ПЕРМСКИЙ НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ ПОЛИТЕХНИЧЕСКИЙ УНИВЕРСИТЕТ

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Fixed Priority Scheduling of xy-tasks

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xy-tasks Model:

1. Special instants (x and y)
2. Vector of external information
3. Quality function and states
4. Release algorithm

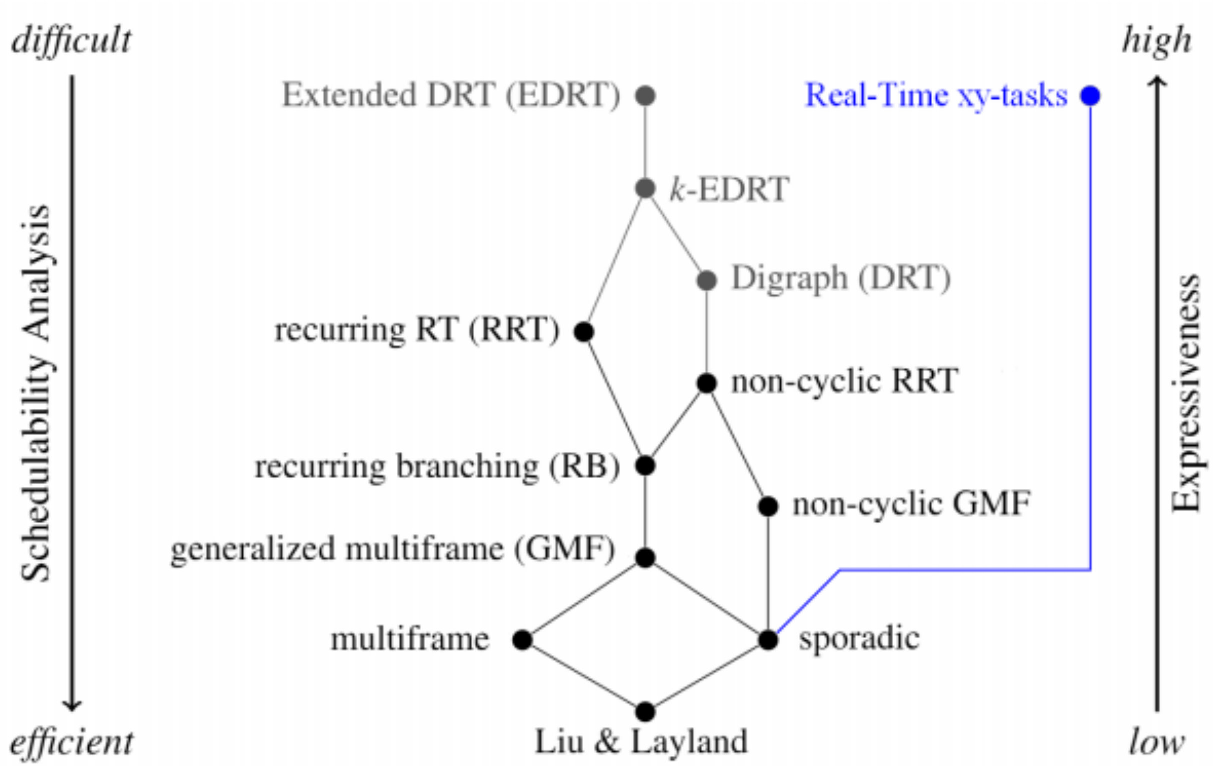
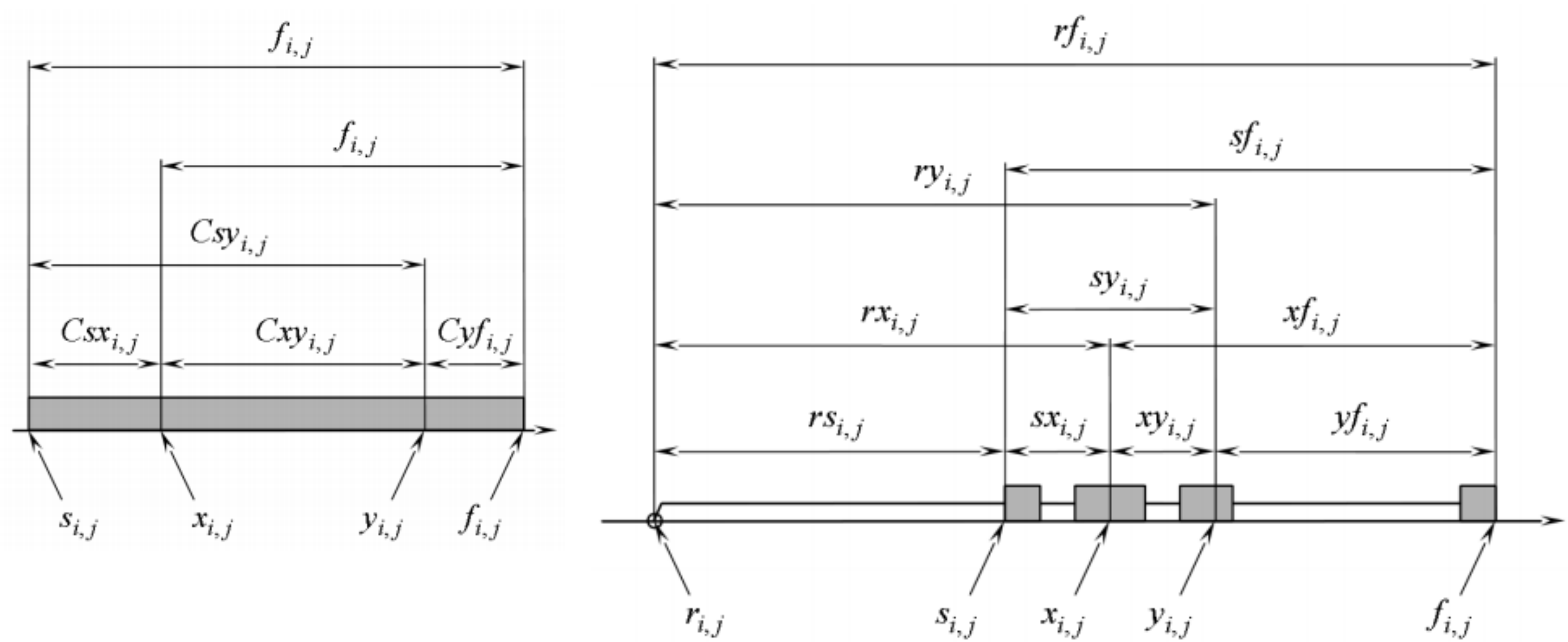


Figure from [Martin Stigge, Real-time workload models: Expressiveness vs. analysis efficiency, 2014] slightly adjusted

xy-tasks Model:
1. Special instants (x and y)



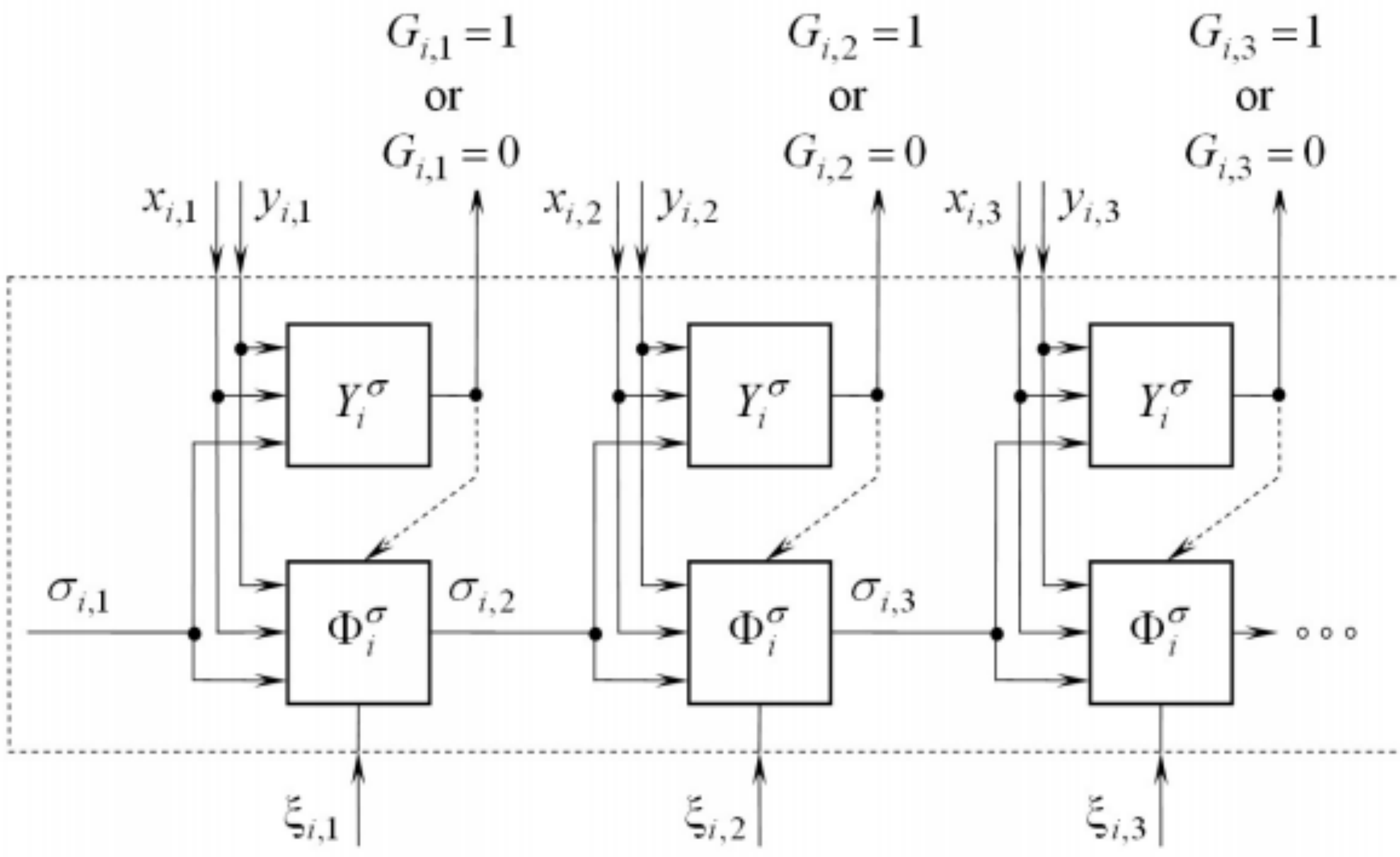
xy-tasks Model:
2. Vector of external information

