Dual Priority Scheduling is Not Optimal

Pontus Ekberg
Uppsala University

ECRTS 2019
Stuttgart, Germany

Dual priority scheduling?

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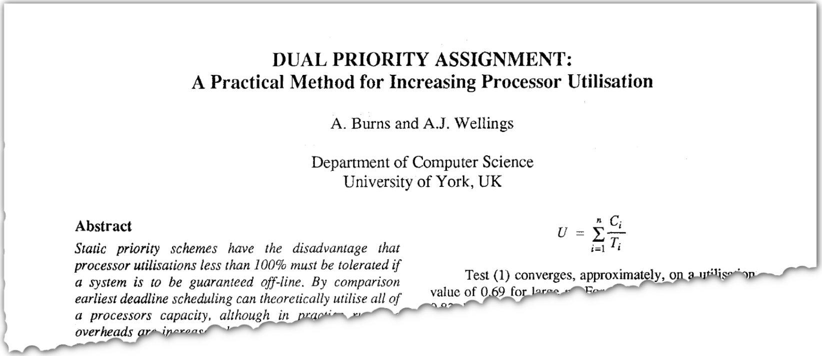

DUAL PRIORITY ASSIGNMENT:
A Practical Method for Increasing Processor Utilisation

A. Burns and A.J. Wellings

Department of Computer Science
University of York, UK

Abstract

Static priority schemes have the disadvantage that processor utilisations less than 100% must be tolerated if a system is to be guaranteed off-line. By comparison earliest deadline scheduling can theoretically utilise all of a processors capacity, although in

$$U = \sum_{i=1}^{n} \frac{C_i}{T_i}$$

Test (1) converges, approximately, on a

value of 0.69 for

(Euromicro Workshop on Real-Time Systems, 1993)

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Assumptions

- Implicit deadline periodic tasks
- Single preemptive processor

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For each task $\tau_i = (e_i, p_i)$, assign

- a phase change point $\delta_i \in \{0, \ldots, p_i\}$
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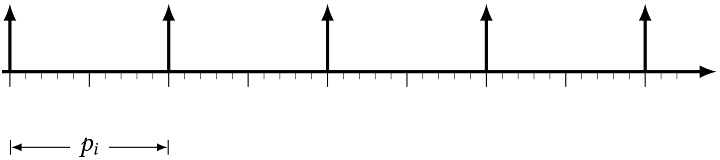

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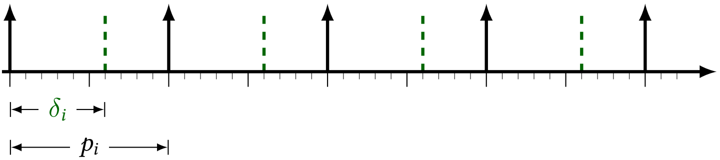

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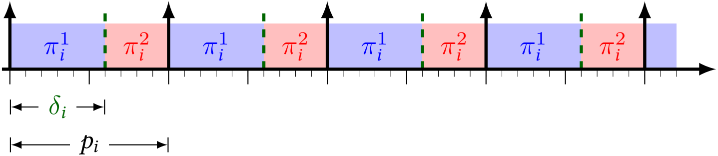
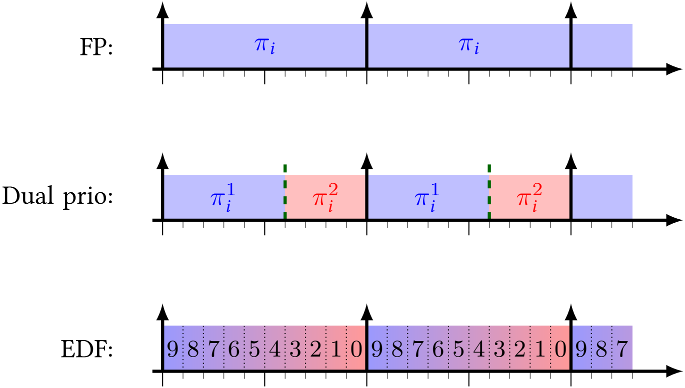


