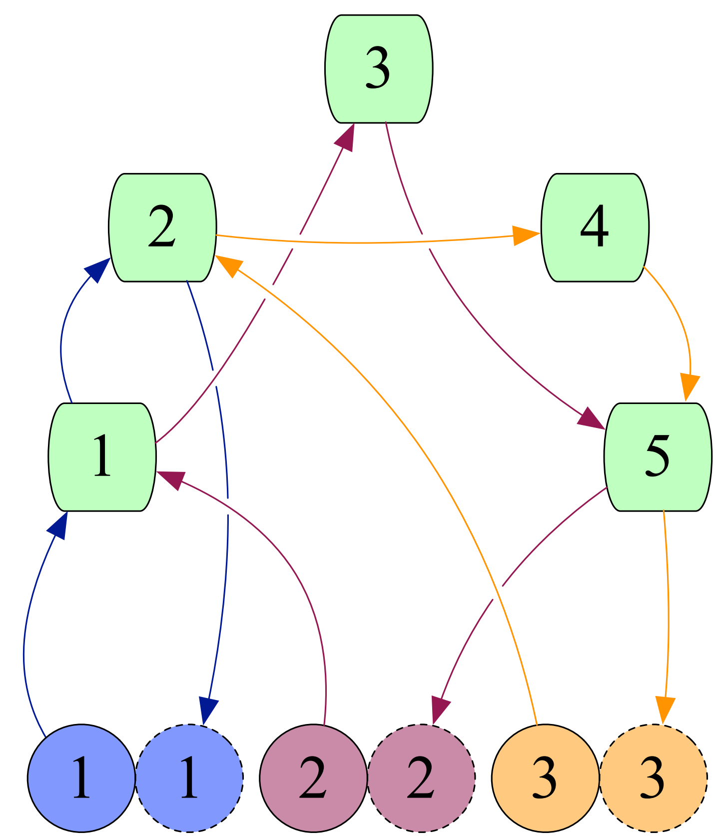
Definition of response-time $R_i(t)$:

The time it takes a packet entering node $i \in \mathscr{V}$ at time t to be processed.



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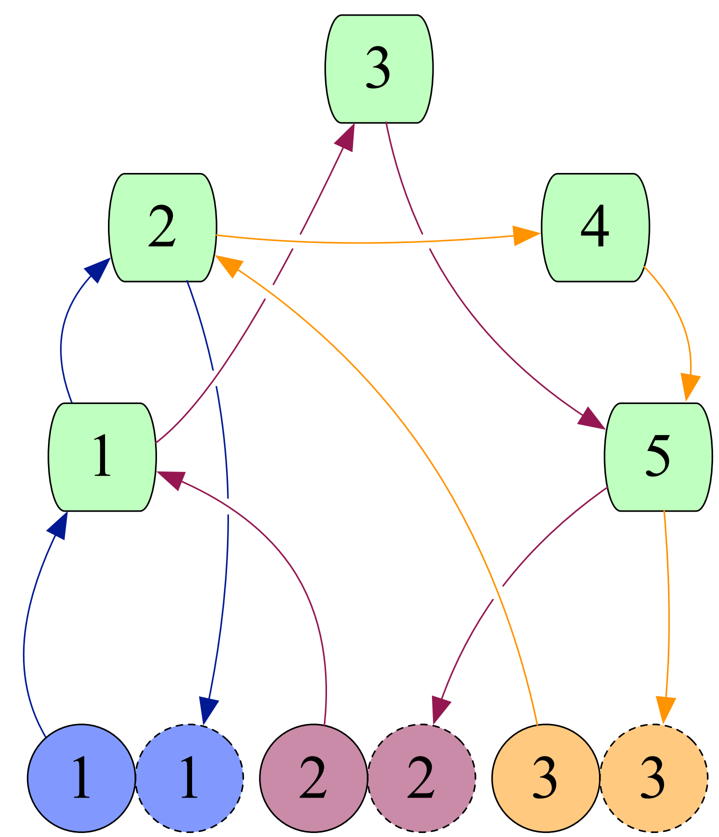

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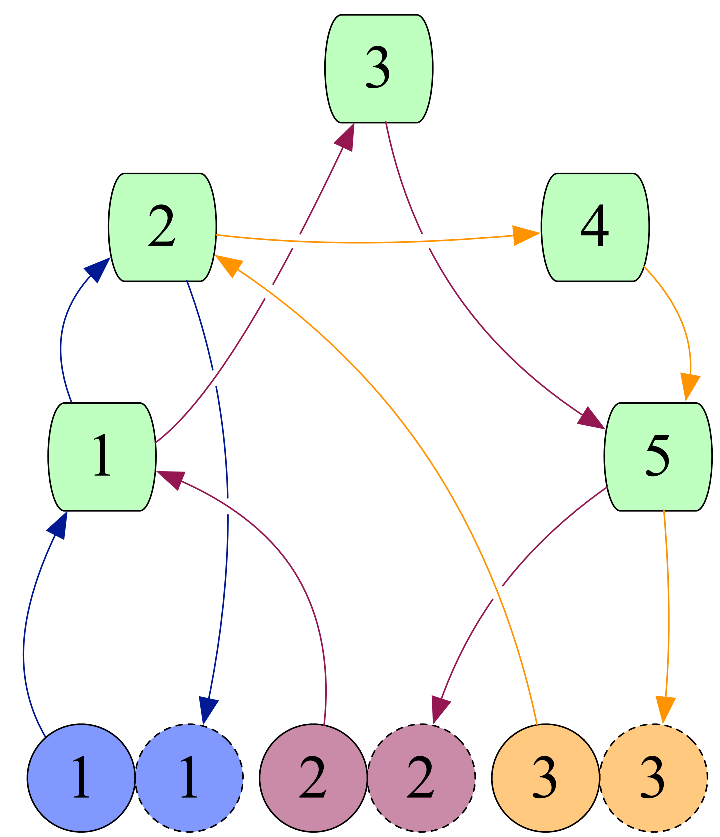

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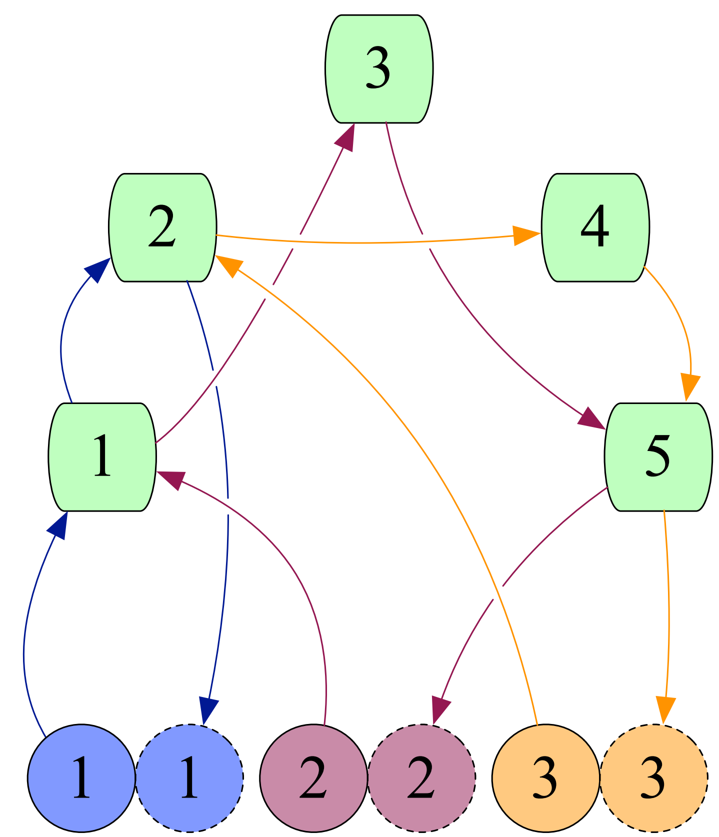

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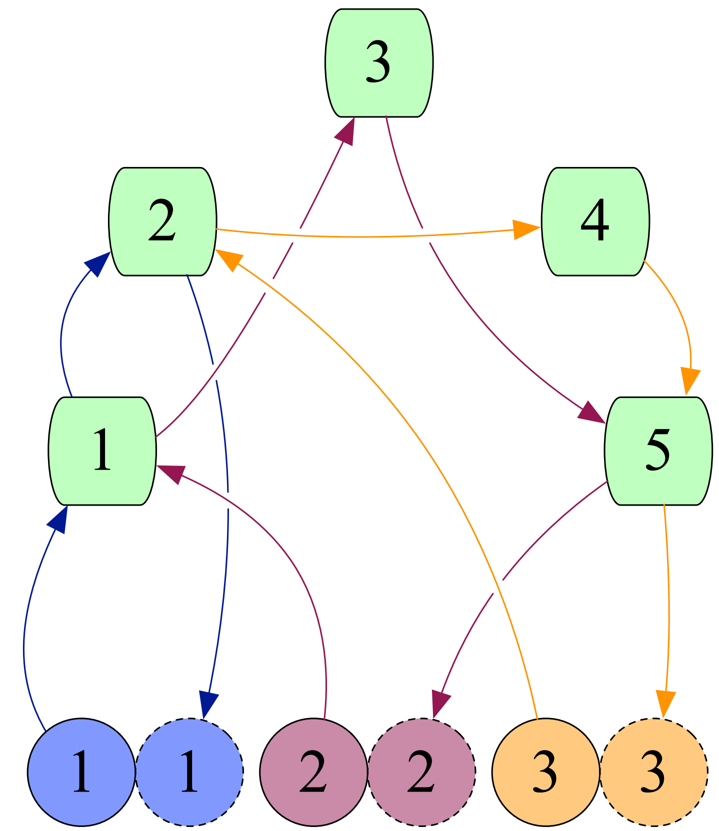