Vector ξ of information about:
- mode changes,
- changes in control law,
- human interference, etc.

Each vector ξ of the job (task instance) is available no later than finish time (f) of the job.

Note: For hard real-time tasks possible values of ξ (and the affect of them on the system) must be known (or estimated) before the system start.

xy-tasks Model:
3. Quality function and states



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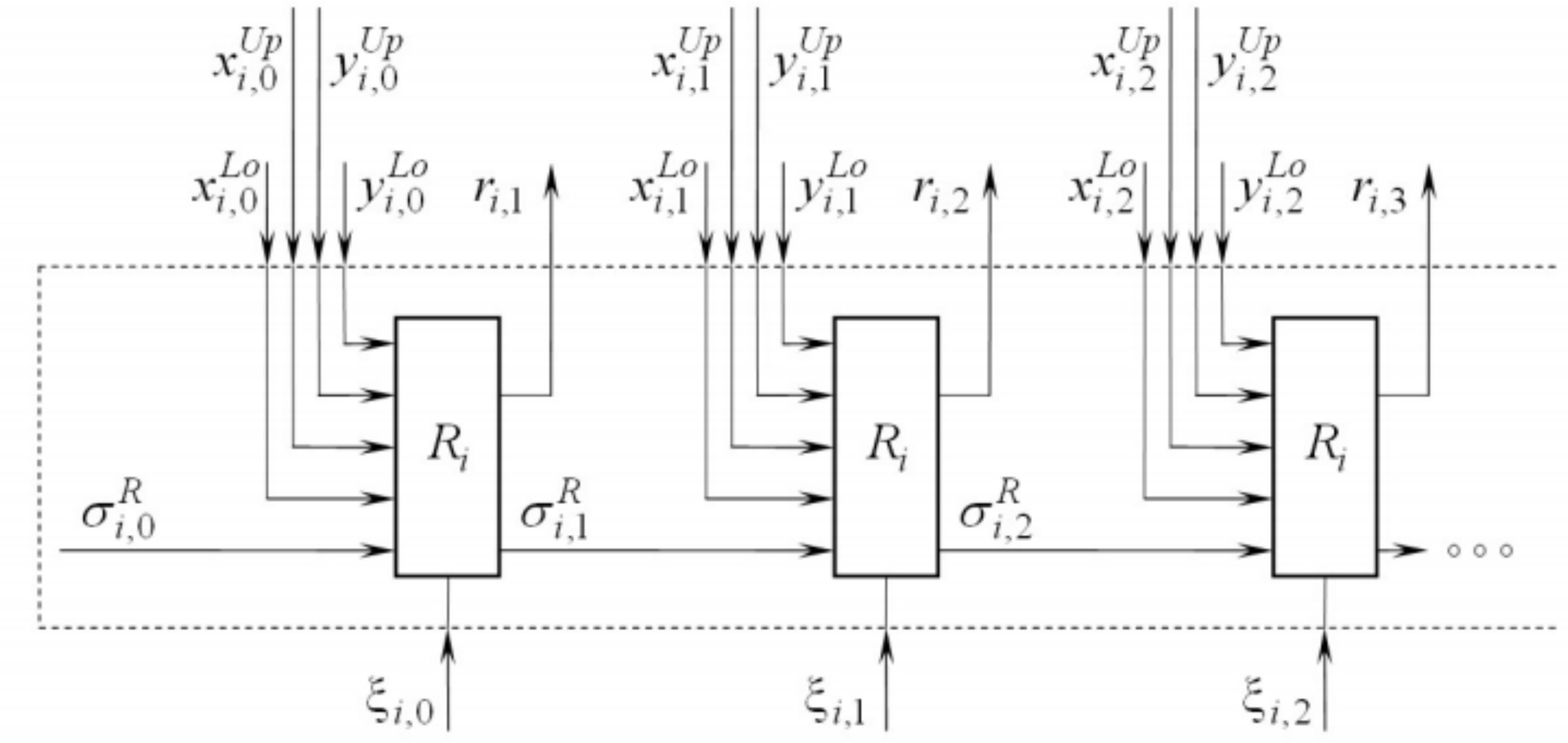
xy-tasks Model:
4. Release algorithm

Release algorithm (*R*) can be seen as a generalization of the periodic release (activation) of a time-triggered task.



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xy-tasks Model:

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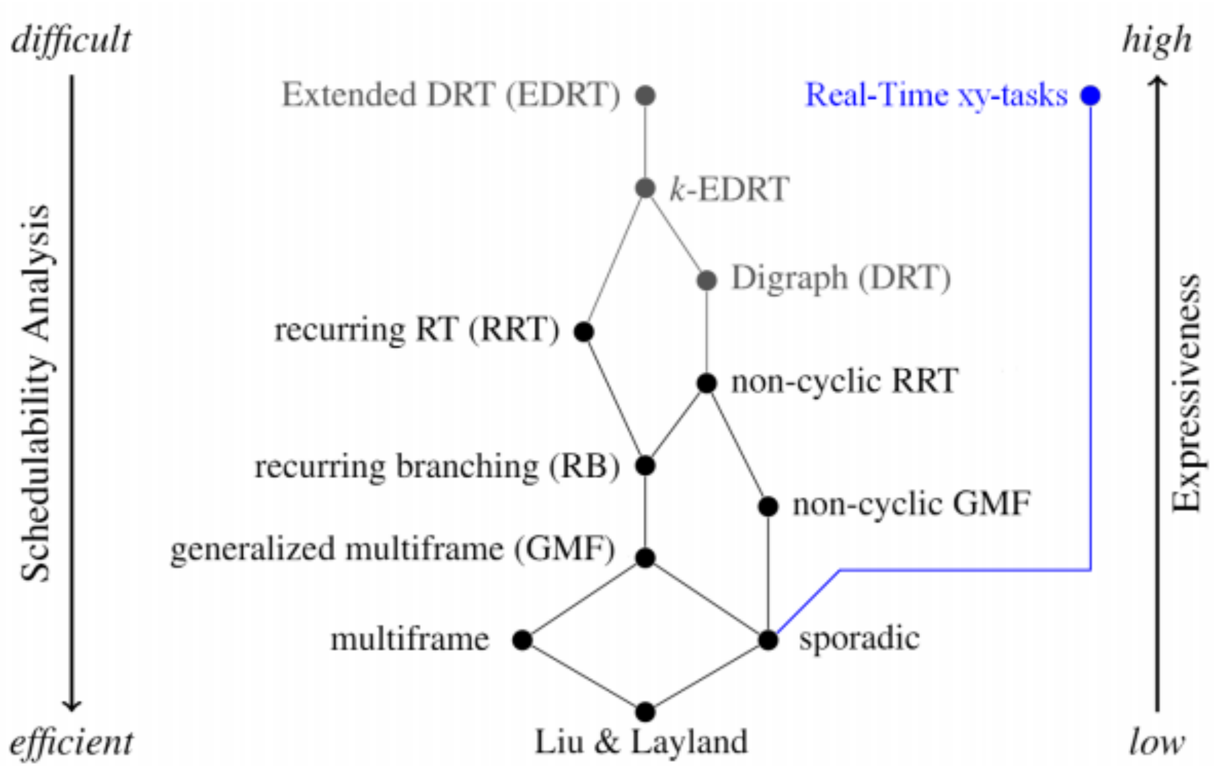