Comparing with FP and EDF



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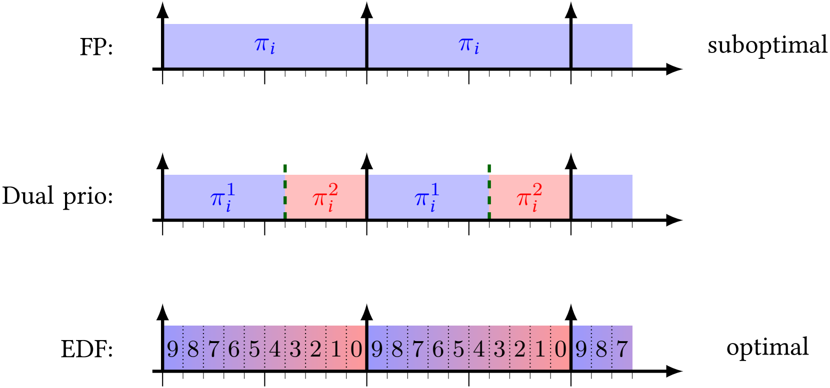

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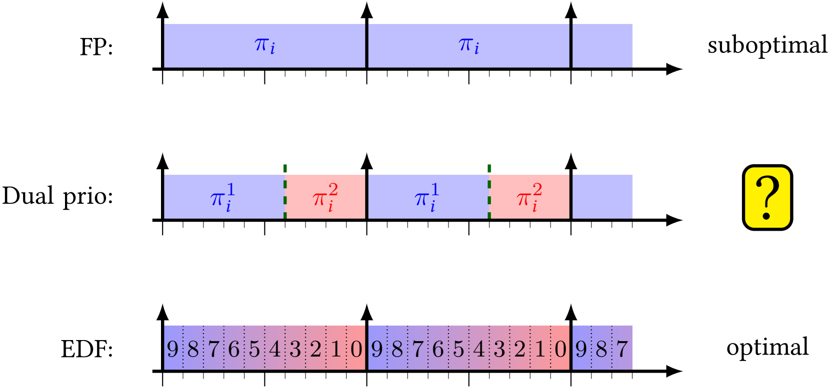

Is dual priority scheduling optimal?

priorities be found.

The answer to the first takes the form of a conjecture:

Conjecture C1

For any task set with total utilisation less than or equal to 100% there exists a dual priority assignment that will meet all deadlines.

Analysis of this conjecture in given in section 4.

τ_3				
τ_4	20	6		
Total Utilisation		100%		
LCM		240		

Table 2: Task Set E2

With no task having a second phase, the system is not

Conjecture 1 (Burns and Wellings, 1993)

Dual priority scheduling is *optimal* for implicit deadline periodic tasks.

(Sadly not)

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Find a task set that is

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- Almost all task sets are schedulable
- Evaluating the schedulability can be *very* costly

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Schedulability test

For every configuration (setting of $\pi_i^1, \pi_i^2, \delta_i$), simulate the hyper-period until a deadline miss.

How many configurations are there?

Assumptions

- All priority levels are unique
  - $\Rightarrow$ A total of $(2n)!$ permutations
- Phase change points (δ_i) are integer
  - $\Rightarrow$ A total of $\prod_{i=1}^{n}(p_i + 1)$ combinations

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A task set $\mathcal{T} = \{\tau_1, \ldots, \tau_n\}$ has

$$(2n)! \times \prod_{i=1}^{n}(p_i + 1)$$

distinct configurations.

A counterexample

Not dual priority schedulable

	e_i	p_i
τ_1	8	19
τ_2	13	29
τ_3	9	151
τ_4	14	197

hyper-period: 16 390 597
utilization: ~ 0.9999971

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$$\#\text{configurations} = (2n)! \times \prod_{i=1}^{n} (p_i + 1) = 728\,082\,432\,000$$

Simulating the full hyper-period for all configurations would take *hundreds of years* on my computer.