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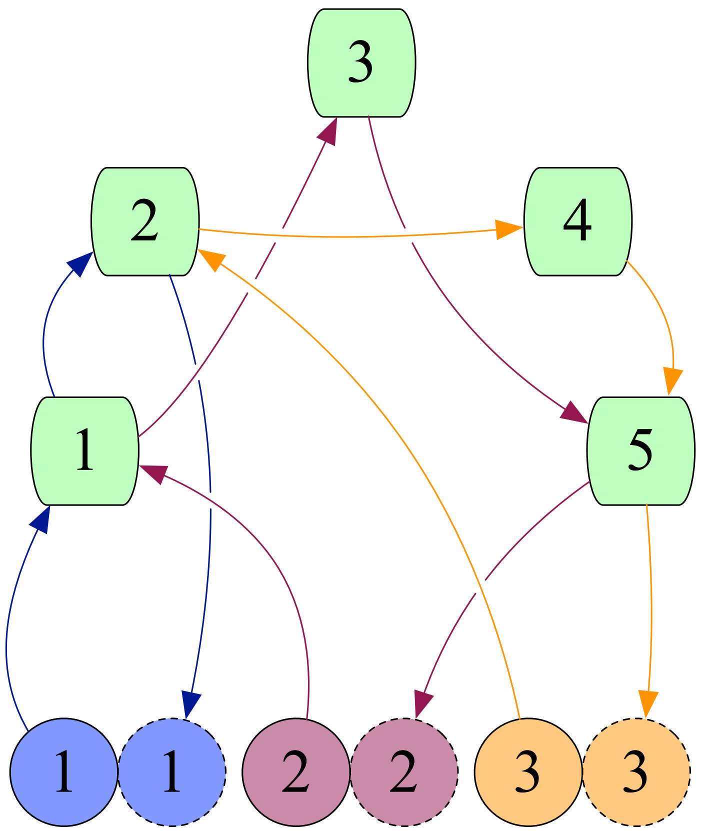
- We do not focus on how this is computed!

Response-time control



Response-time control

Assumption (response-time control):

We assume that every node $i \in \mathscr{V}$ can guarantee that its response-time $R_i(t)$ is less than its node deadline $D_i(t)$:

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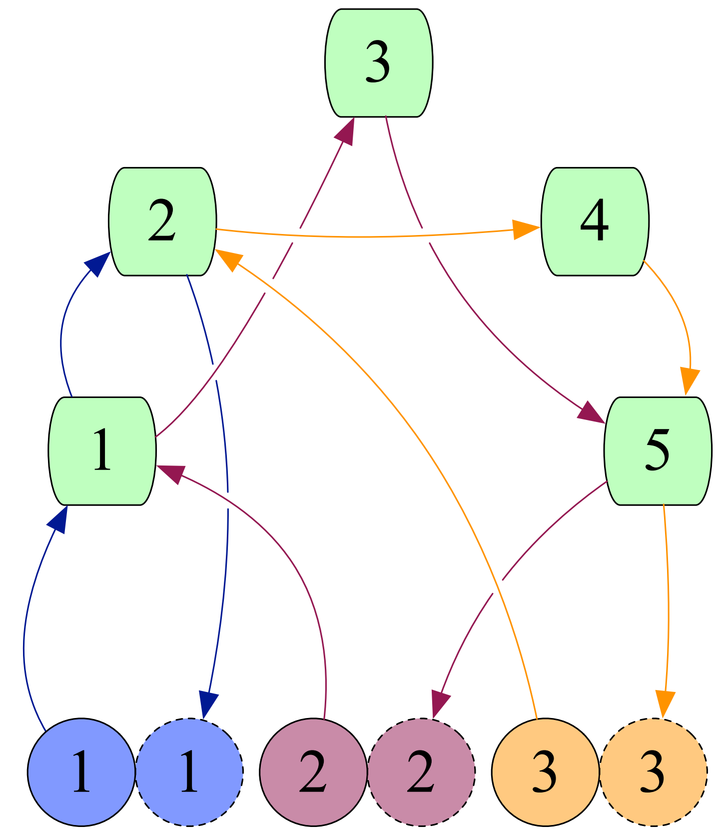

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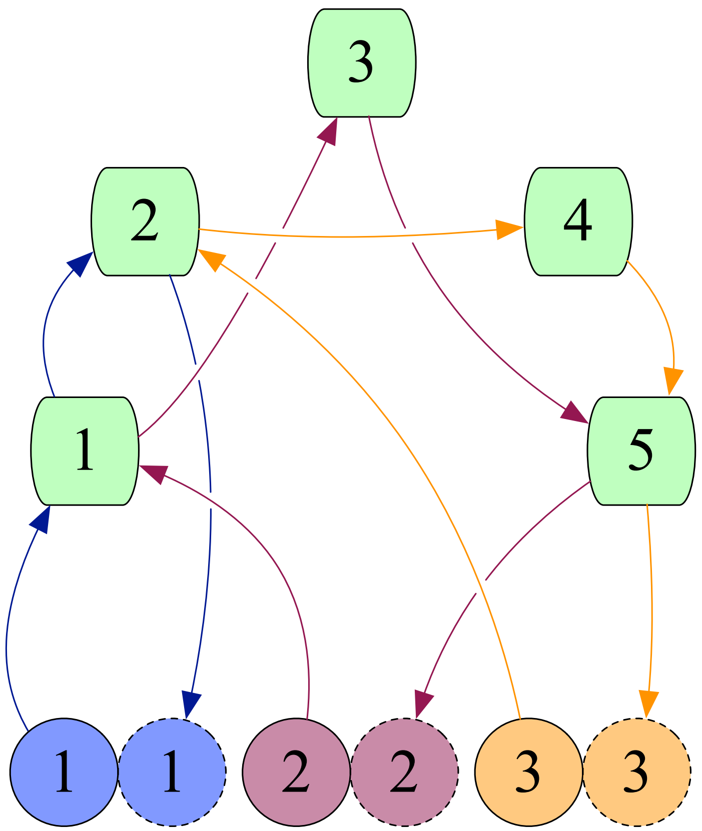

Ways to enforce $R_i(t) \leq D_i(t)$:

- Brown-out control [18].
- Scaling of processing capacity [7].

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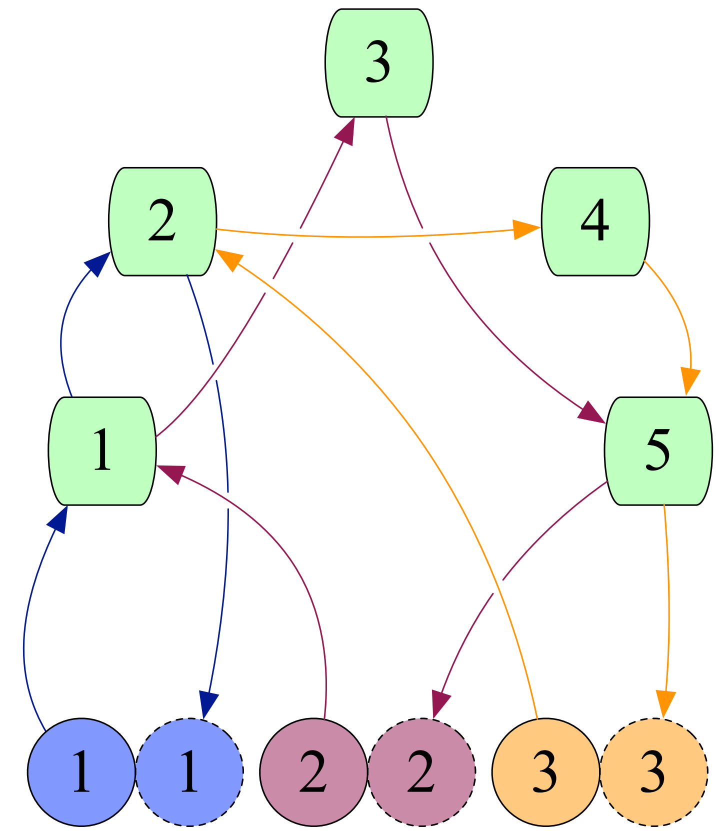

18 Tommi Nylander, Marcus Thelander Andrén, Karl-Erik Årzén, and Martina Maggio. Cloud application predictability through integrated load-balancing and service time control. In *2018 IEEE International Conference on Autonomic Computing (ICAC)*, pages 51–60. IEEE, 2018.

7 Dan Henriksson, Ying Lu, and Tarek Abdelzaher. Improved prediction for web server delay control. In *Real-Time Systems, 2004. ECRTS 2004. Proceedings. 16th Euromicro Conference on*, pages 61–68. IEEE, 2004.

What's the goal here?

