Resource Augmentation Bounds of EDF for Sporadic Tasks with Constrained Deadlines

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Speedup / Resource Augmentation Bound

- A schedulability test has **speedup factor**[a] **s**, **s**≥1, if *any* task set that is *schedulable* by *any* algorithm on platform with processors of speed 1, it will be deemed schedulable by this test upon a platform with processors that are ***s*** *times as fast*.
- Speedup bound means a lower bound of speedup factor
- Major metric & standard tool for evaluating sub-optimality

[a] B. Kalyanasundaram and K. Pruhs, “Speed is as powerful as clairvoyance,” J. ACM, vol. 47, no. 4, pp. 617–643, 2000.

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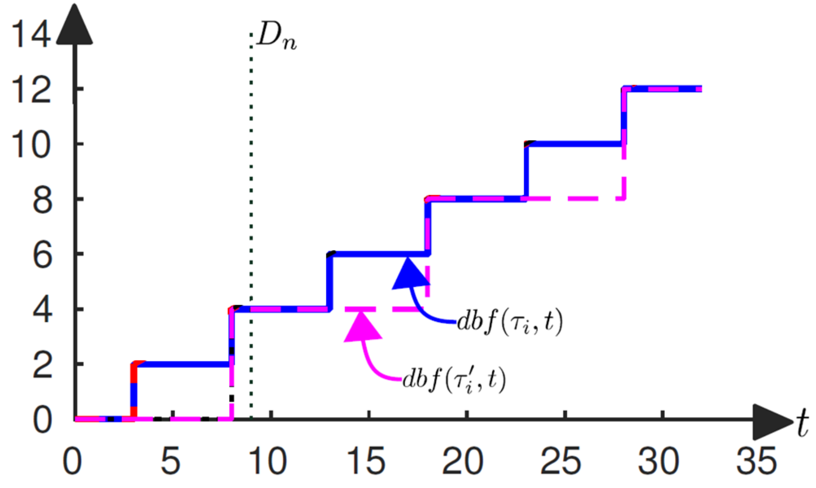
- A schedulability test has **speedup factor**[a] **s**, **s**≥1, if *any* task set that is *schedulable* by *any* algorithm on platform with processors of speed 1, it will be deemed schedulable by this test upon a platform with processors that are ***s*** *times as fast*.
- Speedup bound means a lower bound of speedup factor
- Major metric & standard tool for evaluating sub-optimality
- Potential pitfalls

Z. Guo, "Regarding the optimality of speedup bounds of mixed-criticality schedulability tests," Dagstuhl Seminar 17131, 2017.
J.-J. Chen et al., "On the Pitfalls of Resource Augmentation Factors and Utilization Bounds in Real-Time Scheduling," in ECRTS 2017, pp. 9:1–9:25.
K. Agrawal and S. Baruah, "Intractability issues in mixed-criticality scheduling," in ECRTS'18, to appear.