The saving grace

Most configurations lead to a deadline miss very early.

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Simulation time is ~ 2.5 days on an office computer.

Recognizing the needle: RM+RM

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Definition: RM+RM

A dual priority configuration is called *RM+RM* if

1. phase 1 priorities (π_i^1) are RM
2. phase 2 priorities (π_i^2) are RM
3. $\max_i\{\pi_i^2\} \leqslant \min_i\{\pi_i^1\}$

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Conjecture 2 (George et al., 2014)

RM+RM is an optimal choice of priorities.

(Sadly not, even considering only point 1 above)

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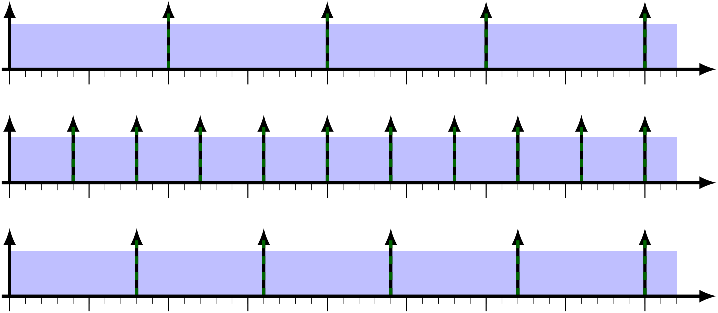

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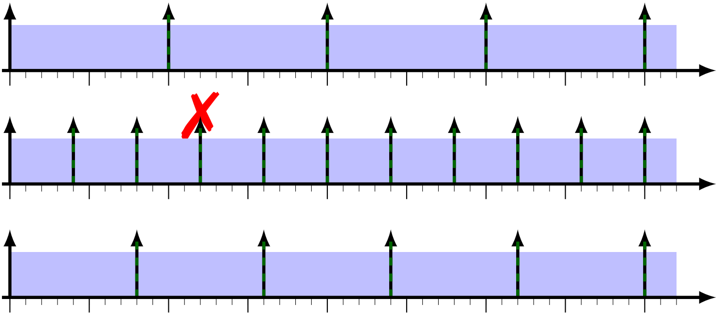

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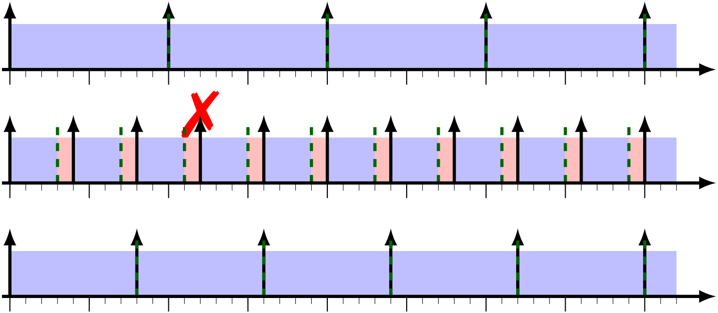

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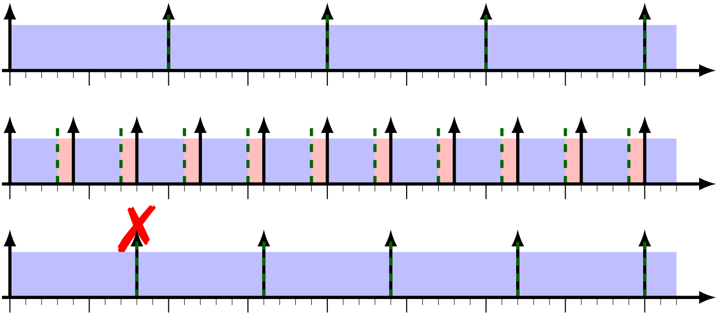