The goal:

1. Guarantee that the flows meet their end-to-end deadlines:

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2. Allow flows to be able to join the network.

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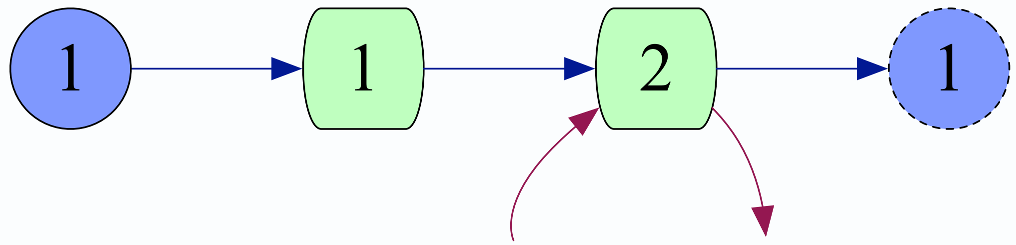
Control knobs:

1. Control the deadlines $D_i(t)$ of the nodes.
2. Control when new flows can join.

Why is this difficult?

2-node example:

Assume a simple system with two nodes and two flows:



End-to-end deadline: $\mathscr{D}_1 = 6.$

Let's assume that node 2 wishes to change its node deadline from 5 to 1 because of a new flow joining the system.

$$D_2(t_1) = 5 \;\rightarrow\; D_2(t_2) = 1$$

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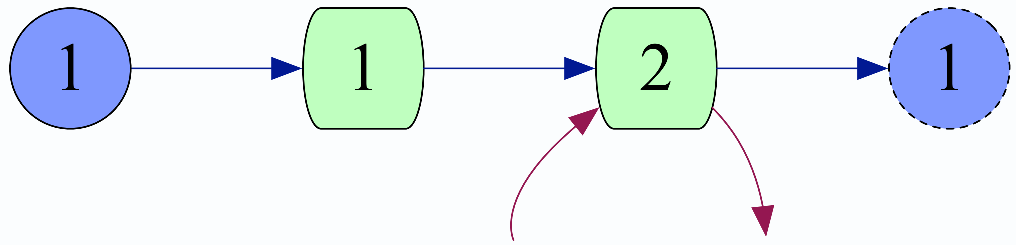

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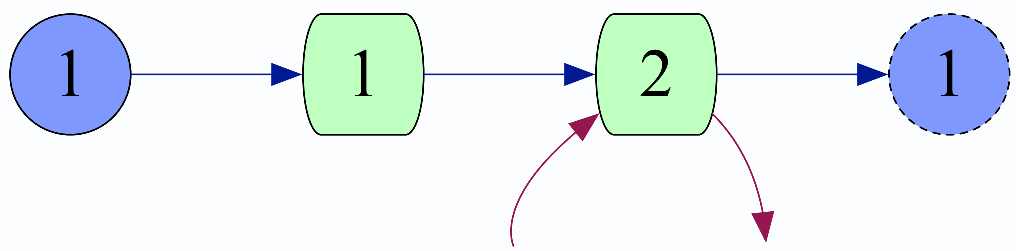

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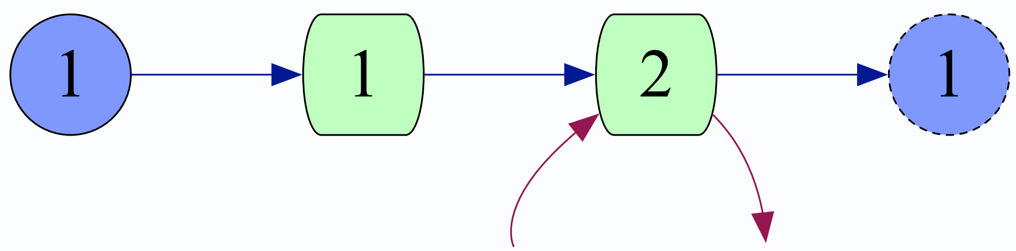

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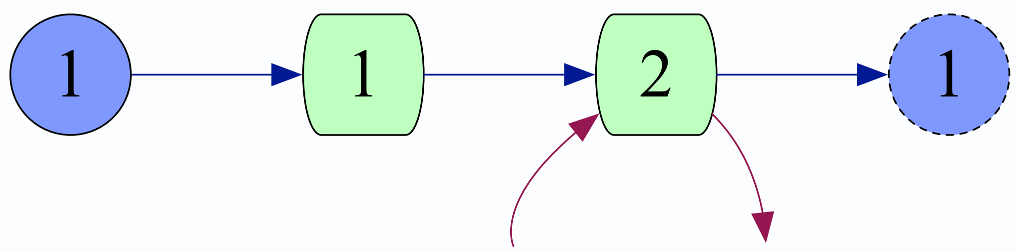

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Change them linearly from $t_1 \rightarrow t_2$

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Let's be evil and assume:

The largest possible response-time:

$$\forall t \geq 0, \quad R_i(t) = D_i(t).$$

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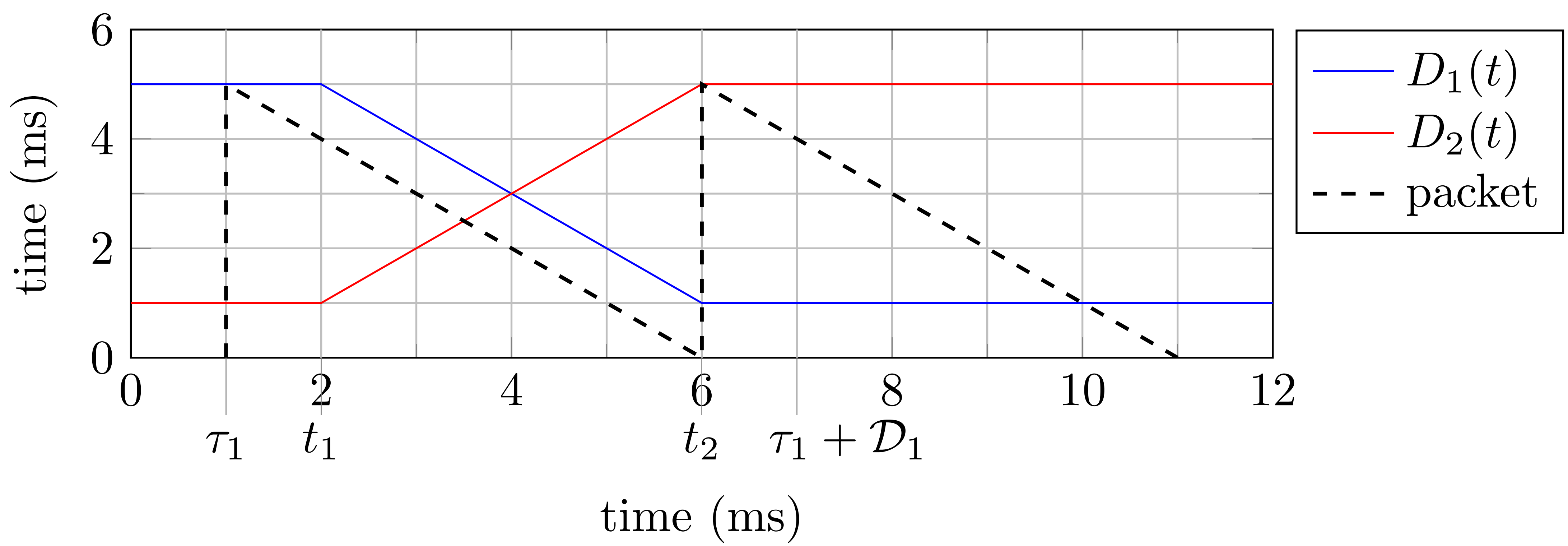

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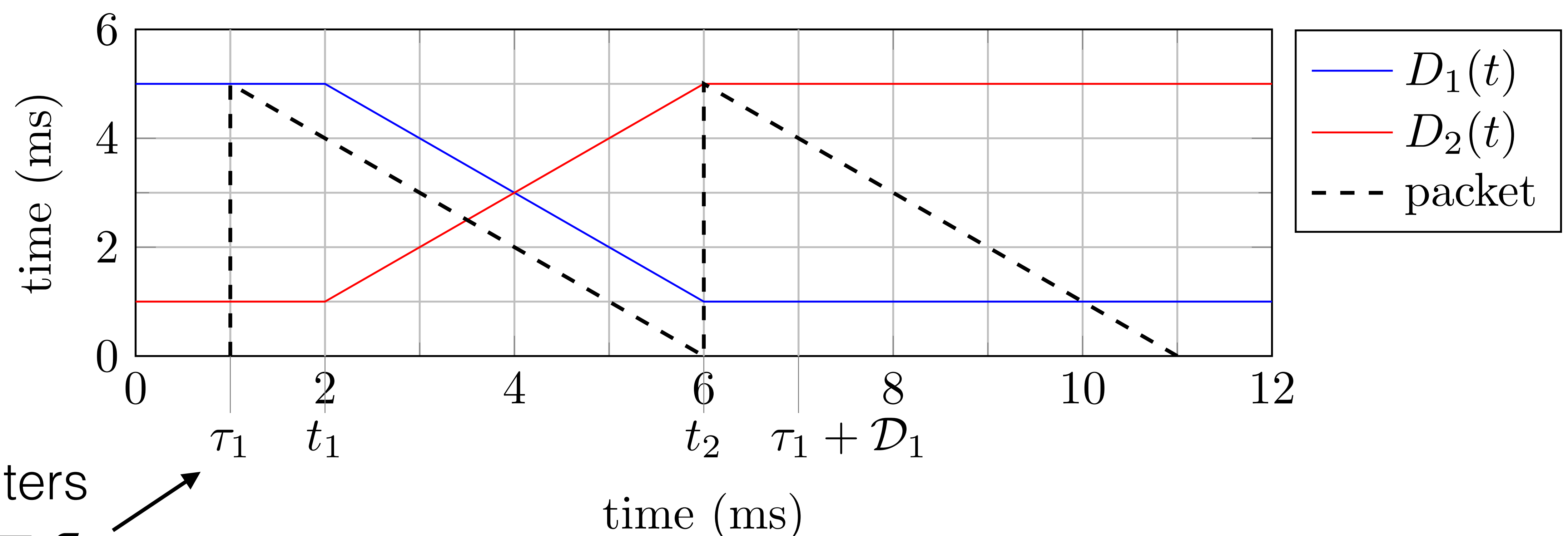


a packet enters node 1 at $t = \tau_1$

Does it work?