Life (RT-Scheduling) Is Often Hard

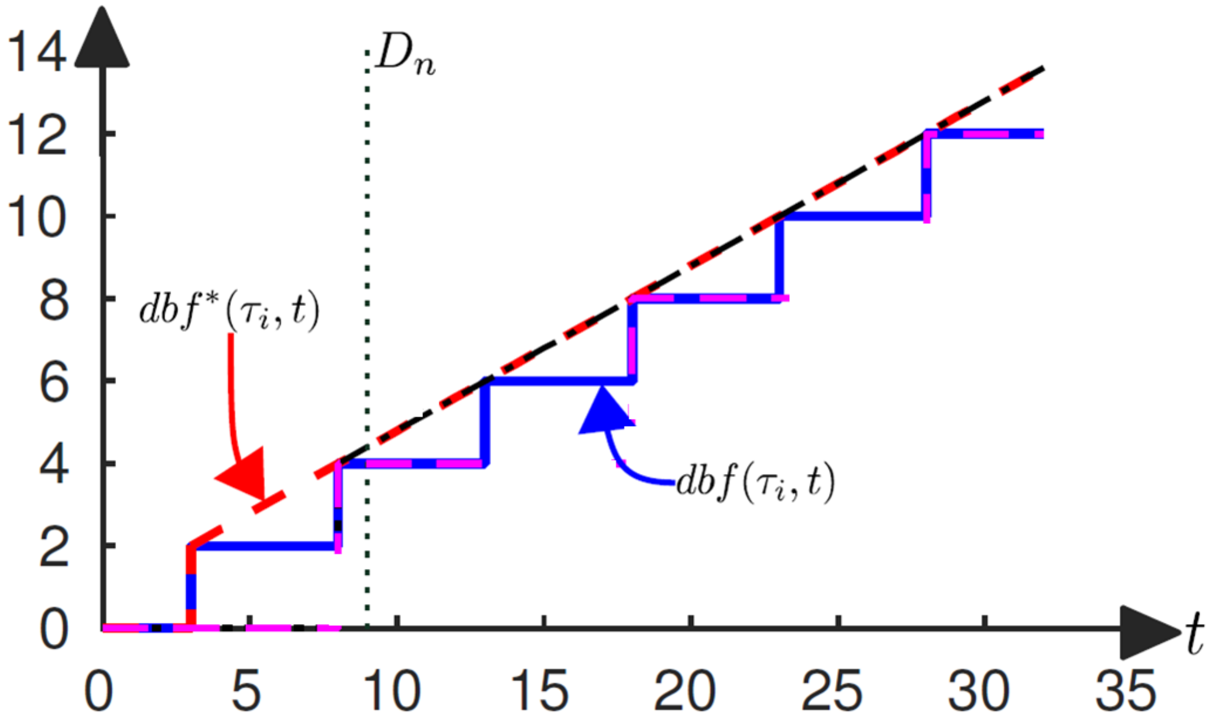
- Uniprocessor, sporadic task set
  - Scheduler: **EDF** is *optimal*
  - Schedulability test
    - Implicit deadlines: U≤1, *optimal* (necessary and sufficient)
    - Constrained deadlines
      - Co-NP-Hard [b]
      - ∀t, dbf(τ,t) ≤ t *optimal,*
      - *Exp. time!*



[b] F. Eisenbrand and T. Rothvoß, "EDF-schedulability of synchronous periodic task systems is co-np-hard," SIAM 2010, pp. 1029–1034.

Life (RT-Scheduling) Is Often Hard

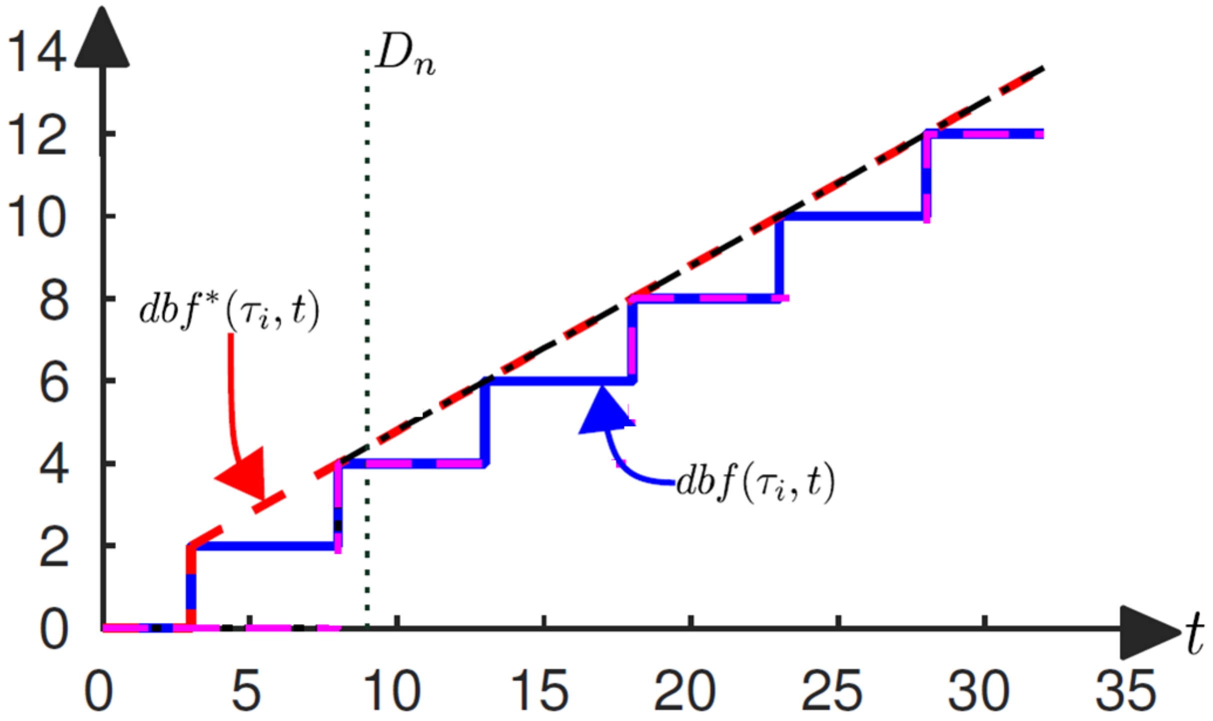
- m (identical) processors, sporadic task set τ
  - pFair is *optimal*…
  - G-EDF or G-FP
  - Partitioned scheduling
    - Speedup = 3 [d]
      - Approximate dbf (dbf*) [c]
      - dbf*(τ,t)/dbf(τ,t) ≤ 2
      - ∀t, τ is any feasible set