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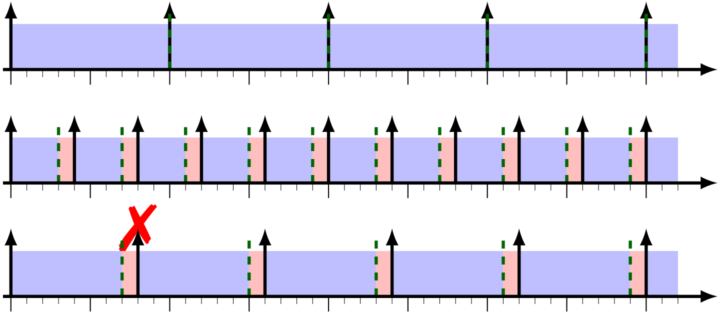

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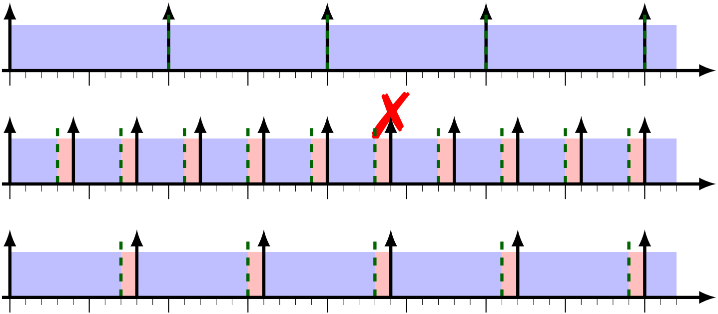

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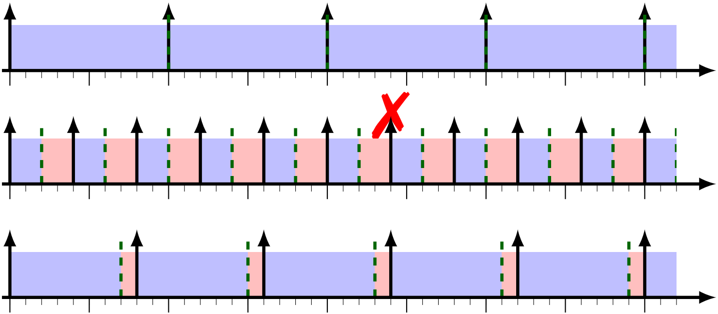

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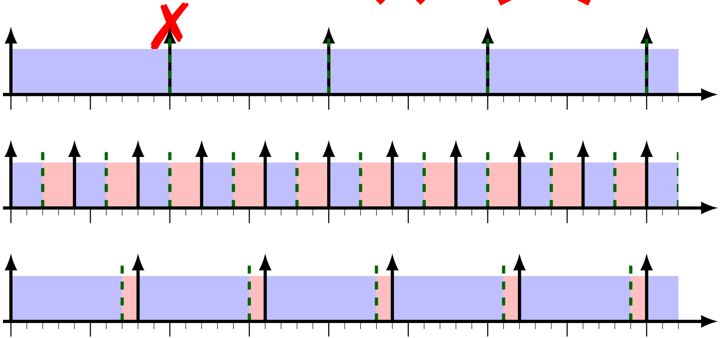

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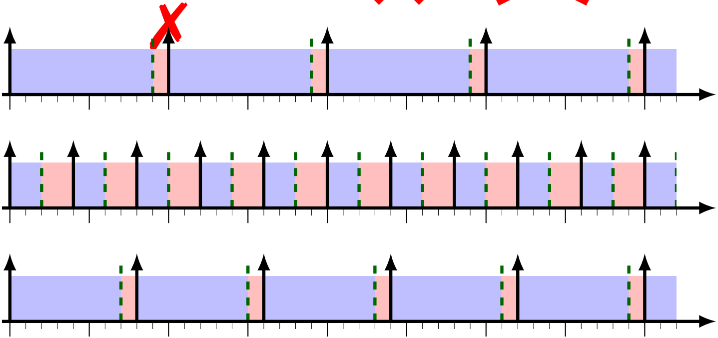