Figure from [Martin Stigge, Real-time workload models: Expressiveness vs. analysis efficiency, 2014] slightly adjusted

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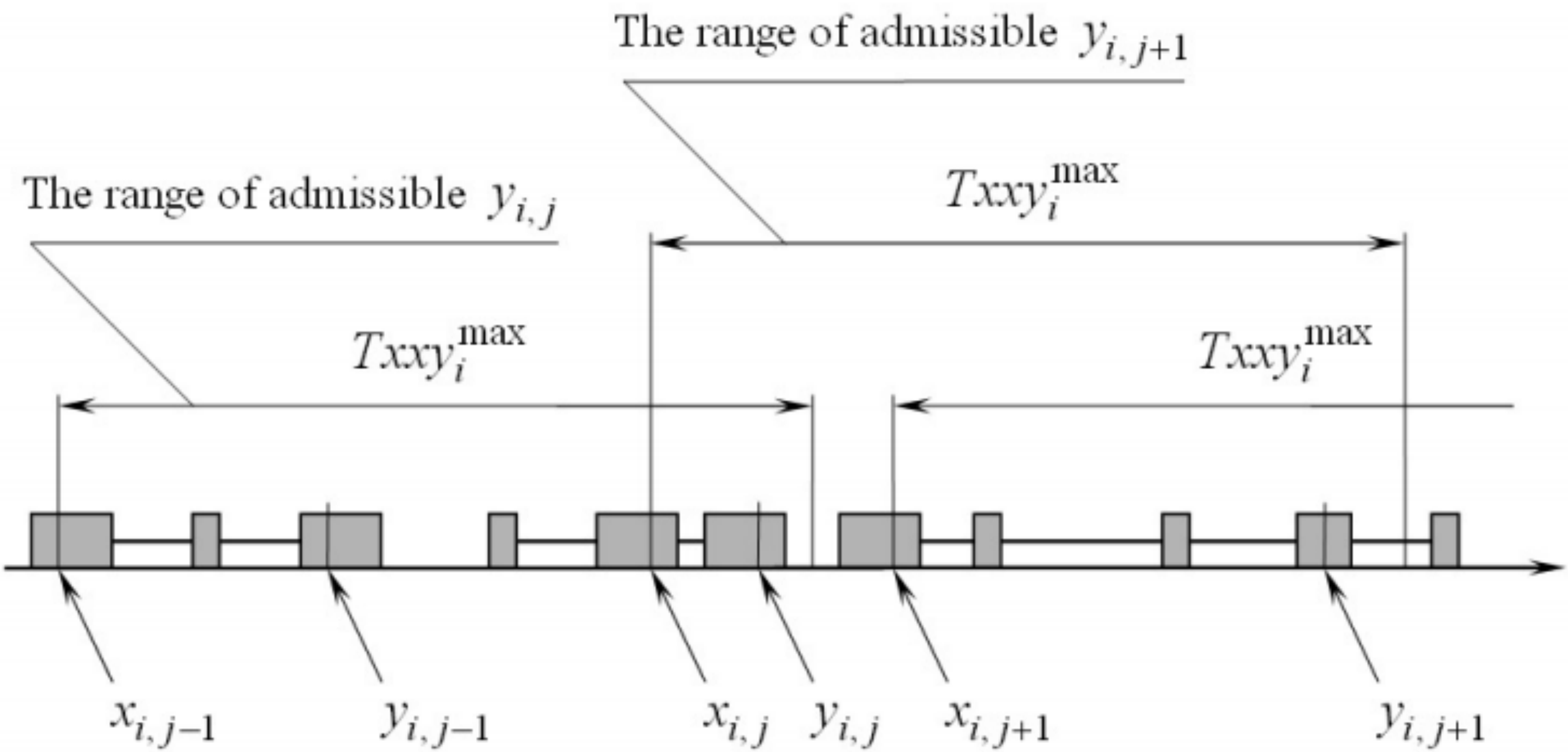
Real-life examples of xy-tasks:

1. Event handling time-triggered task



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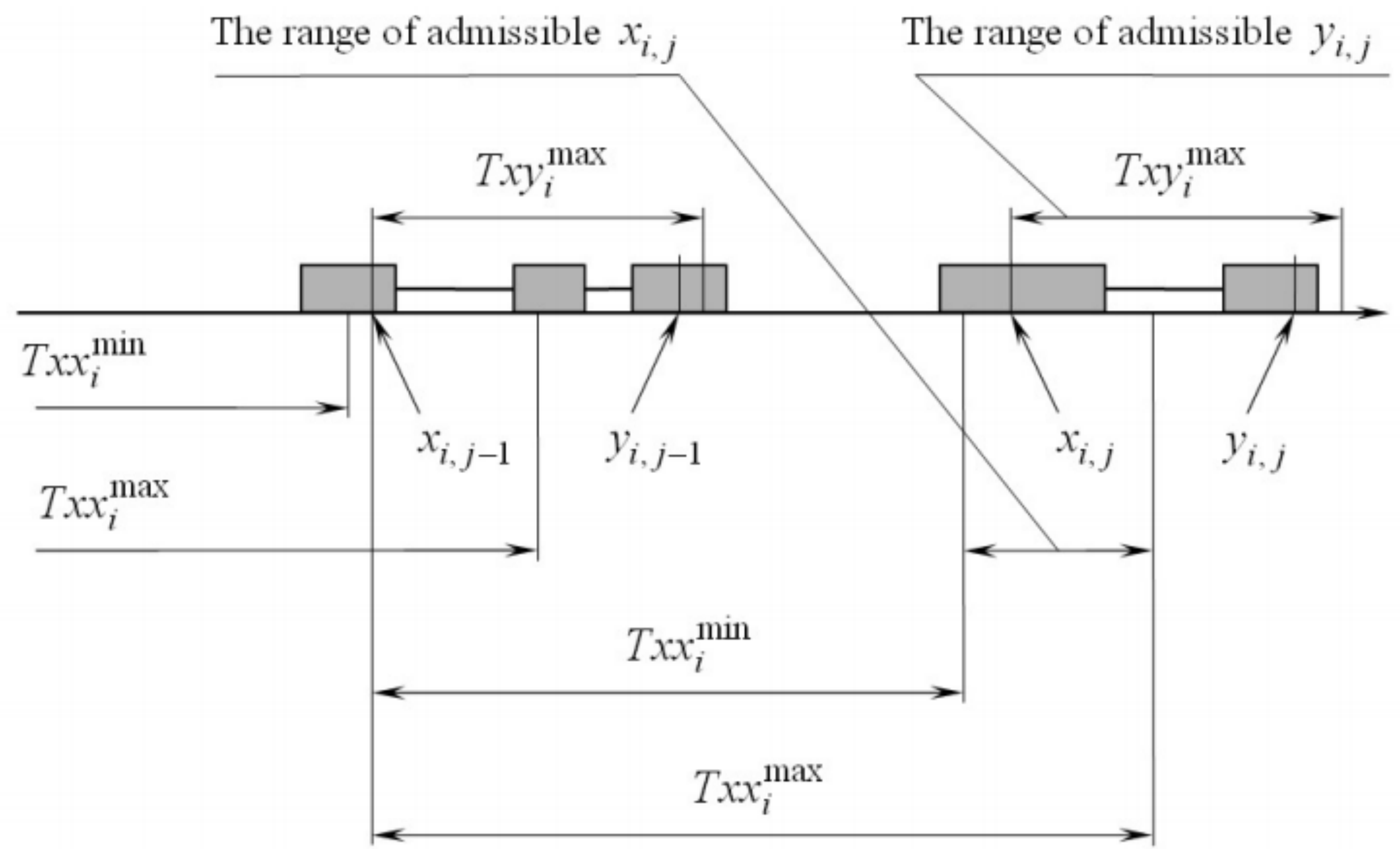
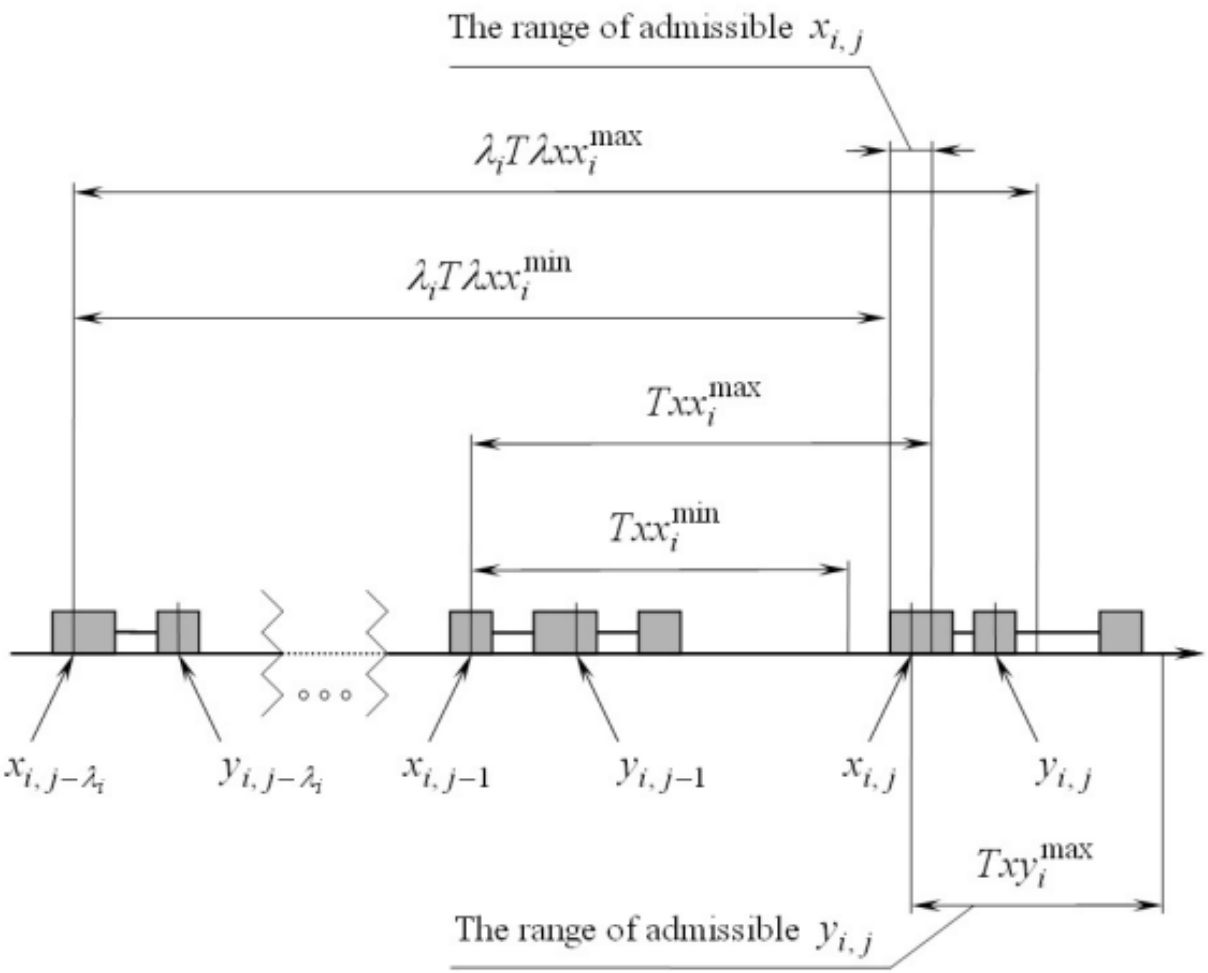
Real-life examples of xy-tasks:

2. Control task with inter-job dependencies



Real-life examples of xy-tasks:

3. Control task with inter-job dependencies and averaged jitters



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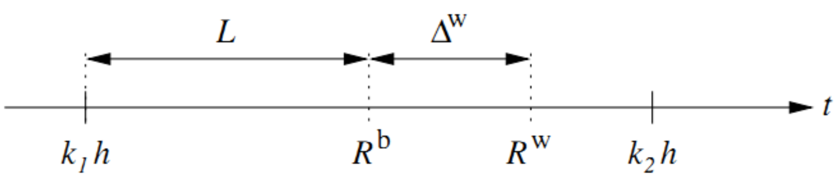
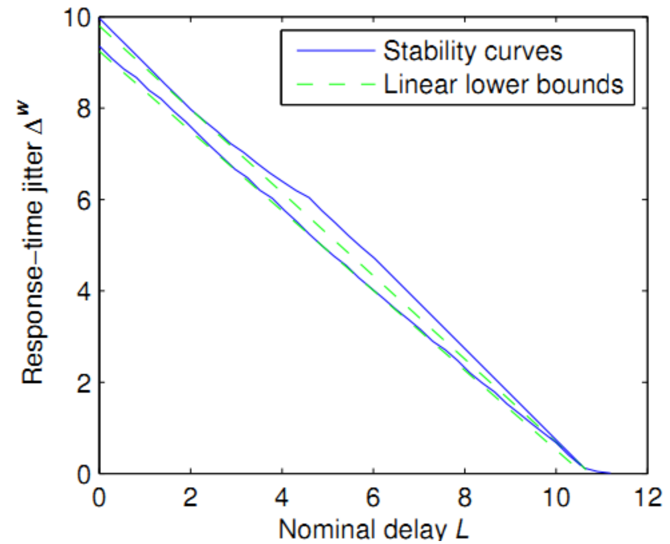
Real-life examples of xy-tasks:

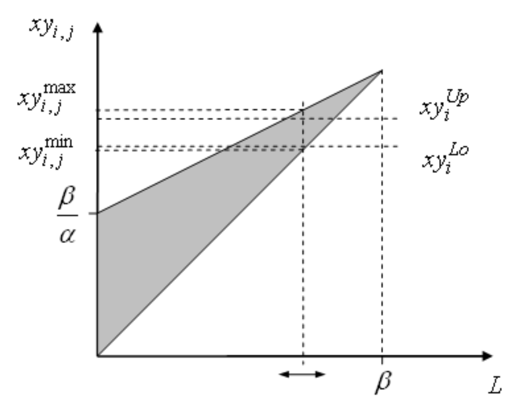
4. Control task with nominal latency and response-time jitter

figures are from [Amir Aminifar, Petru Eles, Zebo Peng, and Anton Cervin, "*Stability-aware analysis and design of embedded control systems*", 2013]





$$
\begin{cases}
x_{i,j} = O_i + (j-1)h \\
xy_{i,j} \geq xy_{i,j}^{\min} = L \\
xy_{i,j} \leq xy_{i,j}^{\max} = (\alpha - 1)L/\alpha + \beta/\alpha \\
L \geq 0 \\
L \leq \beta
\end{cases}
$$



Linear inequalities

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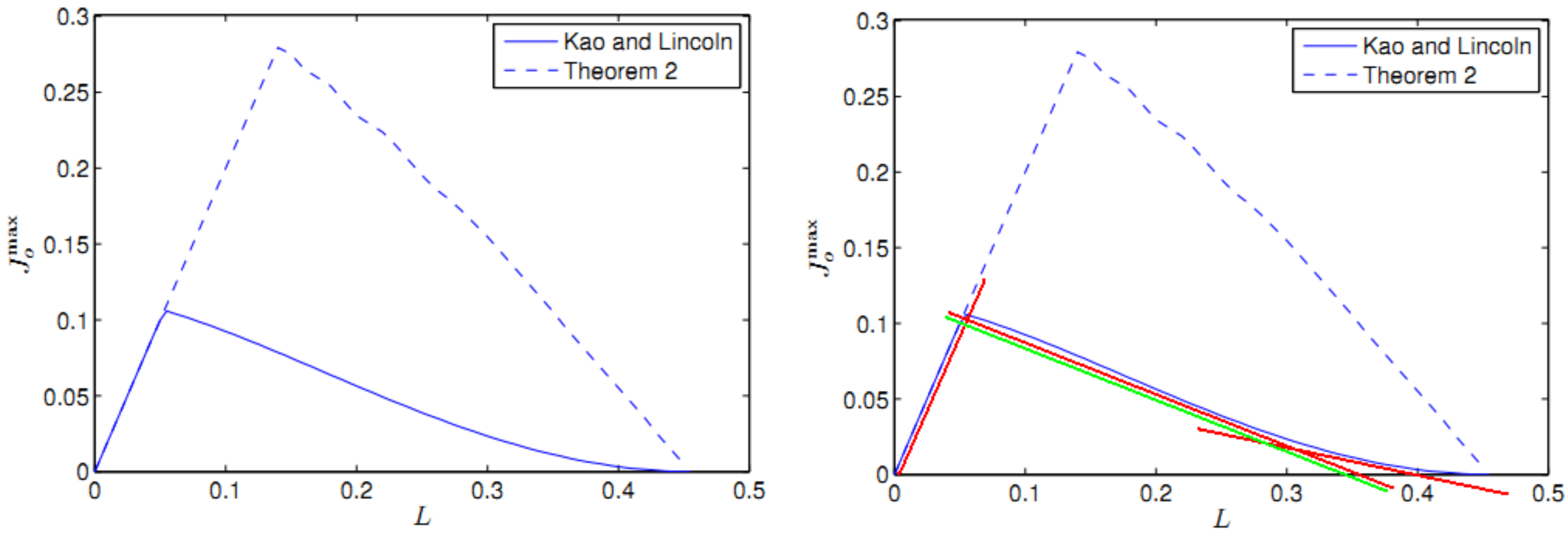
Real-life examples of xy-tasks:

5. Control task that produces more linear inequalities

figure is from [Anton Cervin, “*Stability and worst-case performance analysis of sampled-data control systems with input and output jitter*”, 2012]



But take into account the convexity issue!