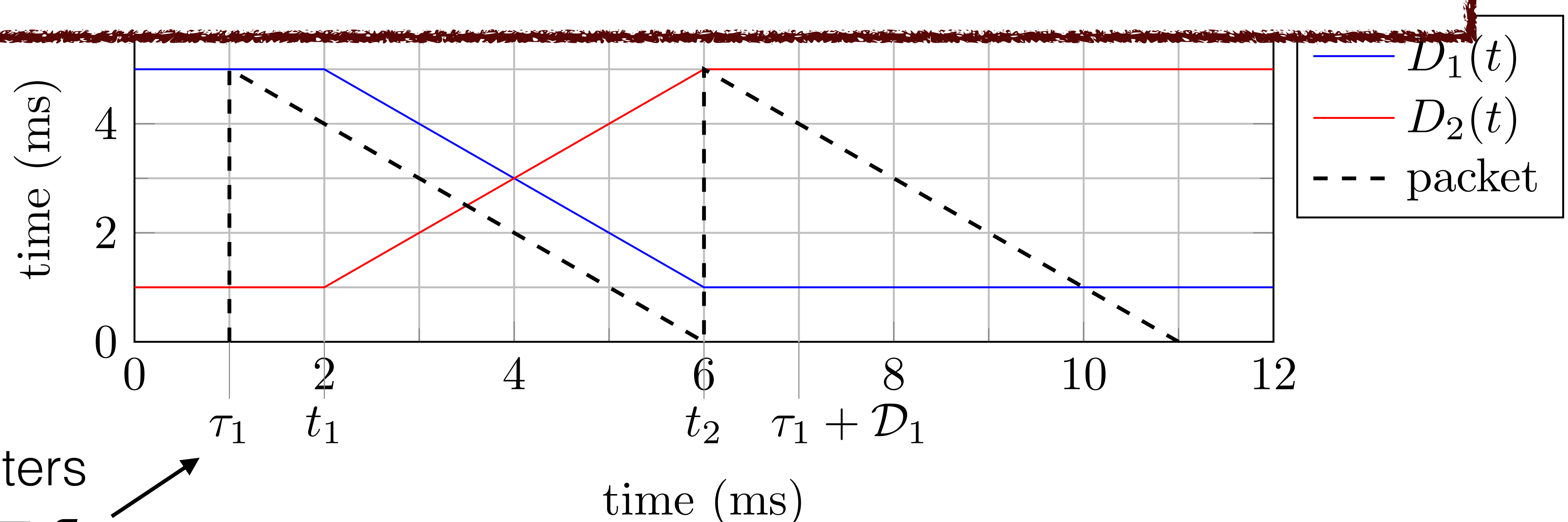
Cha

$D_1(t$

$D_2(t$

$\forall t \geq$

Oops!!!!!



a packet enters node 1 at $t = \tau_1$

Does it work?

Cha

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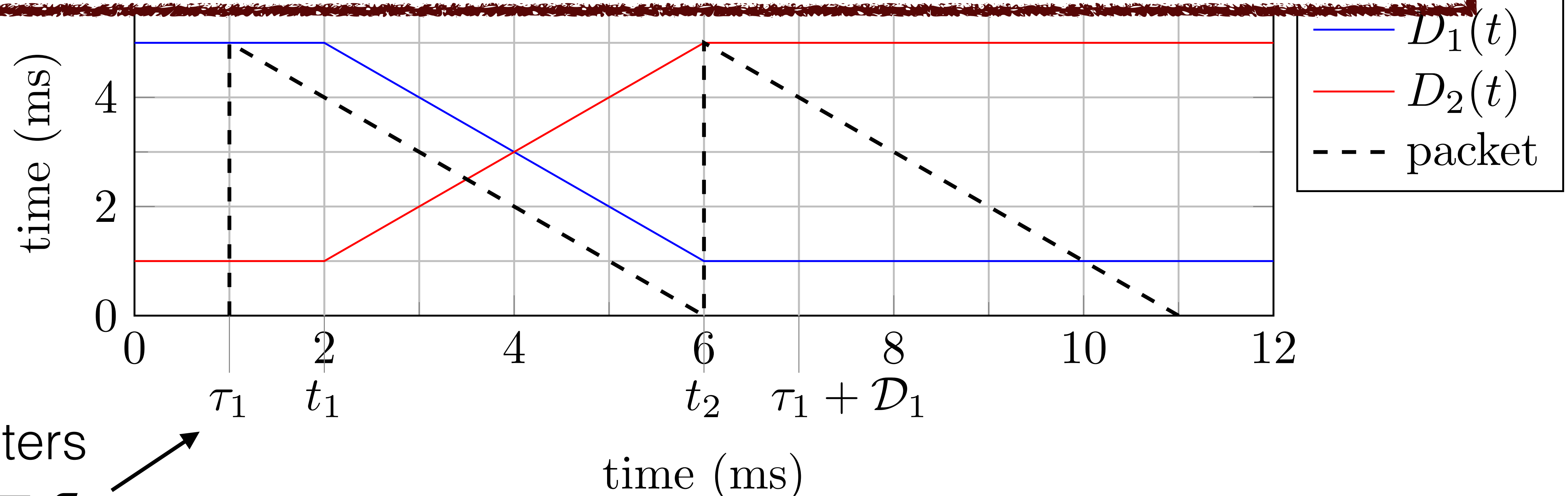

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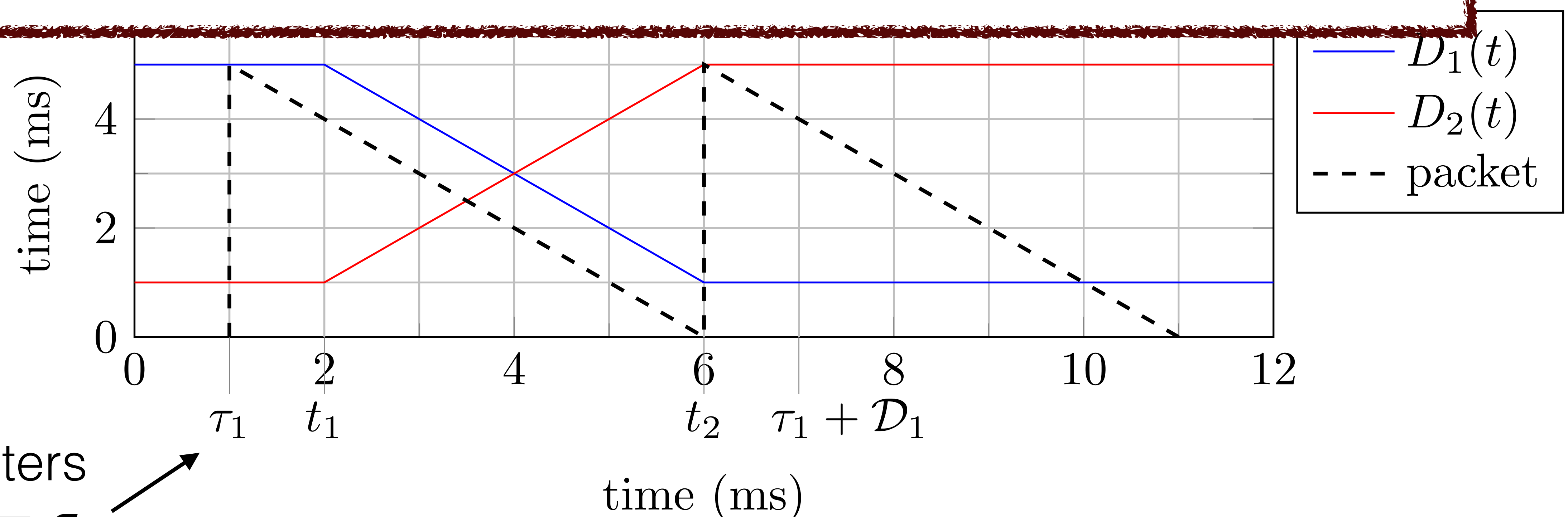