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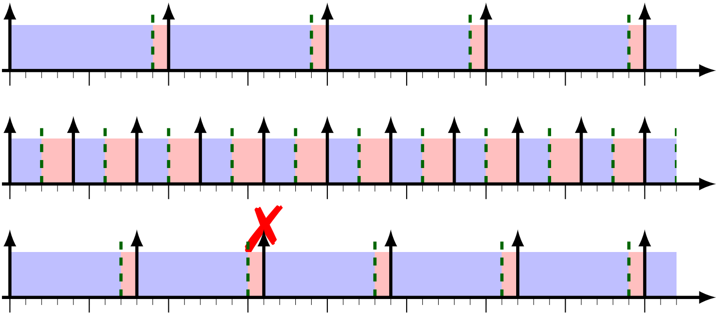

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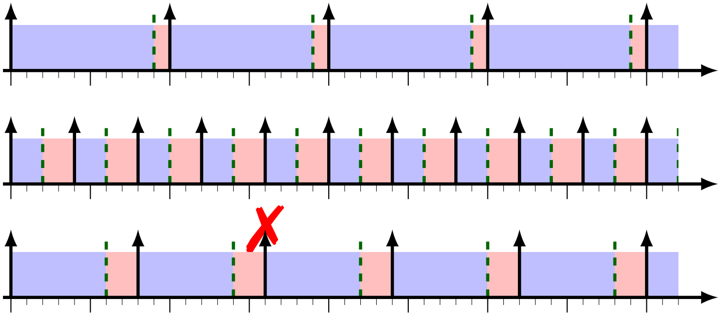

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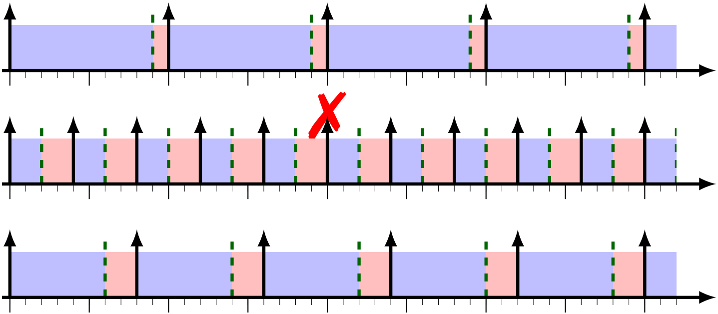

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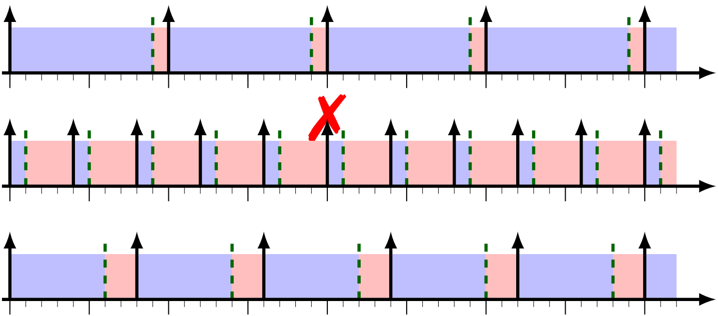

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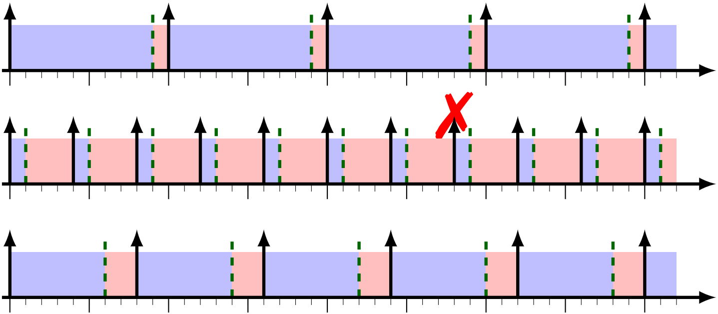