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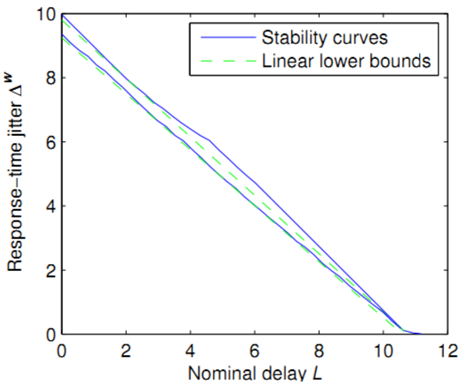
Real-life examples of xy-tasks:

5. Control task that produces quadratic inequalities

figures are from [Amir Aminifar, Petru Eles, Zebo Peng, and Anton Cervin, "*Stability-aware analysis and design of embedded control systems*", 2013]



Linear dependency

$$L + \alpha \cdot \Delta^w \leq \beta$$

but coefficients can be affected by the frequency 1/*h* of the task, e.g., linearly:

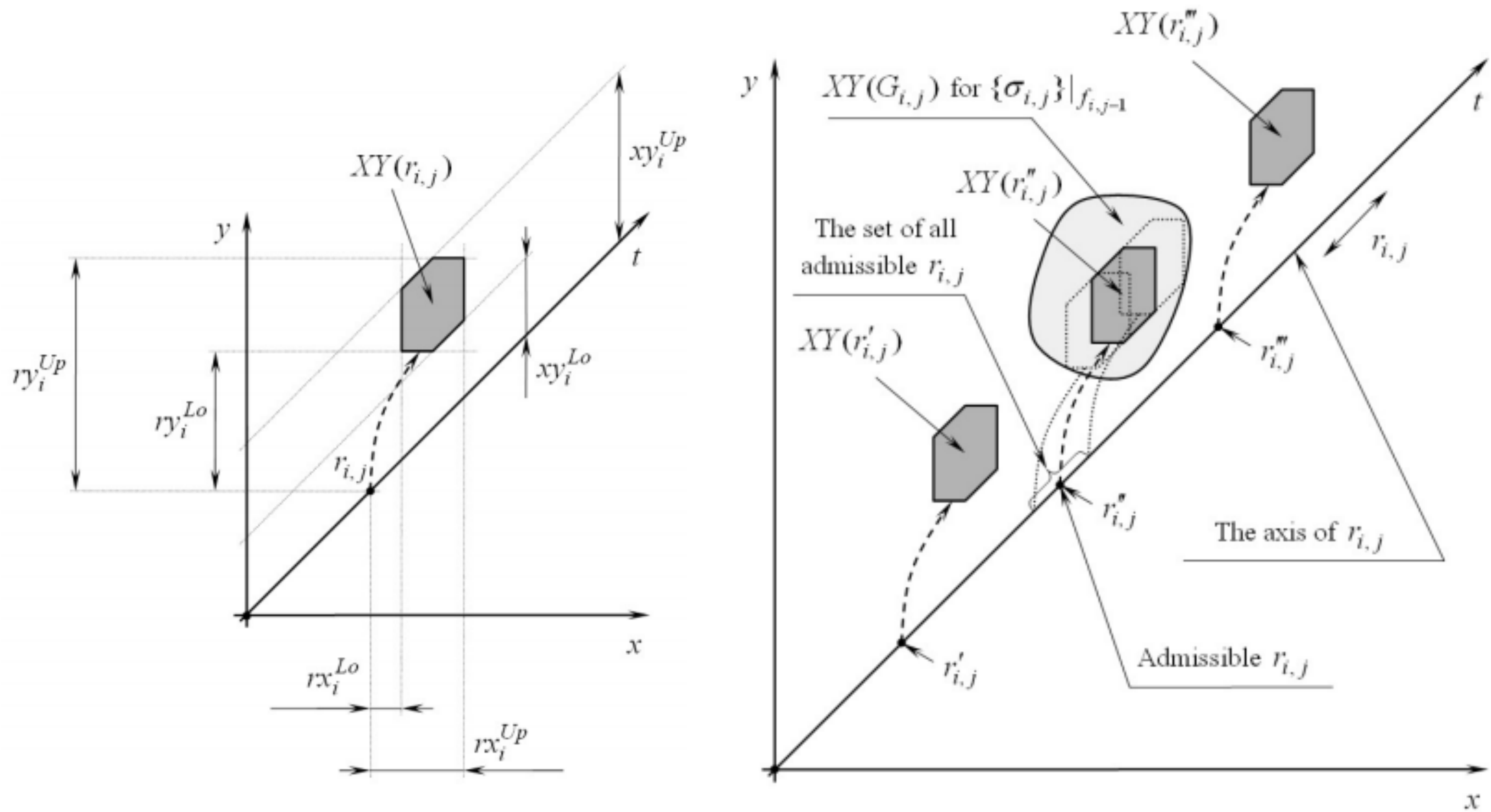
$$\alpha = \alpha_1 / h + \alpha_2$$

$$\beta = \beta_1 / h + \beta_2$$

then we have quadratic inequalities in the timing constraint

$$\begin{cases} x_{i,j} - (j-1) \cdot h = O_i \\ xy_{i,j} - L \geq 0 \\ xy_{i,j} - L - \Delta^w \leq 0 \\ L \geq 0 \\ L \cdot h - \beta_2 \cdot h \leq \beta_1 \\ \Delta^w \geq 0 \\ L \cdot h + \alpha_1 \cdot \Delta^w + \alpha_2 \cdot h \cdot \Delta^w - \beta_2 \cdot h \leq \beta_1 \end{cases}$$

How to schedule fixed priority xy-tasks: admissible release times



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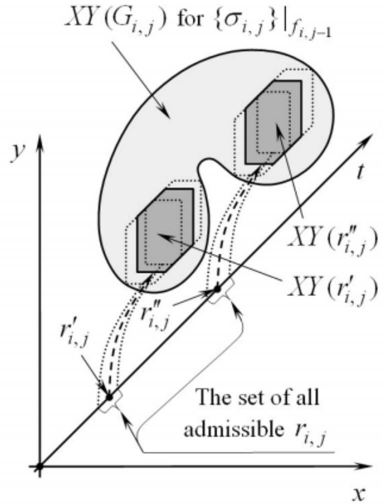
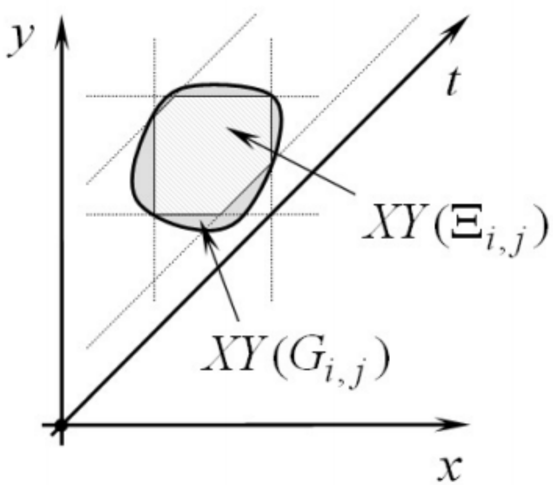
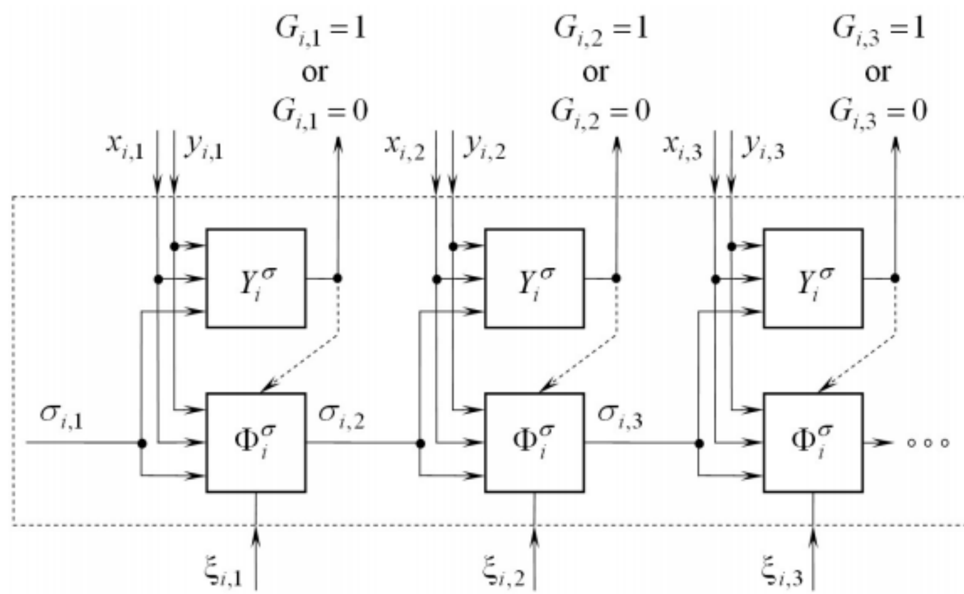
From general form of xy-task timing constraint to more specific, e.g. linear of quadratic