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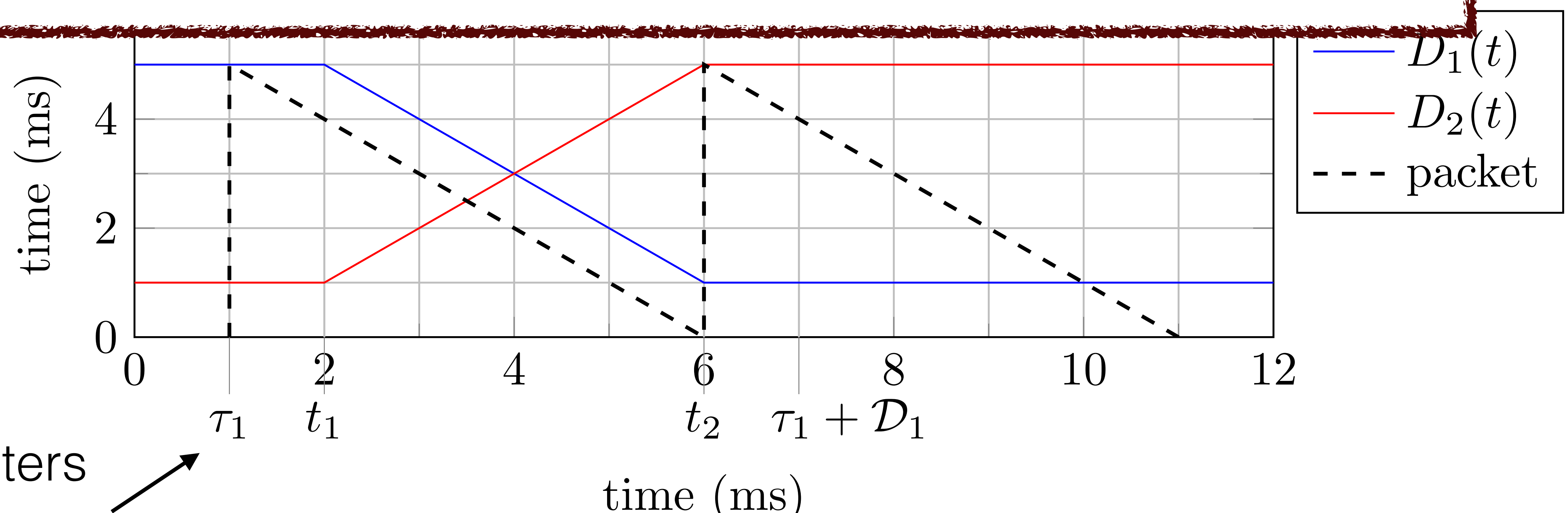

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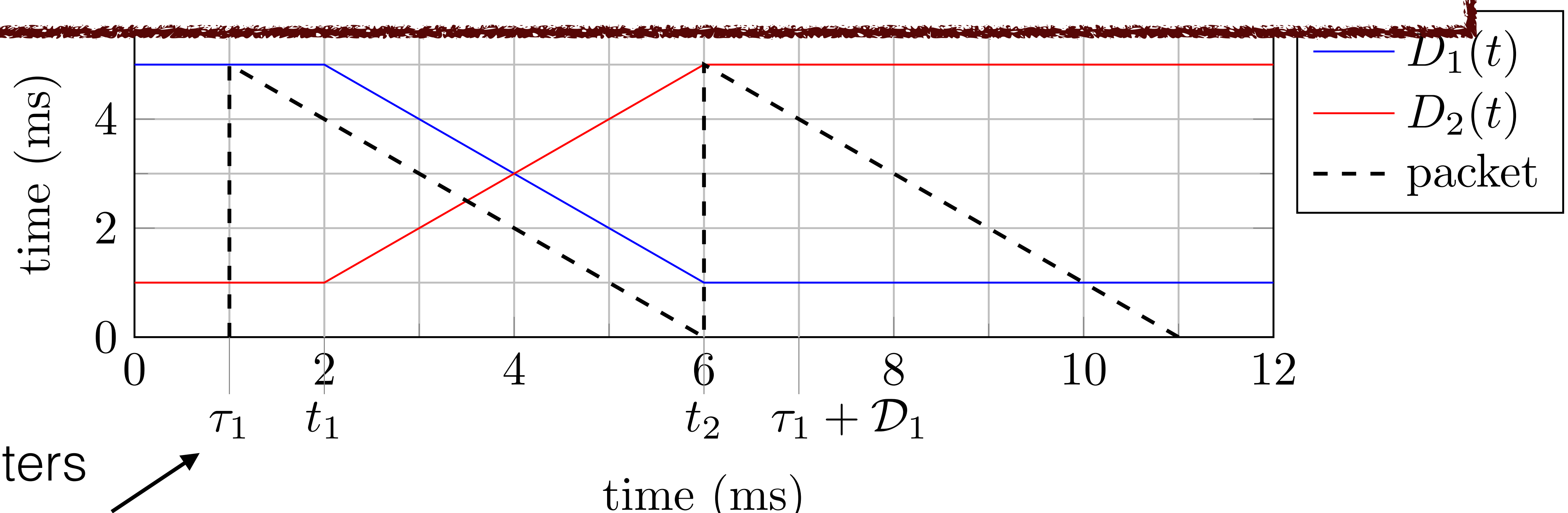

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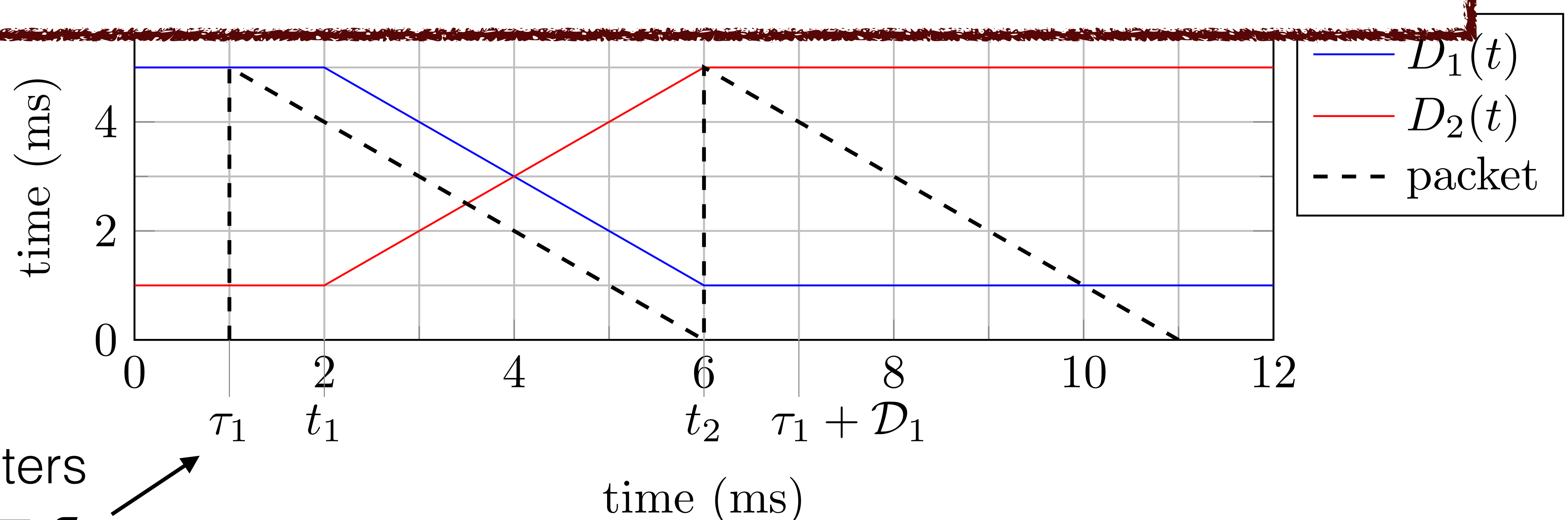

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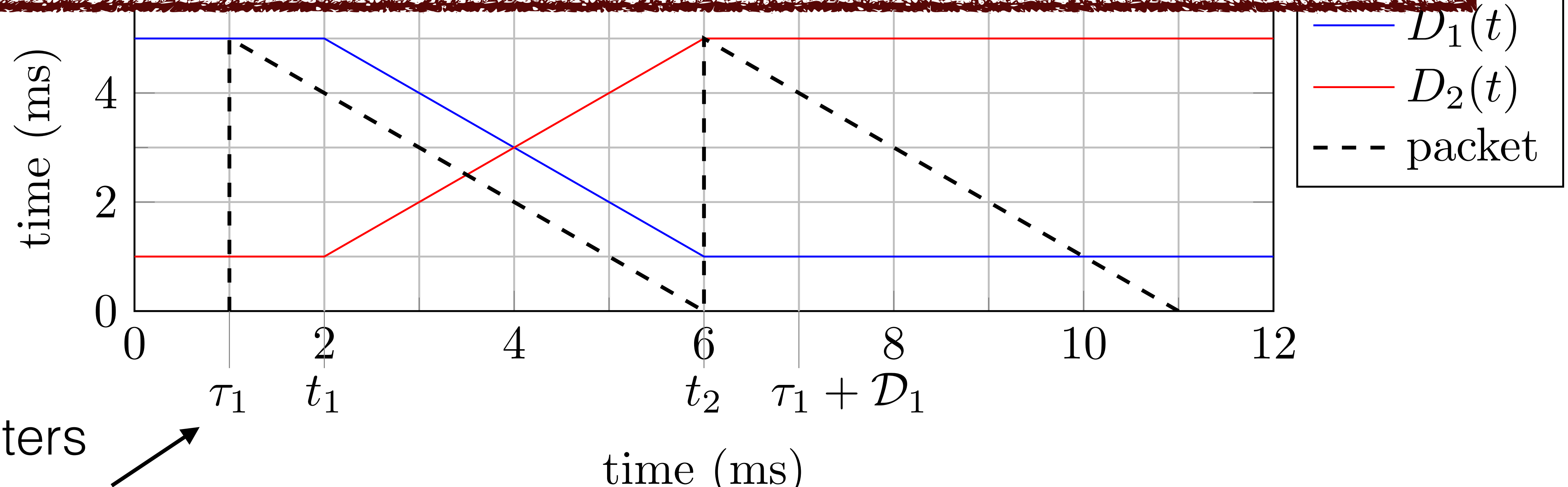

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Movement needs space!