[c] K. Albers and F. Slomka, “An event stream driven approximation for the analysis of real-time systems,” in ECRTS 2004, pp. 187–195.
[d] S. Baruah and N. Fisher, “The partitioned multiprocessor scheduling of sporadic task systems,” in RTSS 2005, pp. 321–329.

Life (RT-Scheduling) Is Often Hard

- m (identical) processors, sporadic task set τ
  - pFair is *optimal*…
  - G-EDF or G-FP
  - Partitioned scheduling
    - Speedup = **α+1** [d]
      - Approximate dbf (dbf*) [c]
      - dbf*(τ,t)/dbf(τ,t) ≤ **α**
      - ∀t, τ is any feasible set



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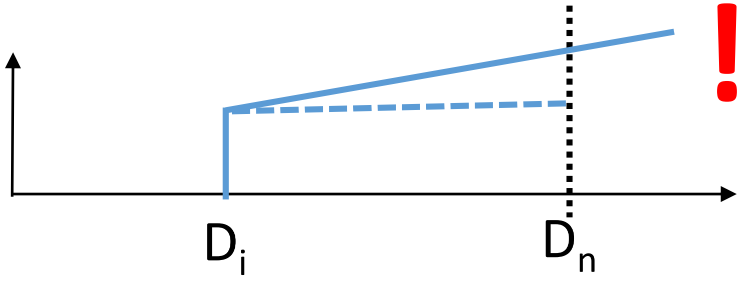
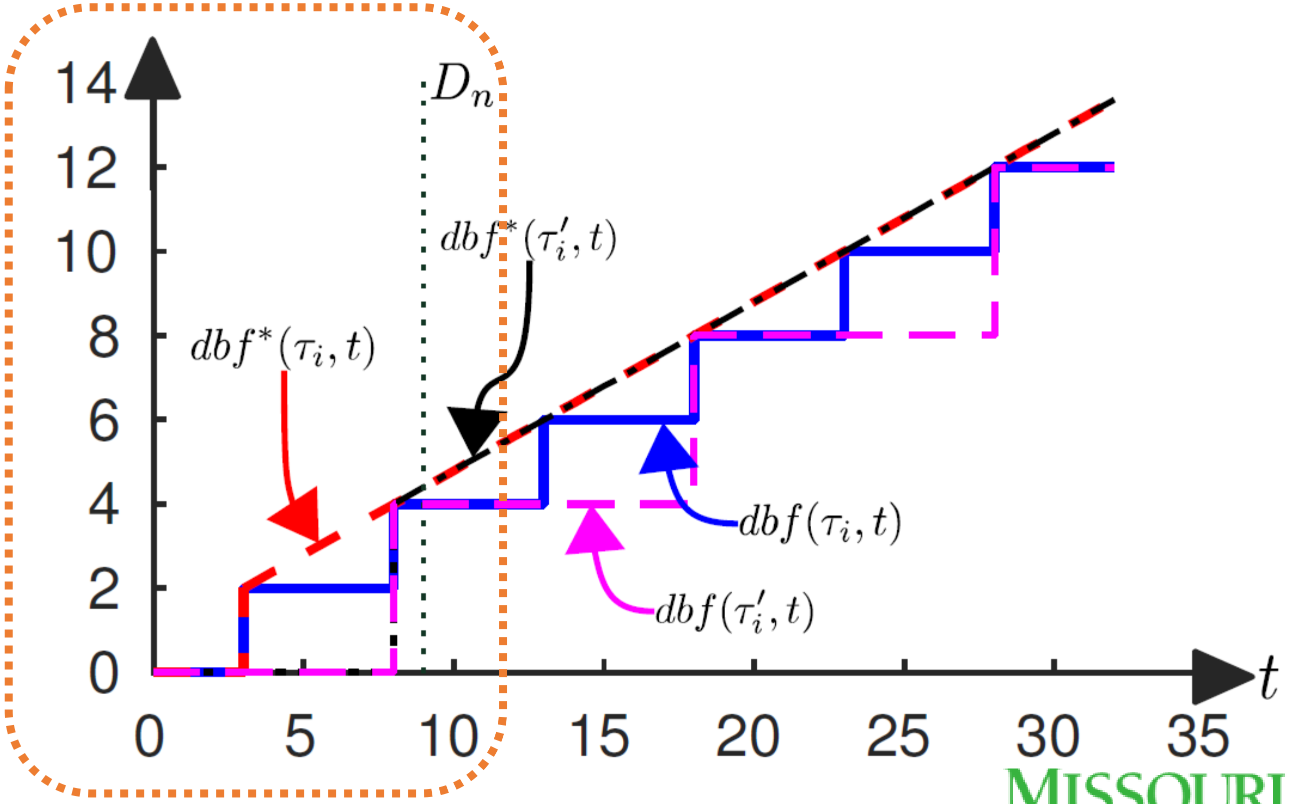
Achieving a Better Bound – Existing Work