How to schedule fixed priority xy-tasks: schedulability test for linear form of xy-task constraint

Definition 6. A set $\{\sigma_{i,j}\}|_{f_{i,j-1}}$ is called *admissible and reachable* if $\{\sigma_{i,j}\}|_{f_{i,j-1}}$ is produced by a sequence of admissible $r_{i,v}$ for $\forall v \in [1, j-1]$ starting from $\sigma_{i,1}$.

Theorem 1. *For a given Ξ_i, xy-task τ_i is guaranteed to be schedulable with* **any** *interval R_i if and only if the following condition holds:*

$$\begin{cases} rx_i^{Up} - rx_i^{Lo} \le \min\limits_{\alpha}(\min\limits_{\beta}(x_{i,j}^{max} - x_{i,j}^{min})) \\ ry_i^{Up} - rx_i^{Lo} \le \min\limits_{\alpha}(\min\limits_{\beta}(y_{i,j}^{max} - x_{i,j}^{min})) \\ ry_i^{Lo} - rx_i^{Up} \ge \max\limits_{\alpha}(\max\limits_{\beta}(y_{i,j}^{min} - x_{i,j}^{max})) \\ ry_i^{Up} - ry_i^{Lo} \le \min\limits_{\alpha}(\min\limits_{\beta}(y_{i,j}^{max} - y_{i,j}^{min})) \\ rx_i^{Up} \le \min\limits_{\alpha}(\min\limits_{\beta}(x_{i,j}^{max}) - (r_{i,j-1} + T_i^R)) \\ ry_i^{Up} \le \min\limits_{\alpha}(\min\limits_{\beta}(y_{i,j}^{max}) - (r_{i,j-1} + T_i^R)) \\ xy_i^{Lo} \ge \max\limits_{\alpha}(\max\limits_{\beta}(xy_{i,j}^{min})) \\ xy_i^{Up} \le \min\limits_{\alpha}(\min\limits_{\beta}(xy_{i,j}^{max})) \end{cases} \tag{16}$$

where α stands for "all admissible and reachable $\{\sigma_{i,j}\}|_{f_{i,j-1}}$ for $\forall j \ge 1$", and β stands for "$\{\sigma_{i,j}\}|_{f_{i,j-1}}$".

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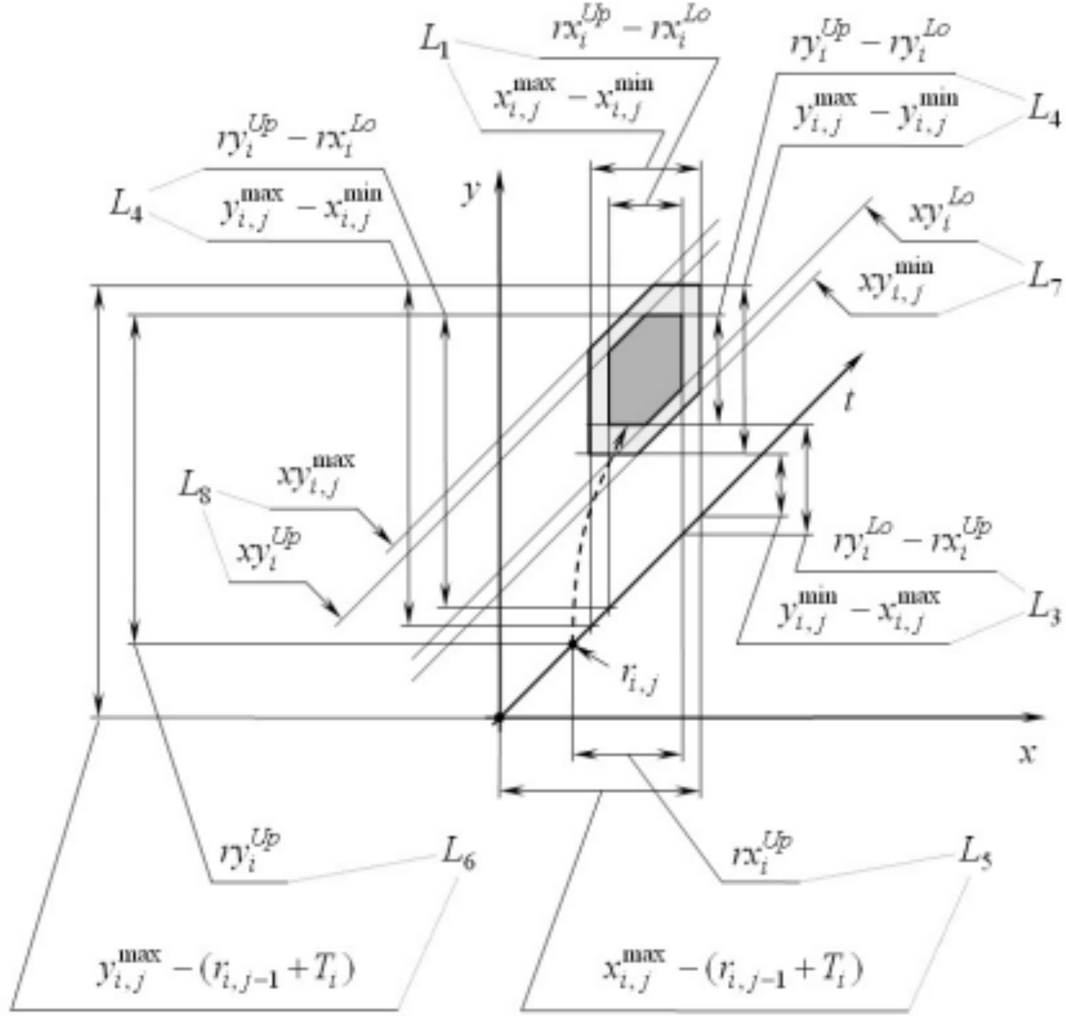
How to schedule fixed priority xy-tasks: schedulability test for linear form of xy-task constraint (graphical interpretation)



How to schedule fixed priority xy-tasks:

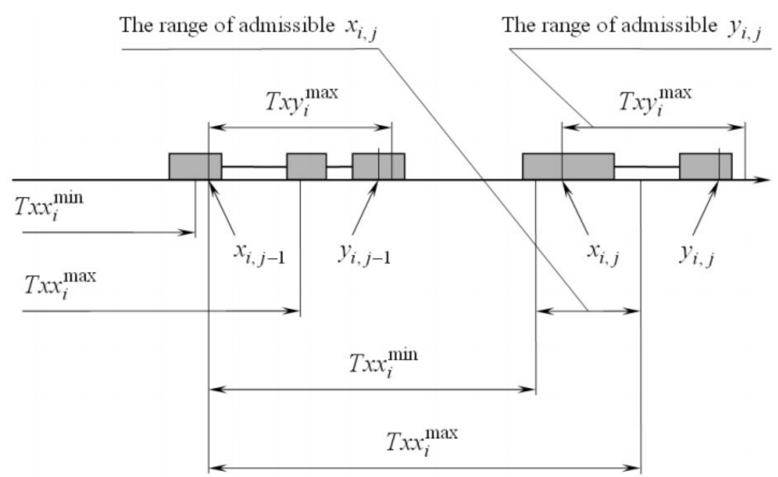
schedulability test for linear form of xy-task constraint (reduces to specific xy-tasks)