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Solution:

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It must hold that:

$$\forall t \geq 0, \quad D_1(t) + D_2\big(t + D_1(t)\big) \leq \mathscr{D}_1$$

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It must hold that:

$$\forall t \geq 0, \quad D_1(t) + D_2\big(t + D_1(t)\big) \leq \mathscr{D}_1$$

Which is guaranteed if:

$$\forall t \geq 0, \quad (1+\alpha)D_1(t) + D_2(t) \leq \mathscr{D}_1$$

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With the requirement of

$$D_2(t_1) = 1 \;\; \rightarrow \;\; D_2(t_2) = 5$$

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With the requirement of

$$D_2(t_1) = 1 \;\; \rightarrow \;\; D_2(t_2) = 5$$

We get

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$$\forall t \geq 0, \quad (1+\alpha)D_1(t) + D_2(t) \leq \mathscr{D}_1$$
$$\forall t \geq 0, \forall i \in \mathscr{V}, \quad |\dot{D}_i(t)| \leq \alpha \in [0,1]$$

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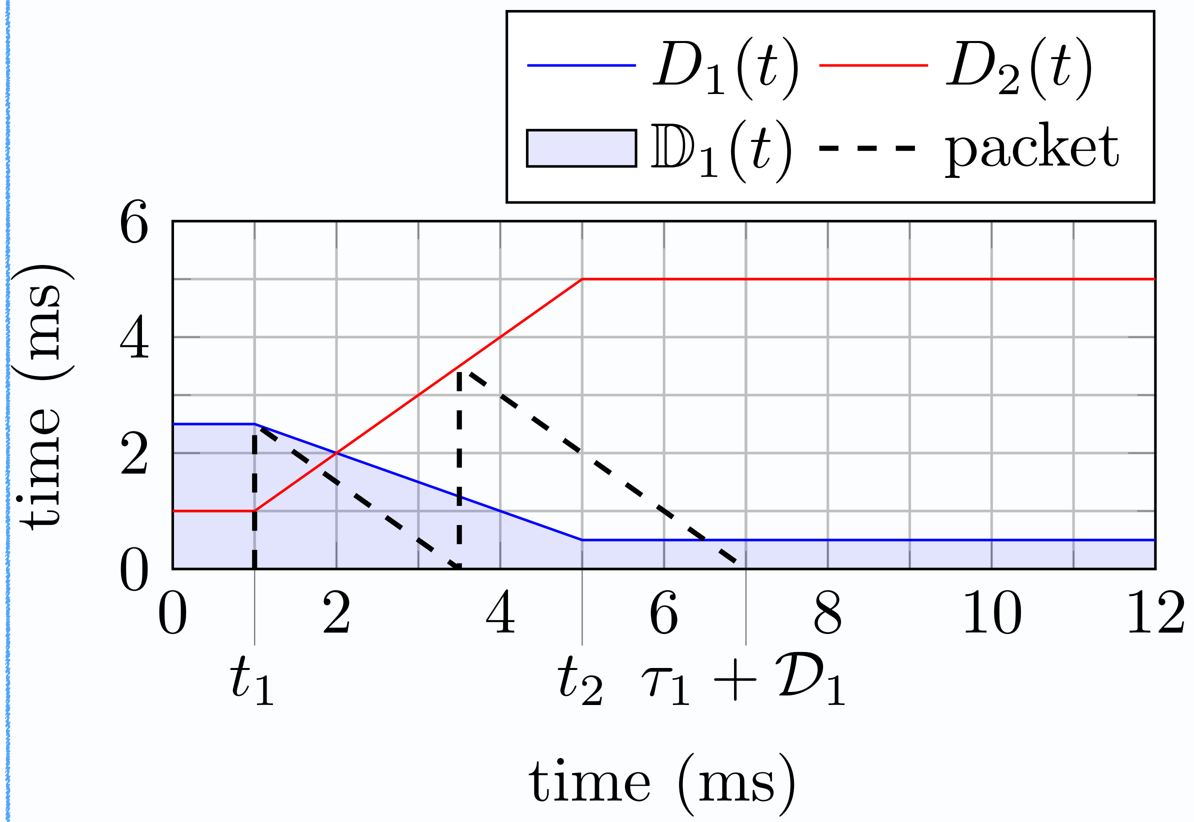

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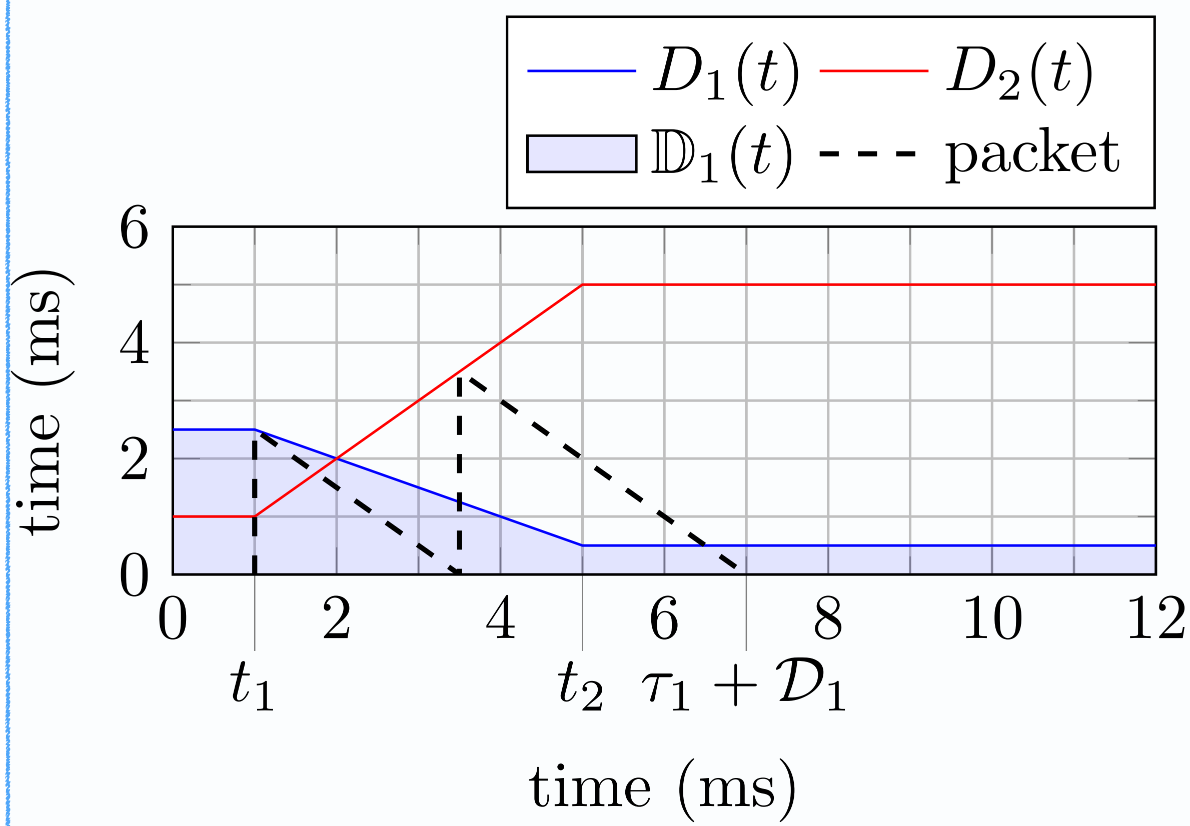