- $\forall t$, $dbf^*(\tau,t)/t \leq 1.632$ [e]
  - τ: **any** uni-proc. feasible set
- Main ideas:
  - Consider only $dbf^*($**τ'**$,D_n)$
  - Normalization:

    For each i, set $T_i' = D_n' - D_i'$
    - $D_1' \leq D_2' \leq \ldots \leq D_n' (=D_n)$





[e] J.-J. Chen and S. Chakraborty, "Resource augmentation bounds for approximate demand bound functions," in RTSS 2011, pp. 272–281.

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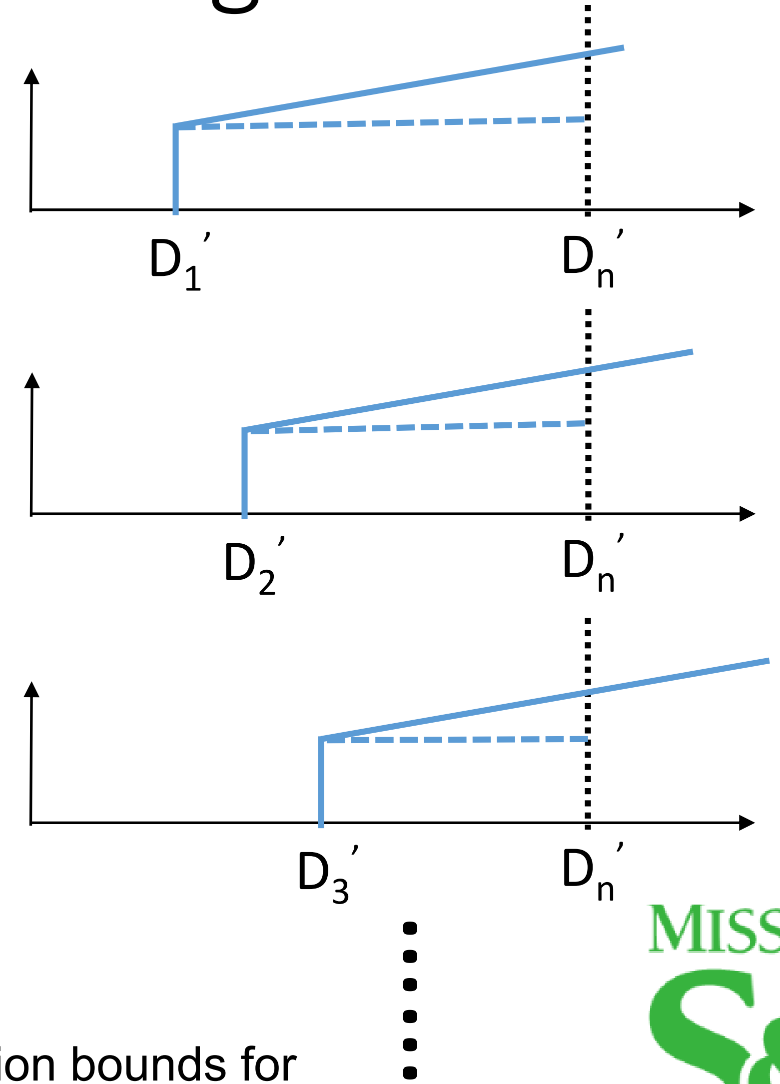
- $\forall t$, $dbf^*(\tau,t)/t \leq$ **1.632** [e]
  - τ: **any** uni-proc. feasible set
- Main ideas:
  - Consider only $dbf^*(\boldsymbol{\tau'},D_n)$
  - Normalization/Relaxation: For each i, set $T_i' = D_n' - D_i'$
  - Issue:
    - $\Sigma_i\, C_i/T_i \leq 1$ may not hold **!**
    - $dbf^*(\boldsymbol{\tau'},D_n') \geq D_n'$