Response-time test for deadline task $rf_i^{Up} \leq D_i$

is a special case of the proposed schedulability test

Sch. test for control task with inter-job dependencies:

$$\begin{cases} T_i^R \geq \max\limits_{\{\xi_{i,j}\}}(Txx_i^{\min}(\xi_{i,j})) - rx_{i,j}^{Lo} + rx_{i,j}^{Up} \\ T_i^R \leq \min\limits_{\{\xi_{i,j}\}}(Txx_i^{\max}(\xi_{i,j})) + rx_{i,j}^{Lo} - rx_{i,j}^{Up} \\ xy_i^{Up} \leq \min\limits_{\{\xi_{i,j}\}}(Txy_i^{\max}(\xi_{i,j})) \end{cases}$$



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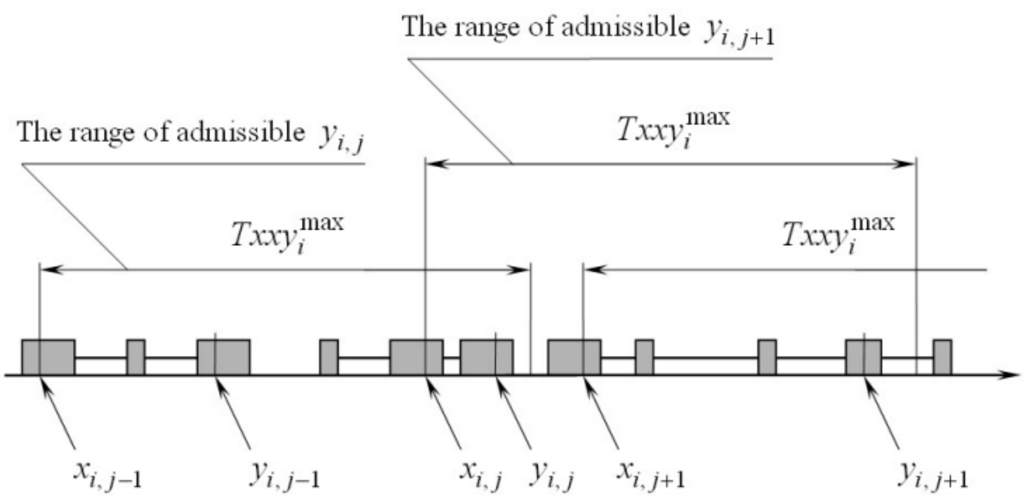
How to schedule fixed priority xy-tasks:

schedulability test for linear form of xy-task constraint (reduces to specific xy-tasks)

Sch. test for event handling time-triggered task:

$$\begin{cases} O_i + ry_i^{Up} \le x_{i,0} + T_i^{xxy\max} \\ T_i + ry_i^{Up} \le rx_i^{Lo} + T_i^{xxy\max} \end{cases}$$



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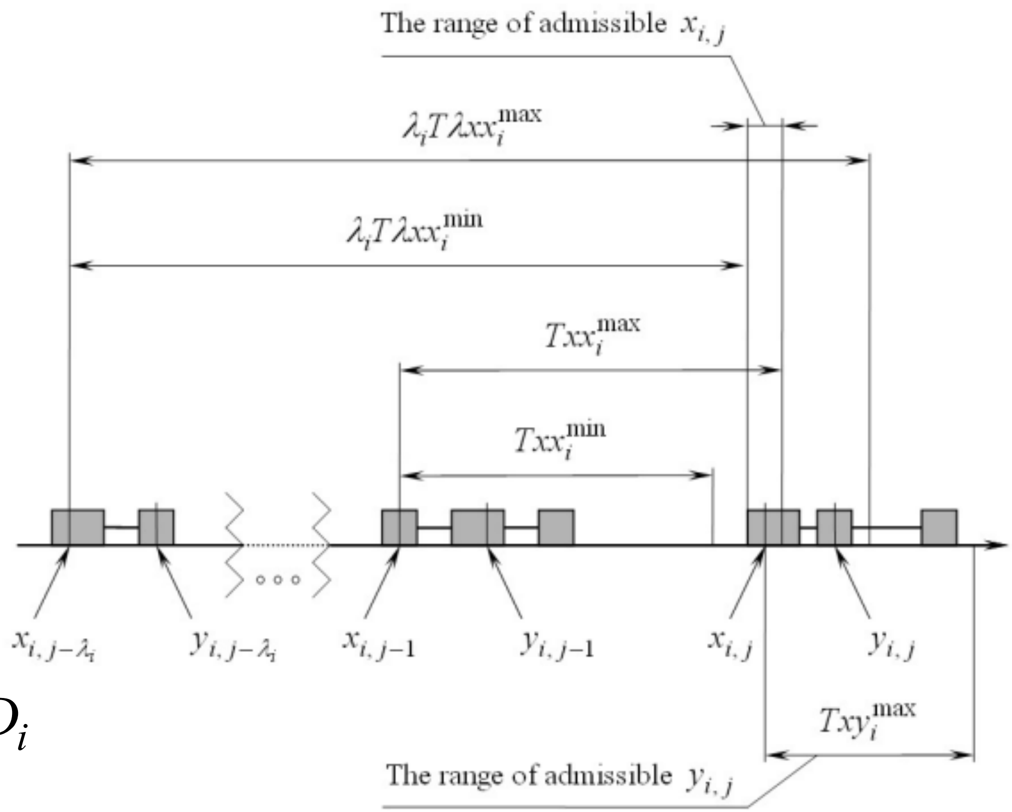
How to schedule fixed priority xy-tasks: schedulability test for linear form of xy-task constraint (reduces to specific xy-tasks)

Sch. test for with inter-job dependencies and averaged jitters:

$$\begin{cases} T_i + rx_i^{Lo} \geq rx_i^{Up} + T_i^{xx\,\min} \\ \lambda_i T_i + rx_i^{Lo} \geq rx_i^{Up} + \lambda_i T_i^{mxx\,\min} \\ T_i + rx_i^{Up} \leq rx_i^{Lo} + T_i^{xx\,\max} \\ \lambda_i T_i + rx_i^{Up} \leq rx_i^{Lo} + \lambda_i T_i^{mxx\,\max} \\ xy_i^{Up} \leq T_i^{xy\,\max} \end{cases}$$



Compare this with response-time test for standard deadline task $rf_i^{Up} \leq D_i$ or $ry_i^{Up} \leq D_i$ when $y_i = f_i$

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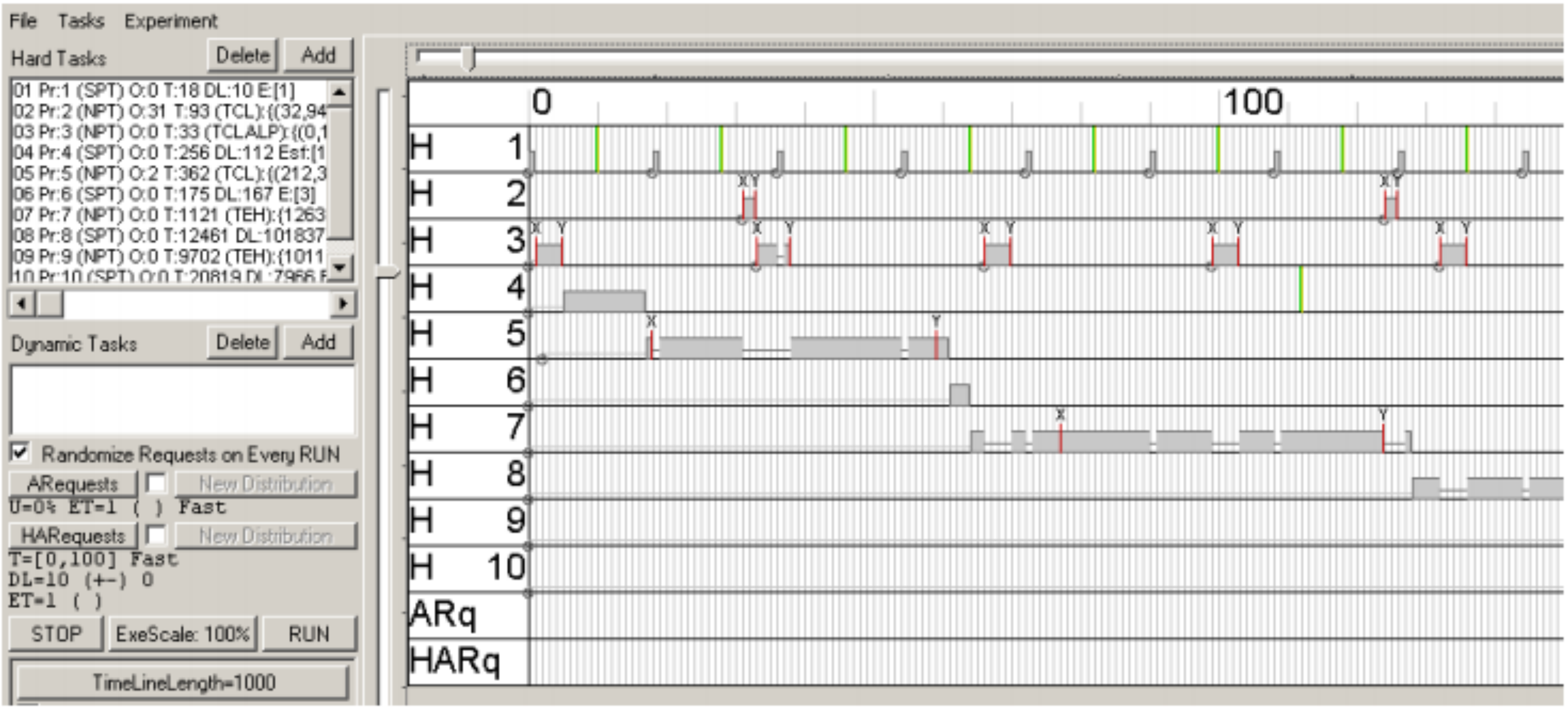
For periodic release pattern, period and priority assignment algorithms have been proposed