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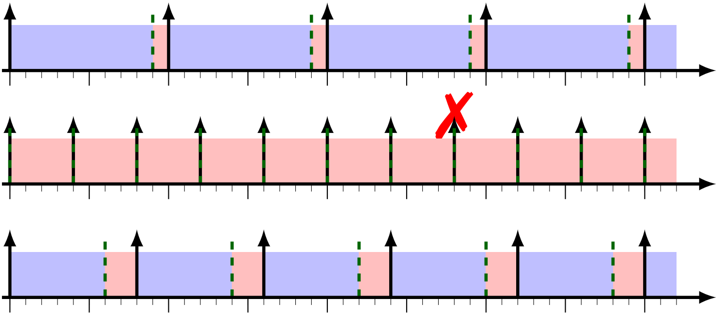

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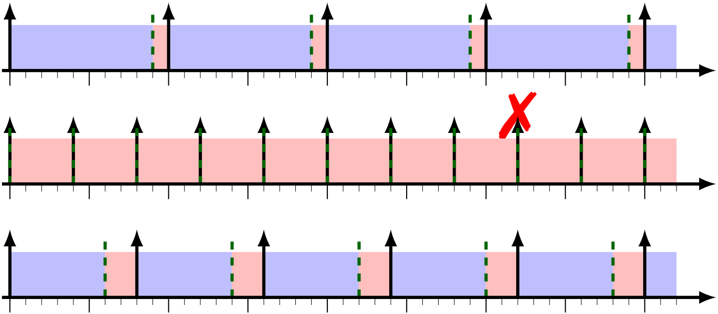

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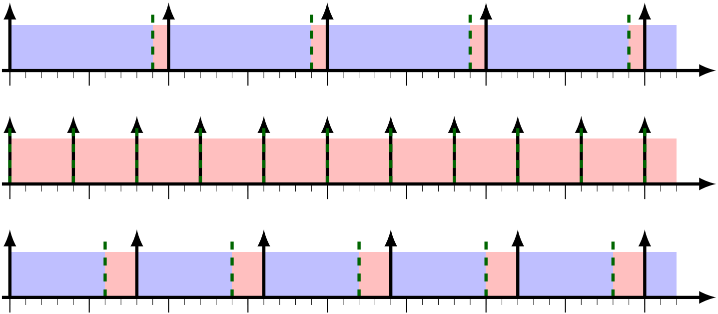

Conjecture 3 (Fautrel et al., 2018)

FDMS always finds optimal phase change points.

(Sadly not)

Recognizing the needle

A simple schedulability analysis strategy

1. Try *RM+RM* priorities with *FDMS*
2. If not successful, check configurations exhaustively

Doing 1 is *much* faster than 2, and works most of the time.

Where is my haystack?

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An ad hoc search space

- 4 tasks
- Utilization $\in [0.99999, 1]$
- Periods chosen from the first 100 primes
- $\prod_{i=1}^{n} p_i \leqslant 35\,000\,000$

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Random task sets were tested from this search space until an unschedulable one was found. This is the breakdown:

	# task sets	% of explored search space
Schedulable with RM+RM using FDMS	129 823	$\sim 99.67\%$
Schedulable with other configurations	431	$\sim 0.33\%$
Unschedulable	1	$\sim 0.0008\%$

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It took *26 years* and evaluating *millions of task sets* to find a single unschedulable one.

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To fully exploit the apparent near-optimality we need efficient tests and methods for finding the right parameters.

Open problems

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Can we efficiently determine if there exists a schedulable configuration?

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Simulation only works for periodic tasks (George et al., 2014)

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What about rational phase change points?

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Open problem 6

What about rational phase change points?

Can't be brute forced!

What if not all priorities are unique?

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We need to define the tie-breaking rule.

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Result

The same counterexample remains unchedulable if priorities can be shared and FIFO or LIFO is used for ties.

(Not in the paper)

Special thanks to

Martina Maggio

Joël Goossens

Artifact evaluators

$\forall$Thank you!

$\Diamond$

$\exists$Questions?