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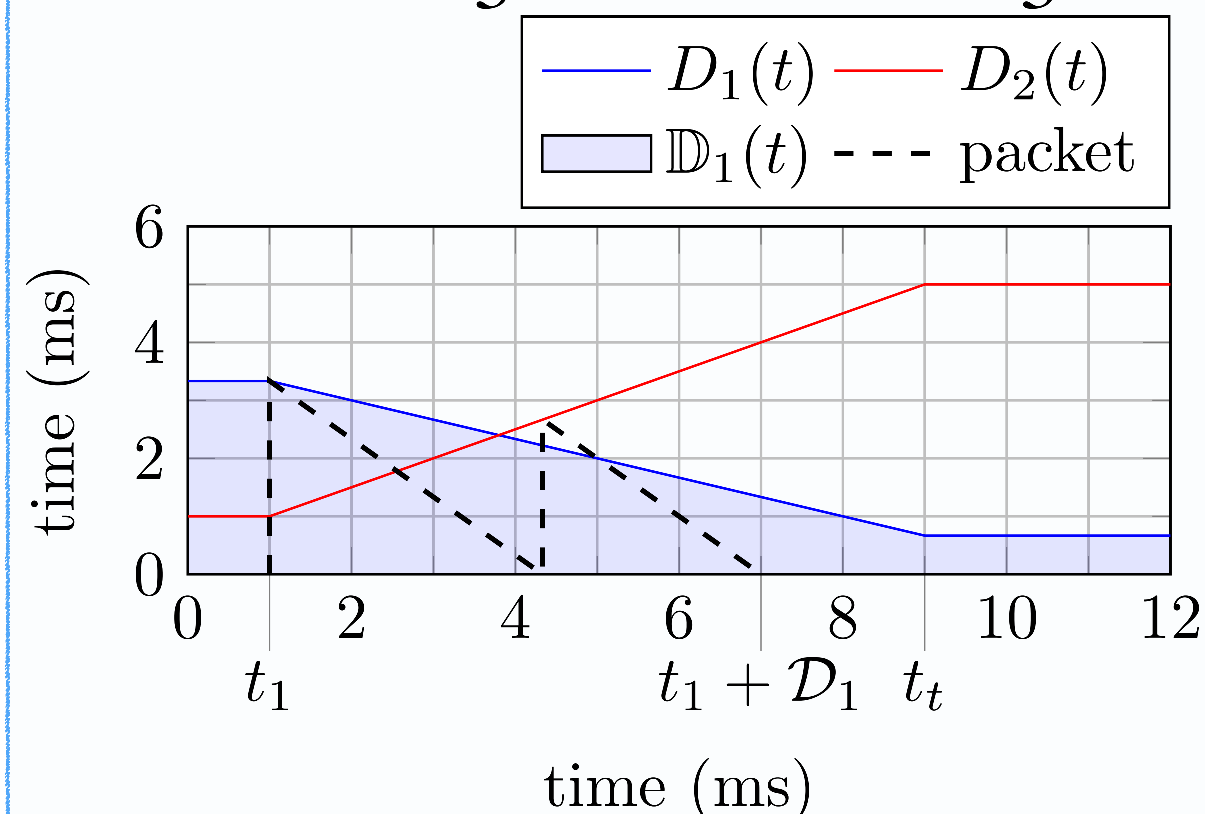


The general case

Theorem in the paper:

If the rate-of-change of the node deadlines is limited by α:

$$\forall t \geq 0,\ \forall i \in \mathscr{V}, \quad |\dot{D}_i(t)| \leq \alpha,$$

and if the node deadlines always remain within the safe space:

$$\forall t \geq 0,\ \forall i \in \mathscr{V}, \quad D_i(t) \in \mathbb{D}(\mathscr{G}),$$

where the *safe space* $\mathbb{D}(\mathscr{G})$ is given by:

$$\mathbb{D}(\mathscr{G}) = \{D_i \in \mathbb{R}^+ \ :\ \forall j \in \mathscr{F},\ \sum_{i=1}^{\ell_j} (1+\alpha)^{\ell_j - i} D_{p_j(i)} \leq \mathscr{D}_j\},$$

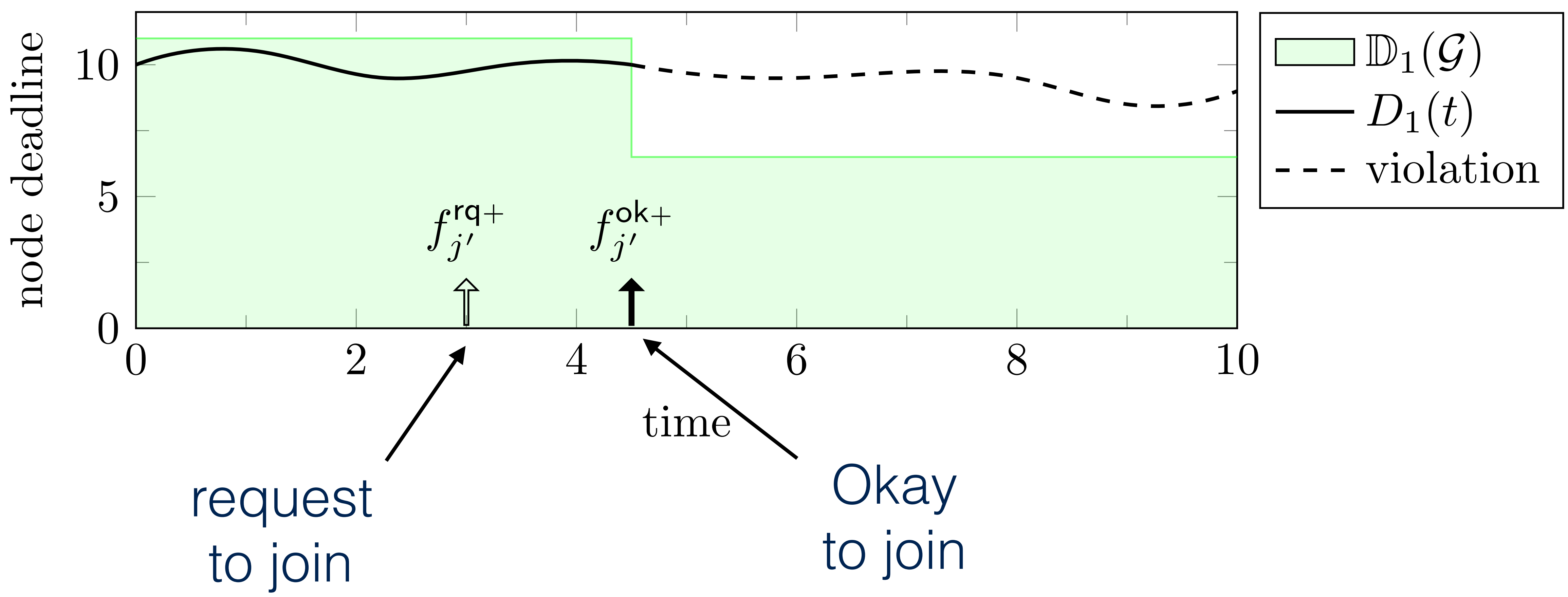
then no end-to-end deadlines will be missed:

$$\forall t \geq 0,\ \forall j \in \mathscr{F}, \quad \mathscr{R}_j(t) \leq \mathscr{D}_j.$$

- $p_j(i)$: the i-th node on path j
- ℓ_j : length of path j
- $\mathscr{G}$: the network
- $\mathscr{F}$: set of flows in the network

What about a flow joining?

New flow ⇒ new end-to-end deadline

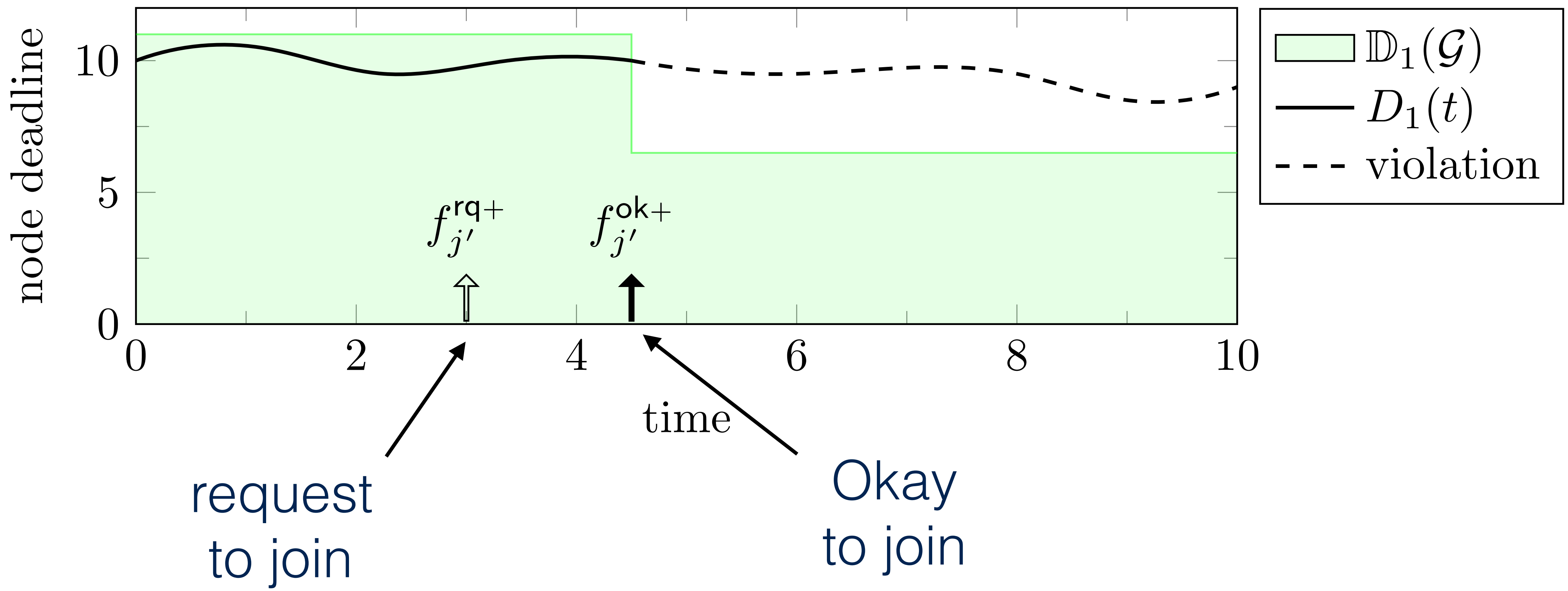


What about a flow joining?