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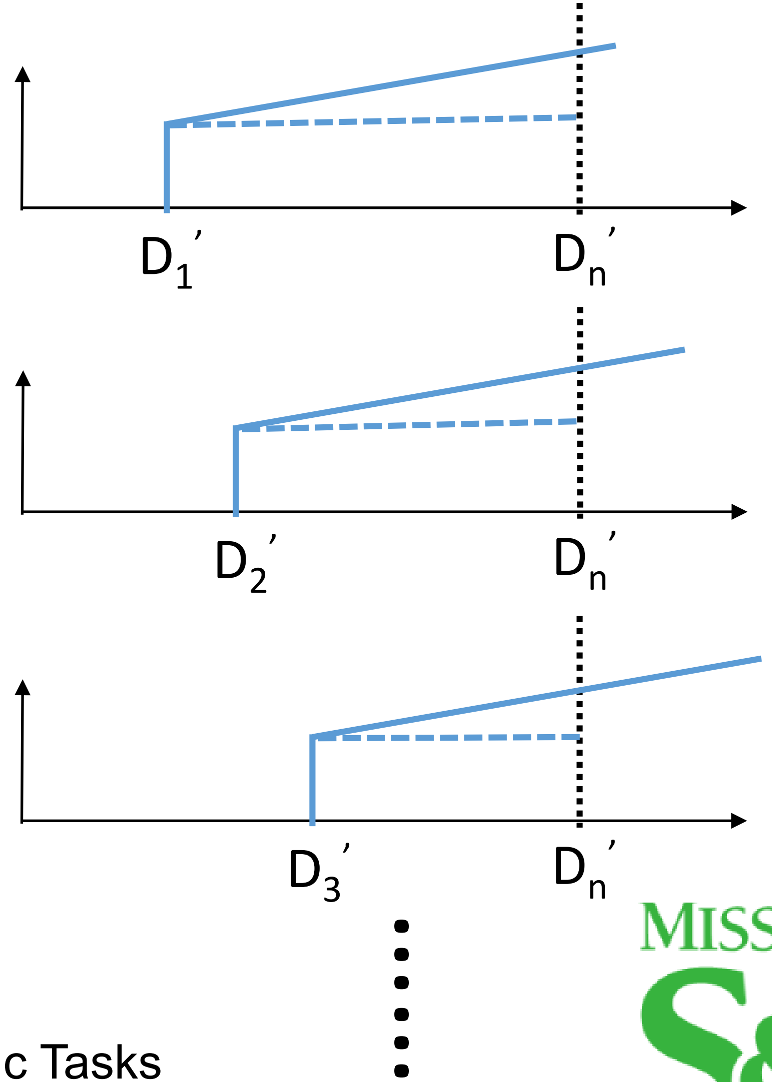
Achieving a Potentially Optimal Bound

- $\forall t$, $dbf^*(\tau,t)/t \le$ **1.555** [f]
  - τ: **any** uni-proc. feasible set
  - Want to maximize: $dbf^*(\boldsymbol{\tau'},D_n')$
- Main ideas:
  - Consider only $dbf^*(\boldsymbol{\tau'},D_n)$
  - Normalization/Relaxation:
    $C_1' = C_2' = \ldots = C_n'$
    Set $D_i' = i/n * D_n'$
    - s.t. $\sum_i T_i' = (n-1)D_n'$