1. (Test Assignment) Modified optimal priority assignment algorithm [Audsley, 1991]. Instead of response-time feasibility test for derived period-deadline task, the proposed schedulability test is used for original xy-task.
2. (Maximal period assignment) First, priorities are assigned according to deadline-monotonic policy for derived period-deadline task (heuristic approach). Then period of each task is maximized being subject to the proposed schedulability test, while taking into account interference of high priority tasks with periods already assigned.

Experimental evaluation



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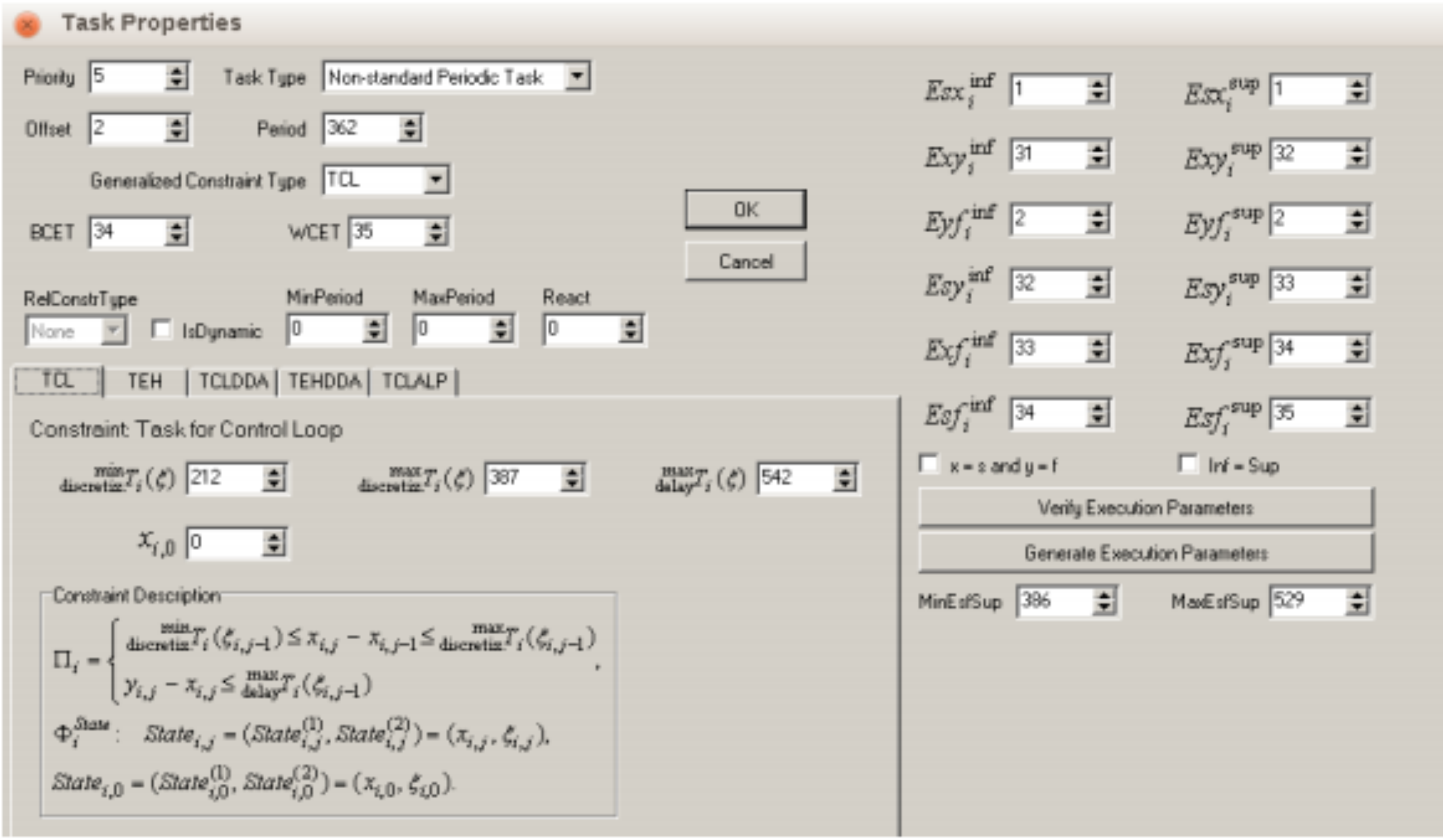
Experimental evaluation



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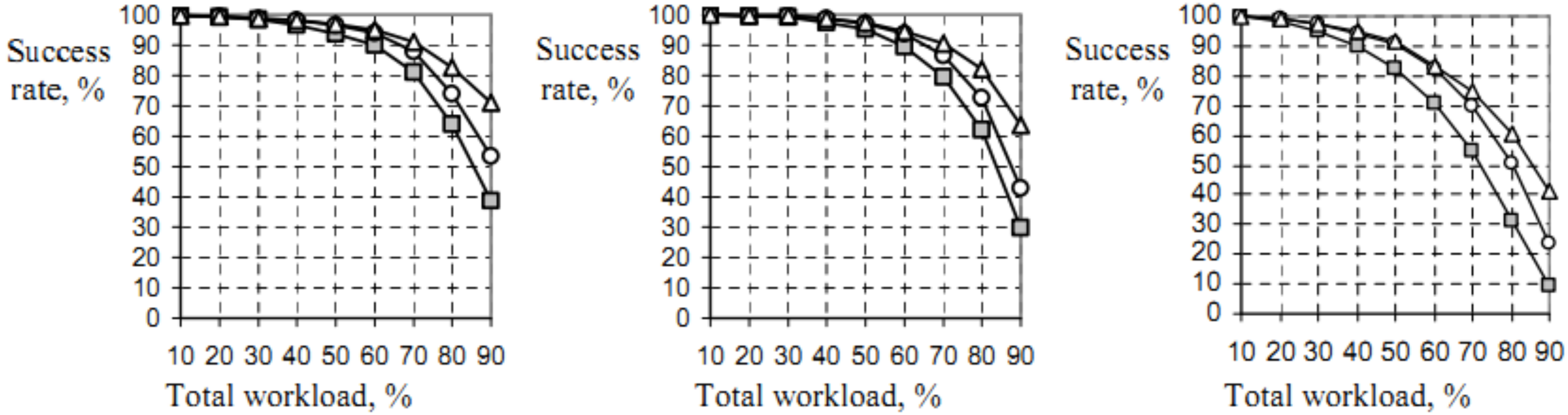
Experimental evaluation



7 TPDConstr; 1 CTIJD; 1 EHT; 1 CTIJDAJ
j)

4 TPDConstr; 2 CTIJD; 2 EHT; 2 CTIJDAJ
k)

1 TPDConstr; 3 CTIJD; 3 EHT; 3 CTIJDAJ
l)

Baseline Assignment — Test Assignment — Maximal Period Assignment

Future research

1. How to find the optimal (or at least, suboptimal) on-line release algorithm R_i or static release pattern for each xy-task under fixed priority policy? Notice that priority assignment is also the problem to solve in this case.
2. How to efficiently schedule the mixture of soft and hard xy-tasks? Notice that R_i can be aware about soft xy-jobs awaiting in the queue.
3. Other approximations of generalized timing constraint of xy-task should be investigated to efficiently resolve the trade-off between the expressiveness of the constraint and the complexity of scheduling, taking into account the practical value of such approximation.