Oops!

The space of feasible node deadlines changed when we admitted the new flow!

=> we are now outside the safe space



What can we do then?

What was the problem?

We left the safe space …

=> cannot guarantee that the end-to-end deadlines will be met

Solution:

Only admit the new flow when you are in the new safe region $\mathbb{D}(\mathscr{G}^{+})$.

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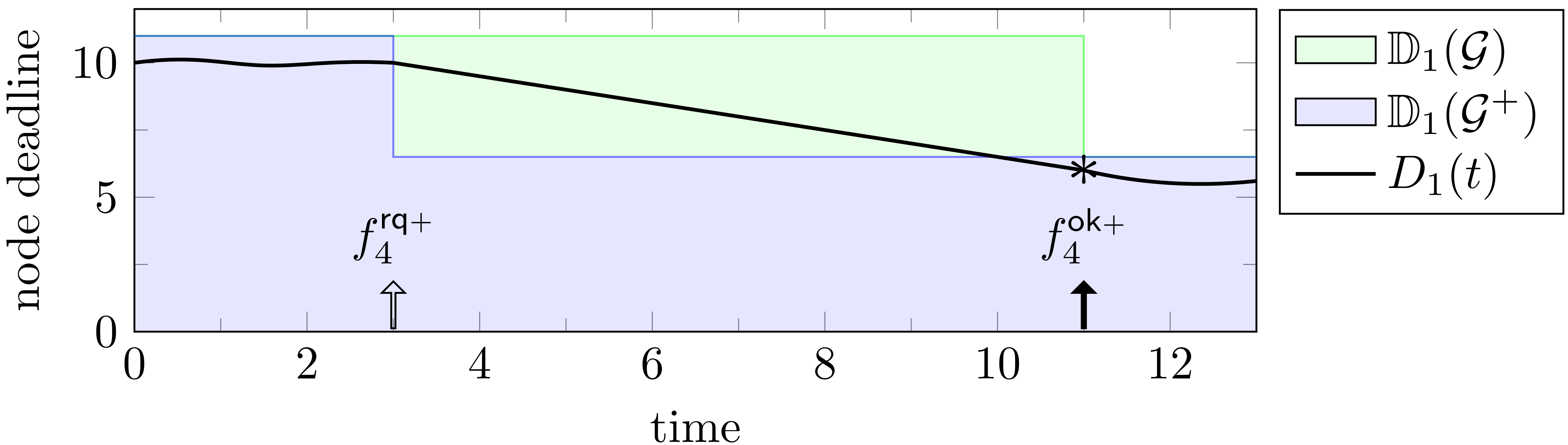

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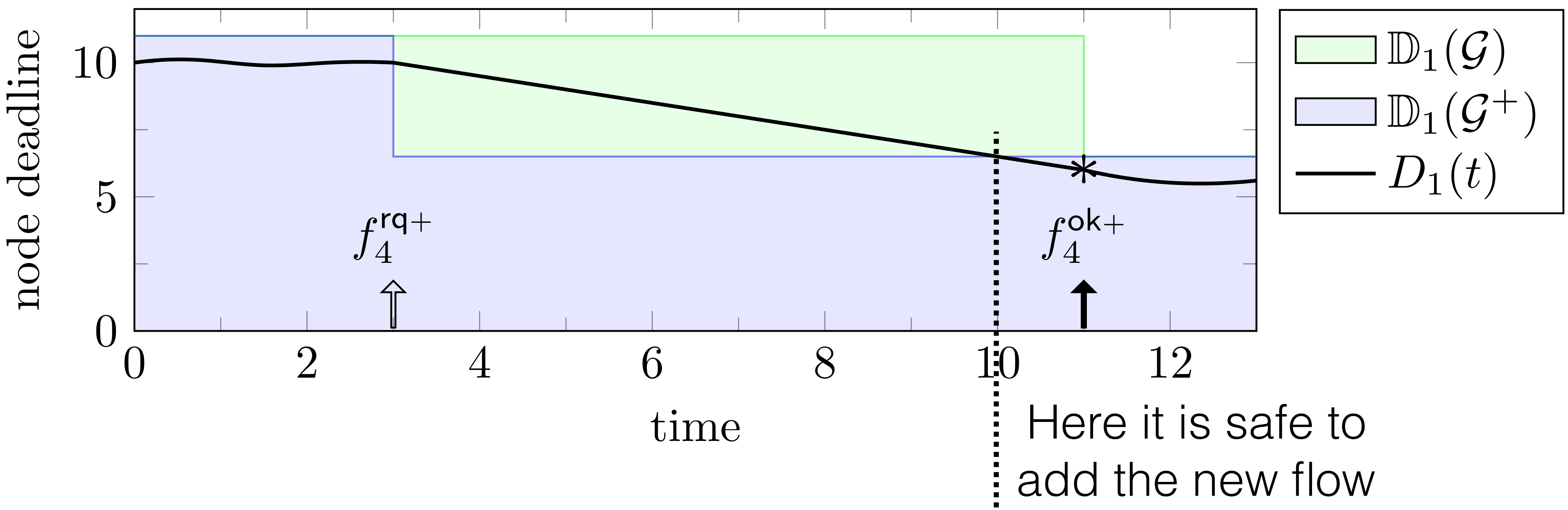

Conclusions

Take-away message:

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- How to allow nodes to leave
- Simulation results
- Protocols for:
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  - admitting new flows
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Future work:

- How to enforce $R_i(t) \leq D_i(t)$

Thank you!

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Automatic Control
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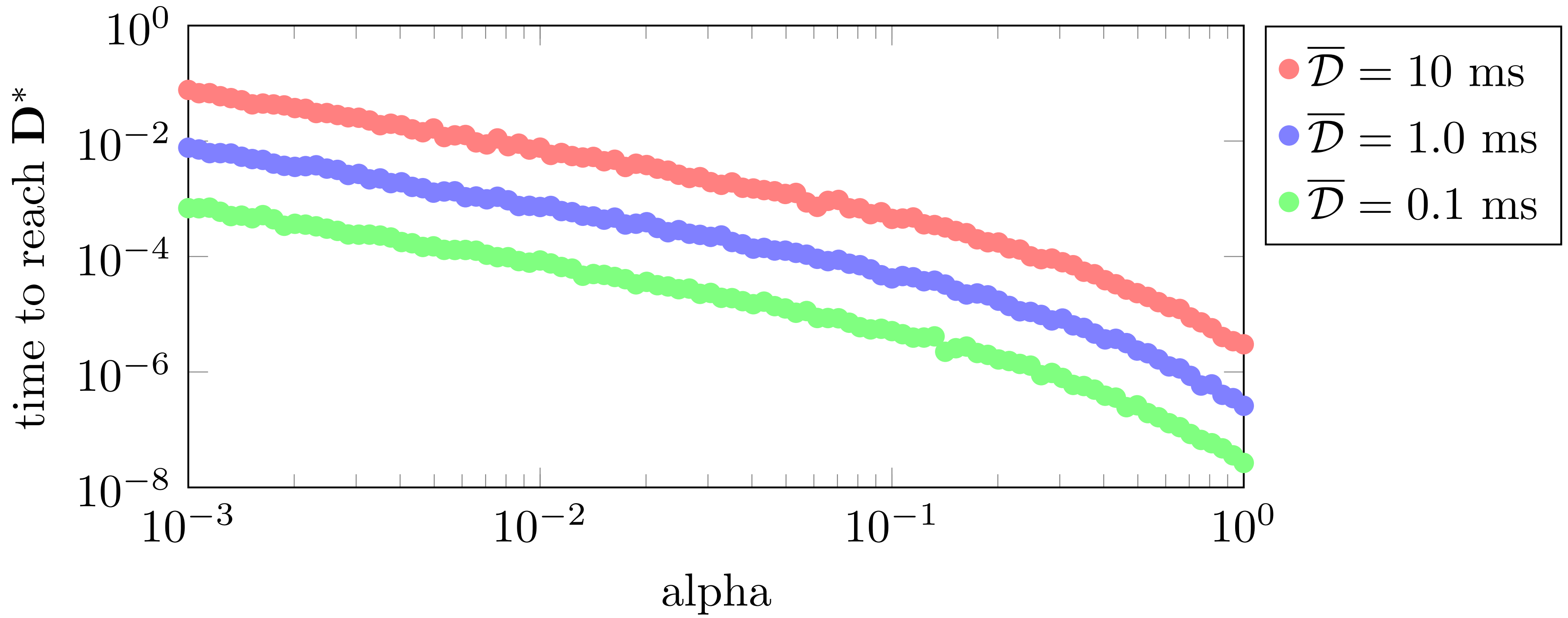
Waiting time for flows…



How does it affect QoS?