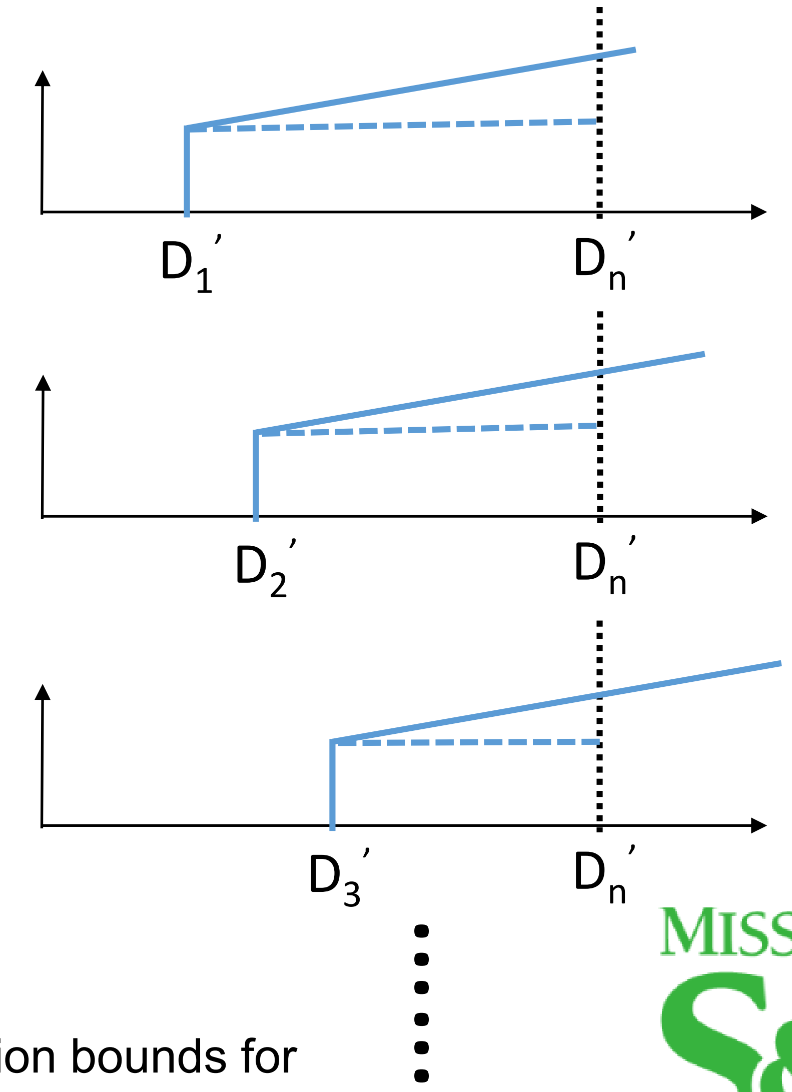


[f] X. Han et al., "An Improved Speedup Factor for Sporadic Tasks with Constrained Deadlines under Dynamic Priority Scheduling," in submission.

Achieving a Potentially Optimal Bound

- $\forall t$, $dbf^*(\tau,t)/t \leq$ **1.5**
  - τ: **any** uni-proc. feasible set
  - Want to maximize: $dbf^*(\tau',D_n')$
- Main ideas:
  - Consider only $dbf^*(\tau',D_n)$
  - Normalization/Relaxation:

    ?
  - 1.5 is the lower bound [e]



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Thank You!

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Achieving a Potentially Optimal Bound

- $\forall t$, dbf*(τ,t)/t ≤ **1.5**
  - τ: **any** uni-proc. feasible set
  - Want to maximize: dbf*(**τ'**,D_n')
- Main ideas:
  - Consider only dbf*(**τ'**,D_n)
  - Normalization/Relaxation:

    ?
  - 1.5 is the lower bound [e]

$D_i \geq 1$ are integers

$\Sigma_{i=1}^{n-1} D_i = (n-1)n$

Lower bound of:

$\lim_{n\to\infty} \Sigma_{i=1}^{n-1} i/D_i$

Guess: n/2

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